VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

DEPARTMENT OF BCA

SUBJECT - COMPUTER MATHEMATICS CHAPTER 3- MATRICES

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LEARNING OBJECTIVES:-

- Cost estimation, Sale projection and factory problems can be solved by using matrix.
- Expressing in vector form
- Expressing day to day life problems in matrix form
- Matrix notation and operations are used in electronic spreadsheet, advanced statistics.
- Expressing simultaneous linear equations in matrix form.

Defination of matrix:-

- A matrix is an ordered rectangular array of numbers that represent some data (Plural = matrices)
- A matrix on its own has no value it is just a representation of data
- Could be data associated with manufactured quantity in a factory, speed of a rocket etc
- Forms the basis of computer programming
- A matrix is used in solving equations that represent business problems

Types of matrix :-

- Row matrix: it having only one row Ex [-1 2 1]
- Column matrix: it having only one column Ex
- **Zero matrix:** A matrix is called a zero matrix if all the entries are 0 Ex

 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

- Square matrix: if number of rows is equal to number of columns
- Note: if number of rows = no of number columns =n, is called square matrix of order n or order n

 $\begin{bmatrix} 5 & 7 \\ -2 & 5 \end{bmatrix}$

 $\begin{bmatrix} 5 & 7 \\ -2 & 5 \end{bmatrix}$ order 3 $\begin{bmatrix} 5 & 7 & 8 \\ 6 & 4 & 8 \\ 1 & 7 & 0 \end{bmatrix}$

Types of matrix :-

- Diagonal matrix: A square matrix is called diagonal matrix, if all of its nondiagonal elements are zero.
- ► EXAMPLE

- ro. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- Scalar matrix: A square matrix is called scalar matrix if diagonal elements are same and other are "0"
- $\begin{array}{c} \text{EXAMPLE} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$
- Identity/ unit matrix : A square matrix is identity if diagonal entries are 1 and other are 0.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

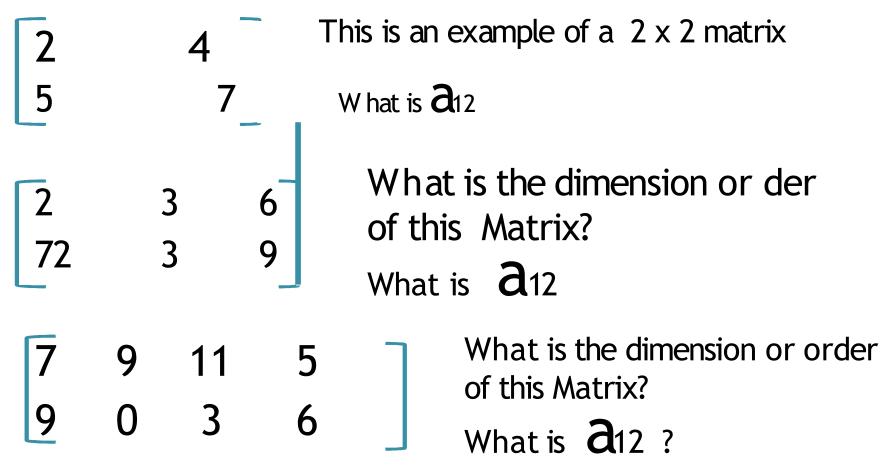
REPRESENTATION OF MATRIX



$\partial ij =$ element in row 'i' and column 'j', where 'a' is an element in the matrix

Eg: **a** 23 = element in 2^{nd} row and 3^{rd} column = 9

Examples of Matrices



Addition /subtraction: when two matrices of same order are added/ subtracted, their corresponding entries are added/subtracted **Addition operation on Matrices**

