VIVEKANAND COLLEGE,KOLHAPUR (AUTONOMOUS)

DEPARTMENT OF BCA

SUBJECT - COMPUTER MATHEMATICS
CHAPTER 3- MATRICES

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LEARNING OBJECTIVES:-

- Cost estimation, Sale projection and factory problems can be solved by using matrix.
- Expressing in vector form
- Expressing day to day life problems in matrix form
- Matrix notation and operations are used in electronic spreadsheet, advanced statistics.
- Expressing simultaneous linear equations in matrix form.

Defination of matrix:-

- A matrix is an ordered rectangular array of numbers that represent some data (Plural = matrices)
- A matrix on its own has no value – it is just a representation of data
- Could be data associated with manufactured quantity in a factory, speed of a rocket etc
- Forms the basis of computer programming
- A matrix is used in solving equations that represent business problems

Types of matrix :-

- **Row matrix:** it having only one row Ex $\begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$
- **Column matrix:** it having only one column Ex $\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$
- **Zero matrix:** A matrix is called a zero matrix if all the entries are 0 Ex

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- **Square matrix:** if number of rows is equal to number of columns $\begin{bmatrix} 5 & 7 \\ -2 & 5 \end{bmatrix}$
- **Note:** if number of rows = no of number columns =n, is called square matrix of order n or order n
- order 2 $\begin{bmatrix} 5 & 7 \\ -2 & 5 \end{bmatrix}$ order 3 $\begin{bmatrix} 5 & 7 & 8 \\ 6 & 4 & 8 \\ 1 & 7 & 0 \end{bmatrix}$

Types of matrix :-

- **Diagonal matrix:** A square matrix is called diagonal matrix, if all of its non-diagonal elements are zero.
- EXAMPLE $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- **Scalar matrix:** A square matrix is called scalar matrix if diagonal elements are same and other are "0"
- EXAMPLE $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$
- **Identity/ unit matrix :** A square matrix is identity if diagonal entries are 1 and other are 0.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

REPRESENTATION OF MATRIX

$$A = [a_{ij}]_{MxN}$$

a_{ij} = element in row 'i' and column 'j', where 'a' is an element in the matrix

Eg: a_{23} = element in 2^{nd} row and 3^{rd} column = 9

Examples of Matrices

$$\begin{bmatrix} 2 & 4 \\ 5 & 7 \end{bmatrix}$$

This is an example of a 2 x 2 matrix

W hat is a_{12}

$$\begin{bmatrix} 2 & 3 & 6 \\ 72 & 3 & 9 \end{bmatrix}$$

What is the dimension or der of this Matrix?

What is a_{12}

$$\begin{bmatrix} 7 & 9 & 11 & 5 \\ 9 & 0 & 3 & 6 \end{bmatrix}$$

What is the dimension or order of this Matrix?

What is a_{12} ?

Addition /subtraction: when two matrices of same order are added/ subtracted, their corresponding entries are added/subtracted

Addition operation on Matrices

$$A = \begin{bmatrix} 2 & 45 & 72 \\ 6 & 3 & 0 \\ 7 & 9 & 10 \end{bmatrix} + B = \begin{bmatrix} 40 & 7 & 9 \\ 6 & 1 & 2 \\ 7 & 2 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} (2+40) & (45+7) & (72+9) \\ (6+6) & (3+1) & (0+2) \\ (7+7) & (9+2) & (10+8) \end{bmatrix} = \begin{bmatrix} 47 & 52 & 81 \\ 12 & 4 & 2 \\ 14 & 11 & 18 \end{bmatrix}$$

Only Matrices of the same order(comparable) can be added!!

Rule 1: A + B =B + A

Question Set 1

1. Add the following matrices:

$$\begin{bmatrix} 32 & 4 & 60 \\ 29 & 2 & 4 \\ 21 & 65 & 7 \end{bmatrix} + \begin{bmatrix} 22 & 5 & 8 \\ 10 & 8 & 12 \\ 9 & 7 & 2 \end{bmatrix}$$

2. Subtract the following matrices:

$$\begin{bmatrix} 18 & 26 & 12 \\ 10 & 11 & 12 \\ 8 & 10 & 16 \end{bmatrix} - \begin{bmatrix} 7 & 2 & 15 \\ 13 & 3 & 5 \\ 5 & 8 & 9 \end{bmatrix}$$

Multiplication of a matrix by a scalar

If K is any number and A is a given matrix,

Then KA is the matrix obtained by multiplying each element of A by K.

