Date: 27/12/2021

Vivekanand College, Kolhapur (Autonomous)

Department of Mathematics

M. Sc. I Sem. I and M.Sc. II Sem III

Internal Examination 2021-22

All the students of M.Sc. I and M.Sc. II are hereby informed that their Internal Examination of Mathematics will be conducted on **as given below timetable.** The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus and timetable for examination will be as mentioned in following table.

Timetable

Date	**Time**	**Class**	**Subject**
06/01/2021	02:00 PM to 03: 00 PM	M.Sc. I	Algebra
	02:00 PM to 03: 00 PM	M.Sc. II	Functional Analysis
08/01/2021	02:00 PM to 03: 00 PM	M.Sc. I	Advanced Calculus
	02:00 PM to 03: 00 PM	M.Sc. II	Advanced Discrete Mathematics
10/01/2021	02:00 PM to 03: 00 PM	M.Sc. I	Complex analysis
	02:00 PM to 03: 00 PM	M.Sc. II	Lattice Theory
12/01/2021	02:00 PM to 03: 00 PM	M.Sc. I	Ordinary Differential Equation
	02:00 PM to 03: 00 PM	M.Sc. II	Number theory
13/01/2021	02:00 PM to 03: 00 PM	M.Sc. I	Classical Mechanics
	02:00 PM to 03: 00 PM	M.Sc. II	Operational Research -I

Syllabus for M. Sc. I Sem. I

Sr. No.	Name of Paper	Topics
1	CP-1170A : Algebra	Unit I
2	CP-1171A: Advanced Calculus	Unit I
3	CP-1172A: Complex analysis	Unit I
4	CP-1173A: Ordinary Differential Equation	Unit I
5	CP-1174A: Classical Mechanics	Unit I

Syllabus for M. Sc. II Sem. III

Sr. No.	Name of Paper	Topics
1	CC-1180C: Functional Analysis	Unit I
2	CC-1181C: Advanced Discrete Mathematics	Unit I
3	CBC-1182C : Lattice Theory	Unit I
4	CBC-1183C: Number theory	Unit I
5	CBC-1184C : Operational Research -I	Unit I

Nature of question paper

Time:-1 Hours **Total Marks: 30**

Q.1) Choose the correct alternative for the following question. [05]

Five questions

Q.2) Attempt any three **[15]**

Four questions

Q.3) Attempt any One **[10]**

Two questions

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(Prof. S. P. Patankar)
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Vivekanand College, Kolhapur (Autonomous)

M.Sc. I Semester-I Internal Examination: 2021-22

MATHEMATICS

Sub: Algebra (CP-1170A)
Date: 06/01/2021

Time:02:00 pm-03:00pm
Total Marks:30

Q1) Select the correct alternatives (5)

1] *Let G be a group & let H & K be two subgroups of G. If both H & K 12 elements, which of the following numbers can not be the cardinality of* $HK = \{hk : h \in H, k \in K\}$?

$a)\ 72$ $b)\ 60$ $c)\ 48$ $d)\ 36$

2] *How many proper subgroups does the group* $\mathbb{Z} \oplus \mathbb{Z}$ *have?*

$a)\ 1$ $b)\ 2$ $c)\ 3$ $d)$ *infinitely many.*

3] *In a group of order* 15 *the number of subgroups of order* 3 *is ...*

$a)\ 3$ $b)\ 5$ $c)\ 1$ $d)\ 2$

4] *If G is an arbitrary group of even order 2n then ...*

a) *G has a proper normal subgroup which is non trivial.*
b) *G admits a quotient group of order n*
c) *G has a subgroup of order* 2
d) *G admits a quotient group of order* 2.

5] *For* $n \geq 2$ *let* $\left(\frac{\mathbb{Z}}{n\mathbb{Z}}\right)^*$ *be the groups of units of* $\frac{\mathbb{Z}}{n\mathbb{Z}}$ *which one of the following is cyclic?*

$a)\ \left(\frac{\mathbb{Z}}{8\mathbb{Z}}\right)^*$ $b)\ \left(\frac{\mathbb{Z}}{15\mathbb{Z}}\right)^*$ $c)\ \left(\frac{\mathbb{Z}}{10\mathbb{Z}}\right)^*$ $d)\ \left(\frac{\mathbb{Z}}{35\mathbb{Z}}\right)^*$

Q2) Solve any THREE of the following. (15)

1] Show that, A_n is generated by 3-cycles for $n \geq 3$.

2] Define ascending central series for a group G. Show that, if H and N are subgroups of G and N is normal in G then $H \cap N$ is normal in H.v

3] State and prove Schreier theorem.

4] Define index of subgroup . Find index of A_n in S_n. Show that A_n is normal in S_n.

Q3) Solve any ONE of the following. (10)

1] Define even permutation. Prove that, no permutation of a finite set can be expressed both as a product of an even number of transpositions and as product of odd number of transpositions.