 2
 45
 72
 40
 7
 9

 6
 3
 0
 6
 1
 2

 7
 9
 10
 7
 2
 8

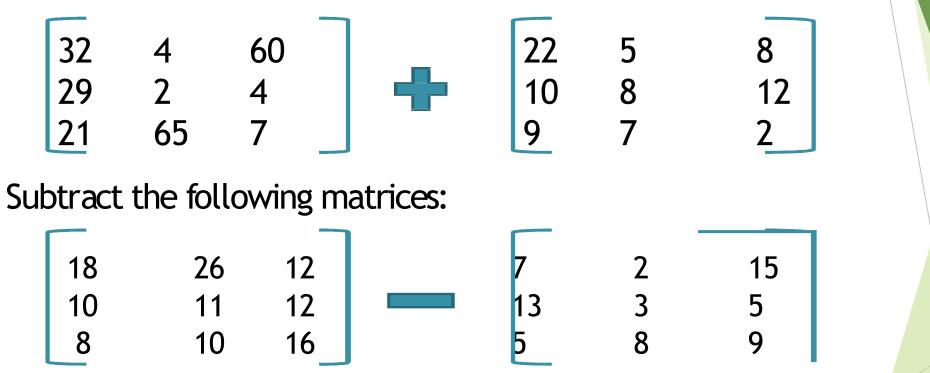
 B Α $= \begin{bmatrix} (2+40) & (45+7) & (72+9) \\ (6+6) & (3+1) & (0+2) \\ (7+7) & (9+2) & (10+8) \end{bmatrix} = \begin{bmatrix} 47 & 52 & 81 \\ 12 & 4 & 2 \\ 14 & 11 & 18 \end{bmatrix}$

> Only Matrices of the same order(comparable) can be added!! Rule 1: A + B = B + A

Question Set 1

2.

1. Add the following matrices:



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Multiplication of a matrix by a scalar

If K is any number and A is a given matrix, Then KA is the matrix obtained by multiplying each element of A by K. K is called 'Scalar'. Eg: if K = 22 4 5 4 8 2 KA = 2 6 A =5 10

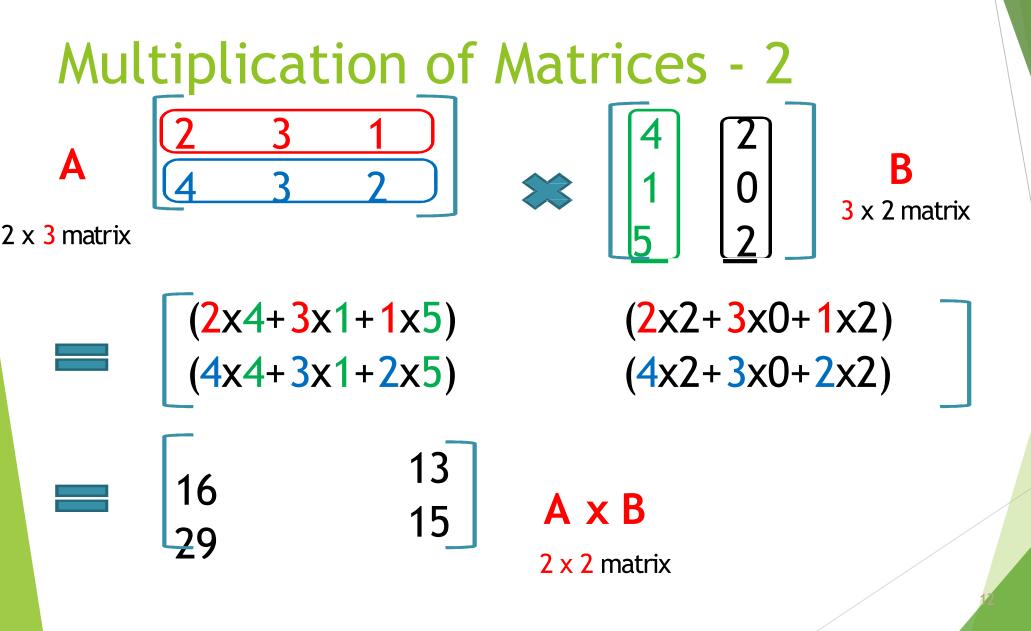
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MULTIPLICATION OF MATRICES

- The product AB of two matrices A and B is defined, if the number of columns of A is equal to the number of B.
- If AB is defined then BA need not be defined. In particular both A and B are square matrices of same order then AB and BA are defined.
- ► In general AB≠BA
- Observation : Two non zero matrices multiplication is zero matrix

Ex:
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2} \times \begin{bmatrix} -1 & 2 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}_{3 \times 2}$$
 here multiplication is not possible.
Ex: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2} \begin{bmatrix} 2 \\ 1 \end{bmatrix}_{2 \times 1}$ here multiplication possible, order of new matrix is 2×1 .