K is called 'Scalar'. Eg: if K = 2

$$A = \begin{bmatrix} 2 & 4 & 5 \\ 1 & 3 & 2 \\ 2 & 5 & 1 \end{bmatrix} \quad KA = \begin{bmatrix} 4 & 8 & 10 \\ 2 & 6 & 4 \\ 4 & 10 & 2 \end{bmatrix}$$

MULTIPLICATION OF MATRICES

- The product AB of two matrices A and B is defined, if the number of columns of A is equal to the number of B.
- If AB is defined then BA need not be defined . In particular both A and B are square matrices of same order then AB and BA are defined.
- In general AB≠BA
- Observation : Two non zero matrices multiplication is zero matrix

Ex: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \times \begin{bmatrix} -1 & 2 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}_{3\times 2}$ here multiplication is not possible.

Ex: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \begin{bmatrix} 2 \\ 1 \end{bmatrix}_{2\times 1}$ here multiplication possible, order of new matrix is 2×1.

If order of first matrix A is $m\times n$ and that of B is $n\times p$, then A.B is possible of order $m\times p$.

Multiplication of Matrices - 2

A

$$\begin{bmatrix} 2 & 3 & 1 \\ 4 & 3 & 2 \end{bmatrix} \times \begin{bmatrix} 4 & 2 \\ 1 & 0 \\ 5 & 2 \end{bmatrix}$$

2 x 3 matrix

B

3 x 2 matrix

$$= \begin{bmatrix} (2x4+3x1+1x5) & (2x2+3x0+1x2) \\ (4x4+3x1+2x5) & (4x2+3x0+2x2) \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 13 \\ 29 & 15 \end{bmatrix}$$

A x B

2 x 2 matrix

MLTIPLICATION OF MATRIX

Procedure for multiplication:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} =$$

$$\begin{bmatrix} a_{11}.b_{11} + a_{12}\,b_{21} + a_{13}\,b_{31} & a_{11}.b_{12} + a_{12}\,b_{22} + a_{13}\,b_{32} & a_{11}.b_{13} + a_{12}\,b_{23} + a_{13}\,b_{33} \\ a_{21}.b_{11} + a_{22}\,b_{21} + a_{23}\,b_{31} & a_{21}.b_{12} + a_{22}\,b_{22} + a_{23}\,b_{32} & a_{21}.b_{13} + a_{22}\,b_{23} + a_{23}\,b_{33} \\ a_{31}.b_{11} + a_{32}\,b_{21} + a_{33}\,b_{31} & a_{31}.b_{12} + a_{32}\,b_{22} + a_{33}\,b_{32} & a_{31}.b_{13} + a_{32}\,b_{23} + a_{33}\,b_{33} \end{bmatrix}$$

EX: $\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1.4 + 2.-1 & 1.0 + 2.2 \\ -1.4 + 0.-1 & -1.0 + 0.2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -4 & 0 \end{bmatrix}$

EX: $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1.1 + 2.3 \\ 1.1 + 1.3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$

Multiplication of Matrices: - 1

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 2 \\ 2 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (1x2 + 3x4+5x6) & (1x1+ 3x5+5x2) & (1x3+3x2+5x3) \\ (2x2 + 4x4+2x6) & (2x1+ 4x5+2x2) & (2x3+4x2+2x3) \\ (2x2 + 5x4+6x6) & (2x1+ 5x5+6x2) & (2x3+5x2+6x3) \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 26 & 24 \\ 32 & 26 & 20 \\ 58 & 39 & 34 \end{bmatrix}$$

Multiplication of Matrices - 3

B 3 x 2 matrix

$$\begin{bmatrix} 4 & 2 \\ 1 & 0 \\ 5 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 1 \\ 4 & 3 & 2 \end{bmatrix}$$

A 2 x 3 matrix

$$= \begin{bmatrix} (4x2+2x4) & (4x3+2x3) & (4x1+2x2) \\ (1x2+0x4 & (1x3+0x3) & (1x1+0x2) \\ (5x2+2x4) & (5x3+2x3) & (5x1+2x2) \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 18 & 8 \\ 2 & 3 & 1 \\ 18 & 21 & 9 \end{bmatrix}$$

B xA

3 x 3 matrix

Rule 2: Ax B ≠ B xA

Question Set 1

3. Multiply the following matrices:

$$\begin{bmatrix} 2 & 3 & 4 \\ 0 & 10 & 3 \\ 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 5 \\ 5 & 6 & 9 \\ 1 & 2 & 0 \end{bmatrix}$$

4. $$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 10 \end{bmatrix}$$

5. $$\begin{bmatrix} 3 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 2 \end{bmatrix}$$

Is it possible to compute No.5?! No!Why?