2] State and prove Caley's theorem.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-I Internal Examination:2021-22

Subject :Advanced Calculus
Time: 02: 00 PM-03:00 pm
Date:08/01/2021 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i. *he series* $\sum_{n=1}^{\infty} a_n sin(nx)$ *converges uniformly on* $\mathbb{R}$ *if*

A) $\sum_{n=1}^{\infty} a_n$ *converges* B) $\sum_{n=1}^{\infty} |a_n|$ *converges*

C) $\sum_{n=1}^{\infty} sin(nx)$ *converges* D) $\sum_{n=1}^{\infty} |sin(nx)|$ *converges.*

ii. *Radius of convergence for the seris* $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$ *is* ________

A) 2 B) 1/2

C) 1 D) *series always diverges.*

iii. *If* $\bar{f}$ *is linear then* $\bar{f}'(\bar{c};\bar{u}) =$ __________

A) 0 B) $\bar{f}(\bar{u})$ C) $\bar{f}(\bar{c})$ D) $\bar{f}'(\bar{u})$

iv. *Stokes theorem relates a surface integral to*

A) *Volume integral* B) *Line integral*

C) *Vector integral* D) *Real integral*

v. *If* $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ & $S: \mathbb{R}^m \longrightarrow \mathbb{R}^p$ *are linear then order of matrix of* $(\bar{S}.\bar{T}) =$ ________

A) $n \times p$ B) $m \times p$ C) $p \times n$ D) $m \times n$

Q.2. Attempt any three of the following: **[15]**

1) Prove that the sequence $\{f_n\}_{n=1}^{\infty}$ *converges pointwise but not uniformly where* $f_n(x) = \frac{1}{nx+1}, 0 < x < 1$

2) *If* $\sum_n a_n$ *converges absolutely then prove that every subseris* $\sum_n b_n$ *also converges absolutely.*

3) Let $\bar{f}$ *be a vector field given by* $\bar{f}(x,y) = \sqrt{y}i + (x^3 + y)j$ *where* $(x,y) \in \mathbb{R}^2,$ $y \geq 0$ *obtain the integral of* $\bar{f}$ *from* $(0,0)$ *to* $(1,1)$ *along the path* $\alpha(t) = ti + tj$

4) *Evaluate* $\iint e^{\frac{y-x}{y+x}} dxdy$ *over triangle bounded by lines* $x + y = 2$ & *two cordinate axes* x & y.

Q.3. Attempt any one of the following: **[10]**

1) *If* $\{f_n\}$ & $\{g_n\}$ *be sequences of Riemann integrable functions defined on* $[a,b]$ &

$\underset{n\to\infty}{\text{l.i.m}} f_n = f$ & $\underset{n\to\infty}{\text{l.i.m}} g_n = g$ *on* $[a,b]$. *Let* $h_n(x) = \int_a^x f_n(t) g_n(t) dt$ &

$h(x) = \int_a^x f(t)\, g(t) dt$

prove that $h_n \rightarrow h$ *uniformely on* $[a,b]$

2) *If* $\bar{f}$ *be differentiable at* $\bar{c}$ *with total derivative* $\bar{T}_{\bar{c}}$. *Prove that* $\bar{f}'(\bar{c};\bar{u}) = \bar{T}_{\bar{c}}(\bar{u})$ *for every* $\bar{u} \in \mathbb{R}^n$

Vivekanand College, Kolhapur (Autonomous)

M.Sc. (Part-I) Semester-I Internal Examination: 2021-22

Subject : Complex Analysis **Time: 02:00 pm-03:00 pm**

Date: 10/01/2021 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i) Which of the following function is analytic?

A) $f(z) = e^{x-iy}$ B) $f(z) = |z|^2$

C) $f(z) = x^2 + iy^2$ D) $f(z) = (z^2 - 2)e^{-x-iy}$

ii) If C is the circle of radius 2 with center at the origin in the complex plane, oriented in the anti - clockwise direction. Then the integral $\oint \frac{dz}{(z-1)^2}$ is equal to.....

A) 1 B) $2\pi i$ C) 0 D) $1/2\pi i$

iii) For the function f(z) $= \frac{z - sinz}{z^3}$, at the point z = 0 is.....

A) Pole of order 3 B) Pole of order 2

C) Essential singularity D) Removable singularity

iv) The excess of the number of zeros over the number of poles of a meromorphic function is called ....

A) Maximum Modulus Principle B) Minimum Modulus Principle

C) Schwarz Lemma D) The Argument Principle

v) The radius of convergence of $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$ is.....

A) -e B) 1/e C) e D) 1/e

Q.2. Attempt any three of the following: **[15]**

1) Find radius of convergence of f(z) = $\sum_{n=1}^{\infty} a_n z^n$.

2) Find radius of convergence of $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$.

3) If $u(x, y) = x^3 + ax^2y + bxy^2 + 2y^3$ is harmonic function and $v(x, y)$ its harmonic conjugate. If $v(0, 0) = 1$, then $|a + b + v(1, 1)|$ is equal to.......