If order of first matrix A is $m \times n$ and that of B is $n \times p$, then A.B is possible of order $m \times p$.



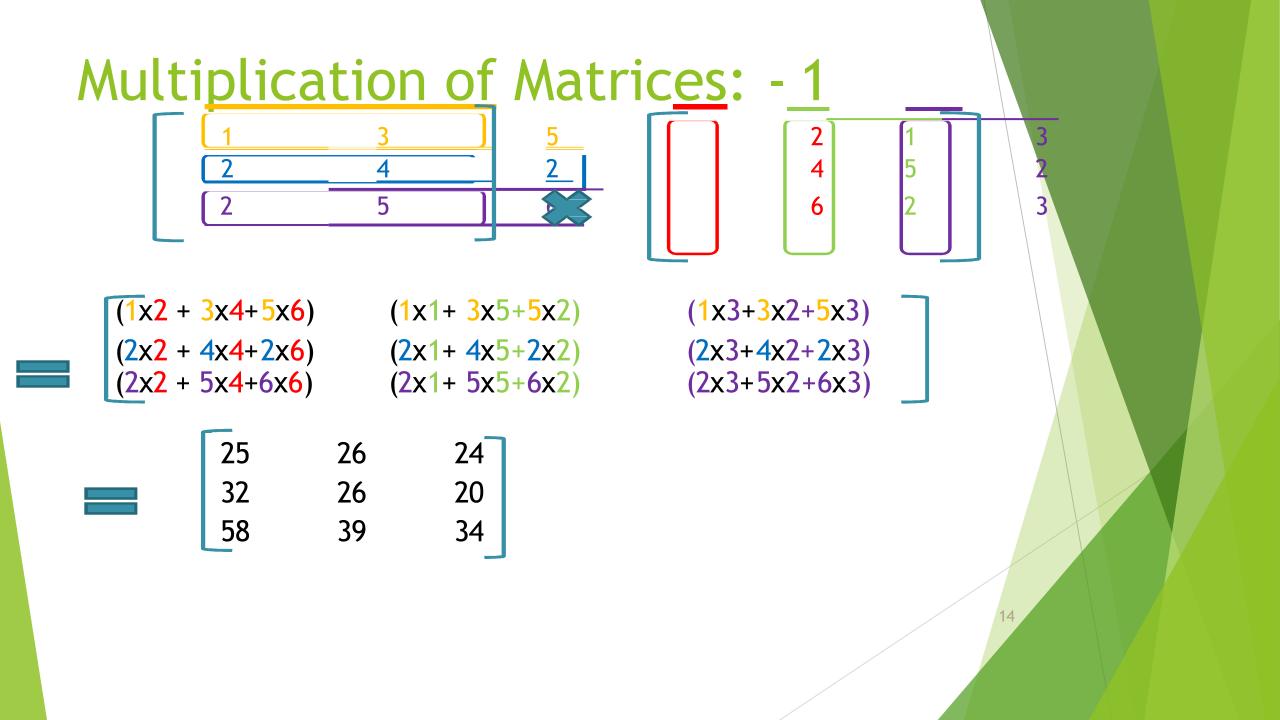
MLTIPLICATION OF MATRIX

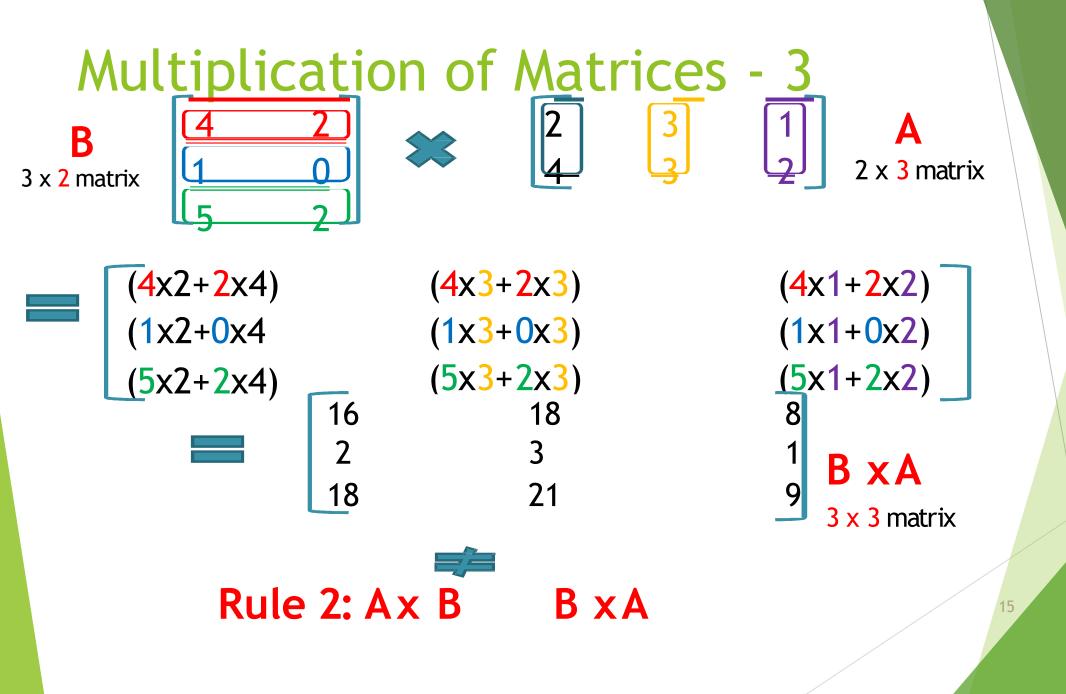
Procedure for multiplication:

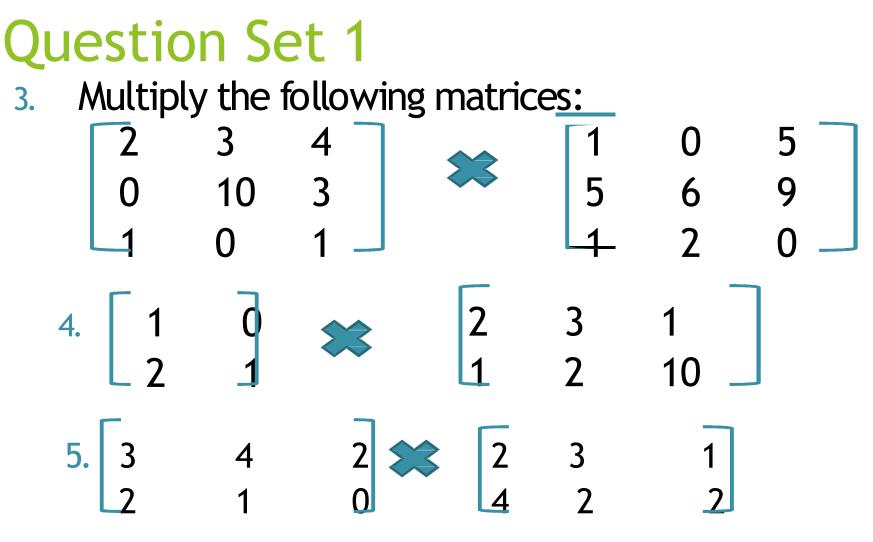
 $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} =$ $\begin{bmatrix} a_{11} \cdot b_{11} + a_{12} b_{21} + a_{13} b_{31} & a_{11} \cdot b_{12} + a_{12} b_{22} + a_{13} b_{32} & a_{11} \cdot b_{13} + a_{12} b_{23} + a_{13} b_{33} \\ a_{21} \cdot b_{11} + a_{22} b_{21} + a_{23} b_{31} & a_{21} \cdot b_{12} + a_{22} b_{22} + a_{23} b_{32} & a_{21} \cdot b_{13} + a_{22} b_{23} + a_{23} b_{33} \\ a_{31} \cdot b_{11} + a_{32} b_{21} + a_{33} b_{31} & a_{31} \cdot b_{12} + a_{32} b_{22} + a_{33} b_{32} & a_{31} \cdot b_{13} + a_{32} b_{23} + a_{33} b_{33} \end{bmatrix}$

EX: $\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1.4 + 2. -1 & 1.0 + 2.2 \\ -1.4 + 0. -1 & -1.0 + 0.2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -4 & 0 \end{bmatrix}$

EX: $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1.1 + 2.3 \\ 1.1 + 1.3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$







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Is it possible to compute No.5?! No!W hy?