Transpose of a Matrix

- Matrix formed by interchanging rows and columns of A is called A transpose (A')

If $A=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}$ then $A'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}$

Q. Verify (A B)' = B' xA'

I. If $A=\begin{bmatrix}2 & 4 & 0\\ 3 & 9 & 6\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4\\ 2 & 8\\ 1 & 3\end{bmatrix}$

Q. Verify (A + B)' = A' + B'

If $A=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}3 & 6 & 5\\ -1 & 5 & 9\end{bmatrix}$

Symmetric matrix: A square matrix A is called symmetric if $A^T = A$

Remark: In a symmetric matrix, the entries opposite to diagonal entries are same. $a_{ij} = a_{ji}$

EX : i} $\begin{bmatrix}1 & -1\\ -1 & 3\end{bmatrix}$ ii} $\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 5\\ 3 & 5 & 4\end{bmatrix}$

Skew symmetric matrix : A square matrix A is called skew symmetric if A^T=-A or $a_{ij} = -a_{ji}$ .In skew symmetric matrix diagonal elements are "0" and entries opposite to main diagonal are same but opposite sign.

EX: $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ ii} $\begin{bmatrix}0 & 2 & 3\\ -2 & 0 & -5\\ -3 & 5 & 0\end{bmatrix}$

Question Set 1

6. Find the transpose of the following matrices and verify that (A+B)' = A' +B'

$$A = \begin{bmatrix} 1 & 2 & 9 \\ 4 & 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 8 & 6 \end{bmatrix}$$

Hint: Find A+B, (A+B)', A' and B' and verify

7. If D is a matrix where first row = number of table fans and second row = number of ceiling fans factories A and B make in one day. If a week has 5 working days compute 5A. What does 5A represent?

$$D = \begin{bmatrix} 10 & 20 \\ 30 & 40 \end{bmatrix}$$

Question Set 1

9. Two shops have the stock of large, medium and small sizes of a toothpaste. The number of each size stocked is given by the matrix A where

	Large	Medium	Small	
A =	150	240	120	shop no.1
	90	300	210	shop no 2

The cost matrix B of the different size of the toothpaste is given by

	Cost
B =	14
	10
	6

Find the investment in toothpaste by each shop

Answer: [3820] -- Investment by shop no 1 5520

-- Investment by shop no 2

Question Set 1

8. For the matrix

$$A = \begin{bmatrix} 4 & 5 & 6 \\ 2 & 1 & 3 \\ -5 & 2 & 2 \end{bmatrix} \quad \text{and } B = \begin{bmatrix} 7 & 9 \\ 10 & 2 \end{bmatrix}$$

Multiply by the Matrix I =

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

What is A.I and I.B ?

Identity Matrix

- If you were to multiply 'a' by '1', you would get 'a'.
  Eg: 2 x 1 =2x1 =2

- The 'identity' matrix (i) is the equivalent of '1' in basic math
  If A is a matrix and I is an identity Matrix,

- Then A x I = A and I x A = A. Identity Matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then A x I = A and I x A = A

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a+0 & 0+b \\ c+0 & 0+d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

To find inverse by using elementary Row transformations.

- **Step 1:** Write **A = IA**
- **Step 2:** Apply various row operations on left hand side and apply same operations to **I** on right side but not to A on right side.
- **Step3:** From step 2 we get a new matrix equation **I = BA** . Hence **B** = $\mathbf{A^{-1}}$.

To find inverse by using elementary Column transformations.

- **Step 1:** Write **A = AI**
- **Step 2:** Apply various Column operations on left hand side and apply same operations to **I** on right side but not to A on right side.
- **Step3:** From step 2 we get a new matrix equation **I = AB** . Hence **B = A^{-1}** .

INVERSE OF ORDER 2 MATRIX

Exmples:

Using elementary row transformation find the inverse of $\begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$.