Transpose of a Matrix

• Matrix formed by interchanging rows and $\lim_{n \to \infty} A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$ columns of A is called A transpose (A') Q. Verify (A + B)' = A' + B'

If
$$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$
 then $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ If $A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 6 & 5 \\ -1 & 5 & 9 \end{bmatrix}$

Q. Verify (A B)' = B' xA'

Symmetric matrix: A square matrix A is called symmetric if $A^T = A$

Remark: In a symmetric matrix, the entries opposite to diagonal entries are same. $a_{ij} = a_{ji}$

EX:
$$\mathbf{i}$$
 $\begin{bmatrix} \mathbf{1} & -1 \\ -1 & \mathbf{3} \end{bmatrix}$ \mathbf{ii} $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 5 \\ 3 & 5 & 4 \end{bmatrix}$

Skew symmetric matrix : A square matrix A is called skew symmetric if A^T =-A or $a_{ij} = -a_{ji}$. In skew symmetric matrix diagonal elements are "0" and entries opposite to main diagonal are same but opposite sign.

EX:
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$
 ii} $\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -5 \\ -3 & 5 & 0 \end{bmatrix}$

Question Set 1 Find the transpose of the following matrices and verify that (A+B)' = A'+B'6. 2 9

Hint: FindA+B, (A+B)', A' and B' and verify

If D is a matrix where first row = number of table fans 7. and second row = number of ceiling fans factories A and B make in one day If a week has 5 working days compute 5A. What does 5Å represent?

B=

3 8

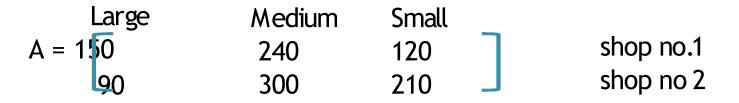
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20 40

A =

Question Set 1

9. Two shops have the stock of large, medium and small sizes of a toothpaste. The number of each size stocked is given by the matrix A where



The cost matrix B of the different size of the toothpaste is given by

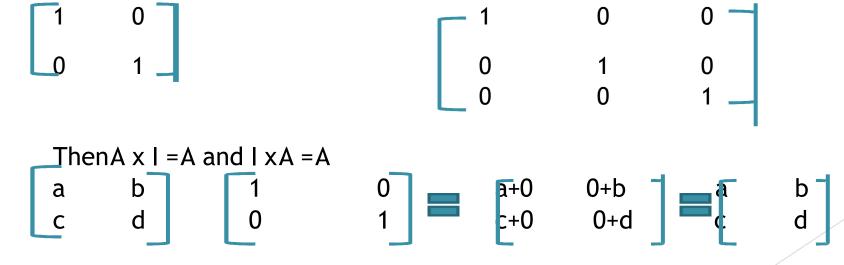


Question Set 1

For the matrix 8. 6-and B = A = 2_ -5 Multiply by the Matrix I = 1 _0 What is A.I and I.B?

Identity Matrix

- If you were to multiply 'a' by '1', you would get 'a'.
 Eg: 2 x 1 = 2x1 = 2
- The 'identity' matrix (i) is the equivalent of '1' in basic math If A is a matrix and I is an identity Matrix,
- Then $A \times I = A$ and $I \times A = A$. Identity Matrices



To find inverse by using elementary Row transformations.

- Step 1: Write A = IA
 Step 2: Apply yarious row
- Step 2: Apply various row operations on left hand side and apply same operations to I on right side but not to A on right side.
- Step3: From step 2 we get a new matrix equation I = BA . Hence B = A⁻¹

To find inverse by using elementary Column transformations.

- Step 1: Write A = AI
 Step 2: Apply various Column operations on left hand side and apply same operations to I on right side but not to A on right side.
- Step3: From step 2 we get a new matrix equation I = AB. Hence B = A⁻¹

INVERSE OF ORDER 2 MATRIX

Exmples:

Using elementary row transformation find the inverse $of \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$.

<u>Ans</u>: Given A = $\begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$ Consider A = IA $\Rightarrow \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ Applying $R_1 \rightarrow R_1 - R_2$ $\begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$ Applying $R_2 \rightarrow R_2$ - 5R₁ $\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -5 & 6 \end{bmatrix} \mathbf{A}$ Applying $R_1 \rightarrow R_1 + R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ -5 & 6 \end{bmatrix} \mathbf{A}$ Applying $R_2 \rightarrow (-1)R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix} \mathbf{A}$ Hence $A^{-1} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix}$

INVERSE OF ORDER 3 MATRIX

. Using elementary row transformation find the inverse of

$$\begin{bmatrix} 2 & 0 & 1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

0

F2

-11

Ans :Given A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ Consider A = IA $\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ Applying $R_1 \rightarrow 3R_1$ $\begin{bmatrix} 6 & 0 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ Applying $R_1 \rightarrow R_1 - R_2$ $\begin{bmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ Applying $R_1 \rightarrow R_1 + R_3$

 $\begin{vmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 3 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ Applying $R_2 \rightarrow R_2$ - 5 R_1 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 0 & 0 & 1 \end{bmatrix} A$ Applying $R_3 \rightarrow R_3 - R_2$ $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 15 & -6 & 6 \end{vmatrix}$ Applying $R_3 \rightarrow \frac{1}{2}R_3$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A \cdot \underbrace{\text{Hence } A^{-1}}_{5} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Inverse of a Matrix

In basic math: 2 2 \Rightarrow 1 and 1/2 x 2 = 1.

Dividing 2 by two is the same as multiplying 2 by 1/2 . The net result is 1.

A similar concept is the 'inverse' of a matrix. If A is a matrix, then A_{i} is the inverse such that $A \times A = I$ (identity matrix)

If A has an inverse (A) then A is said to be 'invertible' A.A=A.A=I

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The determinant is a scalar value that can be computed from the elements of a square matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \text{ then determinant of A or detA or } |A|$$

 $|A| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22}) \neq 0$ then inverse of matrix exist i.e A^{-1}

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 5 \\ 3 & 5 & 4 \end{bmatrix} = 1(-1.4 - 5.5) - 2(2.4 - 3.5) + 3(2.5 - 3. -1) = 11 \neq 0$$

here inverse of A exist. So $A^{-1} = \frac{adjA}{|A|}$, $|A| \neq 0$

$$adjA = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -29 & 7 & 13 \\ 7 & -5 & 1 \\ 13 & 5 & -5 \end{bmatrix}$$
$$A_{11} = -29 A_{12} = 7 A_{13} = 7 A_{13} = 13 A_{21} = 7 A_{22} = -5 A_{23} = 5 A_{31} = 13$$
$$A_{32} = 1 A_{33} = -5$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{11} \begin{bmatrix} -29 & 7 & 13 \\ 7 & -5 & 5 \\ 13 & 1 & -5 \end{bmatrix} = \begin{bmatrix} \frac{-29}{11} & \frac{7}{11} & \frac{13}{11} \\ \frac{7}{11} & \frac{-5}{11} & \frac{1}{11} \\ \frac{13}{11} & \frac{5}{11} & \frac{-5}{11} \end{bmatrix}$$

If the simultaneous equation are of the form

 $a_{11}x + a_{12}y + a_{13}z = b_1$ it can also be represented in matrix form AX=B

$$a_{21}x + a_{22}y + a_{23}z = b_2 \quad \text{where } \mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

 $a_{31}x + a_{32}y + a_{33}z = b_3$ so X= A^{-1} B = $\frac{adjA}{|A|}$ B

EX : Solve by matrix method x+y+z=6, 2x-y+5z=6, 3x+5y+4z=12

Ans Here
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 5 \\ 3 & 5 & 4 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 6 \\ 6 \\ 12 \end{bmatrix}$$

 $|A| = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 5 \\ 3 & 5 & 4 \end{bmatrix} = 1(-1.4 - 5.5) - 2(2.4 - 3.5) + 3(2.5 - 3. -1) = 11 \neq 0$