Ans : Given $A = \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$

Consider $A = IA$

$$\Rightarrow \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 5R_1$

$$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -5 & 6 \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ -5 & 6 \end{bmatrix} A$$

Applying $R_2 \to (-1)R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix} A$$

Hence $A^{-1} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix}$

INVERSE OF ORDER 3 MATRIX

1. Using elementary row transformation find the inverse of $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Ans :Given $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Consider $A = IA$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to 3R_1$

$$\begin{bmatrix} 6 & 0 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_3$

$$\begin{bmatrix}1 & 0 & 0\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix} = \begin{bmatrix}3 & -1 & 1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A$$

Applying $R_2 \rightarrow R_2 - 5R_1$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 1 & 3\end{bmatrix} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\0 & 0 & 1\end{bmatrix}A$$

Applying $R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 3\end{bmatrix} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\15 & -6 & 6\end{bmatrix}A$$

Applying $R_3 \rightarrow \frac{1}{3}R_3$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\5 & -2 & 2\end{bmatrix}A$$. Hence $A^{-1} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\5 & -2 & 2\end{bmatrix}$

Inverse of a Matrix

In basic math: 2 ÷ 2 = 1 and 1/2 x 2 = I.

Dividing 2 by two is the same as multiplying 2 by 1/2 .The net result is 1.

A similar concept is the 'inverse' of a matrix. If A is a matrix, then A^{-1} is the inverse such that $A \times A^{-1}$ = I (identity matrix)

If A has an inverse (A^{-1}) then A is said to be 'invertible'

$A.A^{-1}=A^{-1}.A = I$

The determinant is a scalar value that can be computed from the elements of a square matrix

$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then determinant of A or detA or $|A|$

$|A|=a_{11}(a_{22}a_{33}-a_{32}a_{23})-a_{12}(a_{21}a_{33}-a_{31}a_{23})+a_{13}(a_{21}a_{32}-a_{31}a_{22})\neq 0$ then inverse of matrix exist i.e A^{-1}

$A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 5 \\ 3 & 5 & 4 \end{bmatrix}=1(-1.4-5.5)-2(2.4-3.5)+3(2.5-3.-1)=11\neq 0$

here inverse of A exist. So $A^{-1}=\frac{adjA}{|A|}$, $|A|\neq 0$

$adjA=\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}=\begin{bmatrix} -29 & 7 & 13 \\ 7 & -5 & 1 \\ 13 & 5 & -5 \end{bmatrix}$

$A_{11}=-29$ $A_{12}=7$ $A_{13}=13$ $A_{21}=7$ $A_{22}=-5$ $A_{23}=5$ $A_{31}=13$ $A_{32}=1$ $A_{33}=-5$

$A^{-1}=\frac{adjA}{|A|}=\frac{1}{11}\begin{bmatrix} -29 & 7 & 13 \\ 7 & -5 & 5 \\ 13 & 1 & -5 \end{bmatrix}=\begin{bmatrix} \frac{-29}{11} & \frac{7}{11} & \frac{13}{11} \\ \frac{7}{11} & \frac{-5}{11} & \frac{1}{11} \\ \frac{13}{11} & \frac{5}{11} & \frac{-5}{11} \end{bmatrix}$

If the simultaneous equation are of the form

$a_{11}x+a_{12}y+a_{13}z=b_1$ it can also be represented in matrix form AX=B

$a_{21}x+a_{22}y+a_{23}z=b_2$ where $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

$a_{31}x+a_{32}y+a_{33}z=b_3$ so $X=A^{-1}B=\frac{adjA}{|A|}B$

EX : Solve by matrix method x+y+z=6 ,2x-y+5z=6 ,3x+5y+4z=12

Ans Here $A=\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 5\\ 3 & 5 & 4\end{bmatrix}$ $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ $B=\begin{bmatrix}6\\ 6\\ 12\end{bmatrix}$

$|A|=\begin{vmatrix}1 & 2 & 3\\ 2 & -1 & 5\\ 3 & 5 & 4\end{vmatrix}=1(-1.4-5.5)-2(2.4-3.5)+3(2.5-3.-1)=11\neq 0$