So inverse <u>exist</u>, Previous example it is found that

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{11} \begin{bmatrix} -29 & 7 & 13 \\ 7 & -5 & 5 \\ 13 & 1 & -5 \end{bmatrix} = \begin{bmatrix} \frac{-29}{11} & \frac{7}{11} & \frac{13}{11} \\ \frac{7}{11} & \frac{-5}{11} & \frac{11}{11} \\ \frac{13}{11} & \frac{5}{11} & \frac{-5}{11} \\ \frac{13}{11} & \frac{5}{11} & \frac{-5}{11} \\ \frac{7}{11} & \frac{-5}{11} & \frac{13}{11} \\ \frac{7}{11} & \frac{-5}{11} & \frac{11}{11} \\ \frac{13}{11} & \frac{5}{11} & \frac{-5}{11} \\ \end{bmatrix} \begin{bmatrix} 6 \\ 6 \\ 12 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

So x=1, y=1 and z=1Singular matrix : A square matrix is said to be singular if det(A)=0 Ex : Let $A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$ here det(A)=4-4=0, so given matrix is singular. Non singular Matrix : A square matrix is said to be non singular if det(A) $\neq 0$,

Note : Inverse of a square matrix exist in non singular matrix.

KEY POINTS

- A matrix is an ordered rectangular array of numbers or functions.
- A matrix having m rows and n columns is called a matrix of order mxn
- A is a diagonal matrix if its non diagonal elements are zero.
- A is a identity matrix if diagonal elements are 1 and non diagonal elements are 0
- A is zero matrix if all elements are zero.
- Matrix addition is commutative ,associative over same order. A+B= B+A, (A+B)+C=A+(B+C)

KEY POINTS

- k (A+B)= kA+kB , k is constant A,B are of same order
- ▶ If order of first matrix A is m and that of B is n, then A.B is possible of order m
- Matrix multiplication is not commutative.
- A matrix is symmetric if $a_{ij} = a_{ji}$
- ▶ If a square matrix is invertible if detA \neq 0, or singular matrix
- if detA =0, inverse of a square matrix doesn't exist.or non singular matrix

 $(kA)^{T} = kA^{T}$ $(A \pm B)^{T} = A^{T} \pm B^{T}$ $(A^{-1})^{T} = (A^{T})^{-1}$ $(A^{T})^{T} = A$

CONCEPT MAPPING

Order of a Matrix

A matrix having m rows and n columns is called a matrix of order $m \ge n$.

Types of Matrix

Column Matrix : $A = [a_{ij}]_{m \times 1}$

Row Matrix : $A = [a_{ij}]_{1 \times n}$

Square Matrix : $A = [a_{ij}]_{m \times m}$

Diagonal Matix : $A = [a_{ij}]_{m \times m}$ where $a_{ij} = 0 \forall i \neq j$ Scalar Matrix : $A = [a_{ij}]_{n \times n}$ where $a_{ij} = \begin{cases} 0, \text{ if } i \neq j \\ k, \text{ if } i = j \end{cases}$ for some constant k Zero Matrix : $A = [a_{ij}]_{i}$

Where $a_{ij} = \{0 \forall i = j \text{ and } i \neq j \}$ where $a_{ij} = \{0 \forall i = j \text{ and } i \neq j \}$ Identity Matrix : $\Lambda = [a_{ij}]$ where $a_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \neq j \end{cases}$

Transpose

Transpose is obtained by interchanging rows and columns. If $A = [a_{ij}]_{m \times n}$, then A' or $A^T = [a_{ij}]_{n \times n}$

Special Matrices

Nilpotent Matrix : $A^k = 0$ and $A^{k-1} \neq 0, k \in z^+$ $\Rightarrow |A| = 0$, order = Least value of k

Involutory Matrix : $A^2 = I \implies |A| = \pm 1$

Orthogonal Matrix : $AA^T = A^T A = I$ $\Rightarrow |A| = \pm 1$

Periodic Matrix : $A^k = A$ $\Rightarrow |A| = 0, 1, \text{ order } = k - 1$

Idempotent Matrix : $A^2 = A \implies |A| = 0, 1$

Unitary Matrix : $AA^{\theta} = A^{\theta}A = I$

Symmetric Matrix : A' = A

Skew Symmetric Matrix : A' = -A

Inverse

Properties

(AB)' = B'A'

|A'| = |A|

(A')' = A

 $(A \pm B)'$

 $= A' \perp B'$

If A and B are two square matrices such that AB = BA = I, then B is the inverse matrix of A and is denoted by A^{-1} and A is the inverse of B.