So inverse exist , Previous example it is found that

$$A^{-1}=\frac{adjA}{|A|}=\frac{1}{11}\begin{bmatrix}-29 & 7 & 13\\ 7 & -5 & 5\\ 13 & 1 & -5\end{bmatrix}=\begin{bmatrix}\frac{-29}{11} & \frac{7}{11} & \frac{13}{11}\\ \frac{7}{11} & \frac{-5}{11} & \frac{1}{11}\\ \frac{13}{11} & \frac{5}{11} & \frac{-5}{11}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=X=A^{-1}B=\begin{bmatrix}\frac{-29}{11} & \frac{7}{11} & \frac{13}{11}\\ \frac{7}{11} & \frac{-5}{11} & \frac{1}{11}\\ \frac{13}{11} & \frac{5}{11} & \frac{-5}{11}\end{bmatrix}\begin{bmatrix}6\\ 6\\ 12\end{bmatrix}=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$$

So x=1,y=1 and z=1

Singular matrix : A square matrix is said to be singular if $\det(A)=0$

Ex : Let $A=\begin{bmatrix}2 & 4\\ 1 & 2\end{bmatrix}$ here $\det(A)$=4-4=0 , so given matrix is singular.

Non singular Matrix : A square matrix is said to be non singular if $\det(A)\neq 0$,

Note : Inverse of a square matrix exist in non singular matrix.

KEY POINTS

- A matrix is an ordered rectangular array of numbers or functions.
- A matrix having m rows and n columns is called a matrix of order mxn
- A is a diagonal matrix if its non diagonal elements are zero.
- A is a identity matrix if diagonal elements are 1 and non diagonal elements are 0
- A is zero matrix if all elements are zero.
- Matrix addition is commutative ,associative over same order. A+B= B+A, (A+B)+C=A+(B+C)

KEY POINTS

- k (A+B)= kA+kB , k is constant A,B are of same order
- If order of first matrix A is m and that of B is n, then A.B is possible of order m
- Matrix multiplication is not commutative.
- A matrix is symmetric if $a_{ij} = a_{ji}$
- If a square matrix is invertible if detA≠0, or singular matrix
- if detA =0,inverse of a square matrix doesn't exist.or non singular matrix

$$(kA)^T = kA^T$$
$$(A \pm B)^T = A^T \pm B^T$$
$$(A^{-1})^T=(A^T)^{-1}$$
$$(A^T)^T=A$$

CONCEPT MAPPING

Order of a Matrix
A matrix having m rows and n columns is called a matrix of order $m \times n$.

Transpose
Transpose is obtained by interchanging rows and columns. If $A = [a_{ij}]_{m \times n}$, then A' or $A^T = [a_{ji}]_{n \times m}$

Properties
- $(A')' = A$
- $(AB)' = B'A'$
- $(A \pm B)' = A' \pm B'$
- $|A'| = |A|$

Types of Matrix

- Column Matrix : $A = [a_{ij}]_{m \times 1}$
- Row Matrix : $A = [a_{ij}]_{1 \times n}$
- Square Matrix : $A = [a_{ij}]_{m \times m}$
- Diagonal Matix : $A = [a_{ij}]_{m \times m}$ where $a_{ij} = 0 \ \forall \ i \neq j$
- Scalar Matrix : $A = [a_{ij}]_{n \times n}$ where $a_{ij} = \begin{cases} 0, \text{ if } i \neq j \\ k, \text{ if } i = j \end{cases}$ for some constant k
- Zero Matrix : $A = [a_{ij}]$, where $a_{ij} = \{0 \ \forall \ i = j$ and $i \neq j$
- Identity Matrix : $A = [a_{ij}]$ where $a_{ij} = \{1$ if $i = j$

Special Matrices

- Nilpotent Matrix : $A^k = 0$ and $A^{k-1} \neq 0, k \in z^+$ $\Rightarrow |A| = 0$, order = Least value of k
- Involutory Matrix : $A^2 = I \Rightarrow |A| = \pm 1$
- Orthogonal Matrix : $AA^T = A^T A = I$ $\Rightarrow |A| = \pm 1$
- Periodic Matrix : $A^k = A$ $\Rightarrow |A| = 0, 1$, order $= k - 1$
- Idempotent Matrix : $A^2 = A \Rightarrow |A| = 0, 1$
- Unitary Matrix : $AA^\theta = A^\theta A = I$
- Symmetric Matrix : $A' = A$
- Skew Symmetric Matrix : $A' = -A$

Inverse

If A and B are two square matric[...] such that $AB = BA = I$, then B is th[...] inverse matrix of A and is denote[...] by A^{-1} and A is the inverse of B.