4) If γ is a contour with parameter interval [a, b] and $f(z) = u(x, y) + iv(x, y)$ is continuous function on the contour γ with $|f(z)| \leq M, \forall z \in \gamma$, then prove that $| \int_C f(z)dz | \leq ML$ where L is the length of contour given by $\int_a^b |\gamma'(t)| dt$

Q.3. Attempt any one of the following: **[10]**

1) $0 \leq R < \infty$ called the radius of convergence with the following properties

i) $\sum a_n z^n$ converges absolutely for every z with |z| < R.

ii) If | z| > R, the terms of power series become unbounded and so the series diverges.

find the radius of convergence of the power series $\sum_{n=0}^{\infty} (\frac{1}{n})^n z^n$.

2) If γ is a contour with parameter interval [a, b] and $f(z) = u(x, y) + iv(x, y)$ is continuous function on the contour γ with $|f(z)| \leq M, \forall z \in \gamma$, then prove that $| \int_C f(z)dz | \leq ML$ where L is the length of contour given by $\int_a^b |\gamma'(t)| dt$. If R is a positive real number greater than 1 and γ_R is the contour $Re^{it}, 0 < t < \pi$, then show that $\lim_{R\to\infty} \int_{\gamma_R} \frac{e^{it}}{z^2} dz = 0$

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-I
Internal Examination(2021-22)
Ordinary Differential Equations

Subject: Ordinary Differential Equations **Total Marks: 30**
Date: 12/01/2021 **Time: 02:00PM to 03:00PM**

Q.1) Choose the correct alternative for the following question. [05]

i) The General solution of $y'' + y' - 2y = 0$ is .....

A) $c_1e^{-x} + c_2e^{-2x}$ B) $c_1e^{x} + c_2e^{-2x}$

C) $c_1e^{x} + c_2xe^{x}$ D) $c_1e^{-x} + c_2e^{2x}$

ii) Which of the following is not solution of $y''' - 3r_1y'' + 3{r_1}^2y' - {r_1}^3y = 0$, where r_1 is constant

A)$\emptyset(x) = e^{r_1x}$ B) $\emptyset(x) = x^2e^{r_1x}$
C) $\emptyset(x) = xe^{r_1x}$ D) $\emptyset(x) = x^3e^{r_1x}$

iii)The singular point of equation $x^2(x-4)^2y'' + 3xy' - (x-4)y = 0$ is.....

A) 0 B)-4 C)1 D)2

iv) If $f(x,y) = x + y^2$, $R = \{(x,y)||x| < \infty, |y| \leq b\}$ and K is Lipschitz constant then ......

A) F satisfies Lipschitz Condition on R with $k = 2b$

B) F satisfies Lipschitz Condition on R with $k = 0$

C) F satisfies Lipschitz Condition on R with $k = 1$

D) F do not satisfy Lipschitz Condition on R

v) In Legendary equation If α is a non-negative even integer, then $\emptyset_1$ is polynomial of degree 'n' containing only ..... powers of x.

A) Odd B)Even

C)Odd and zero D)Even and zero

Q.2) Attempt any three [15]

i) If $\emptyset_1$and $\emptyset_2$ are two solutions of L(y)=0 on an interval I containing point x_0 than show that $W(\emptyset_1, \emptyset_2)(x) = e^{-a_1(x-x_0)}\, W(\emptyset_1, \emptyset_2)(x_0)$

ii) Show that there exist 'n' linearly independent solutions of $L(y) = 0$ on any interval I. where, $L(y) = 0$ is homogeneous differential equation with variable coefficient.

iii) Classify the singular points in the finite plane $x^2(x^2 - 4)y'' + 2x^3y' + 3y = 0$

iv) Show that $\emptyset(x) = \frac{d^n}{dx^n}[(x^2 - 1)^n]$ satisfies the Legendre equation hence show that $\emptyset(1) = 2^n n!$

Q.3) Attempt any One [10]

i) Find the power series solutions $\emptyset_1(x)$ and $\emptyset_2(x)$ of Legendre equation given $(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$ where α is constant and $|x| < 0$

ii) Solve the Euler's equation $x^2y'' - 5xy' + 9y = x^3, (x > 0)$

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-I
Internal Examination(2021-22)
Classical Mechanics

Time: 2:00PM–3:00PM **Total Marks: 30**

Date:13/01/2021

Q.1) Choose the correct alternative for the following question. [05]

1) The equation of motion of a single particle is given by.......

A) $\bar{F} = \bar{P}$ B) $\bar{F} = \dot{\bar{P}}$

C) $\bar{F} = \ddot{\bar{P}}$ D) none of these

2) Which of the following is not example of Holonomic constraints?

A) rigid body B) simple pendulum

C) gas molecule moving in the closed containe D) particle moving on parabola $y^2 = 4ax$

3) The number of generalized co-ordinates in simple pendulum is.......

A) 1 B) 2

C) 3 D) 4

4) Let $\bar{r}_i = \bar{r}_i(q_j, t); j = 1, 2, 3 \ldots.. n$. Then $\delta\bar{r}_i =$...........

A) $\sum_{k=1}^{\infty} \frac{\delta\bar{r}_i}{\delta q_k}\delta q_k$ B) $\sum_{k=1}^{n} \frac{\delta\bar{r}_i}{\delta q_k}\delta q_k$

C) $\sum_{k=1}^{n} \frac{\delta\bar{r}_i}{\delta q_k}\delta q_k + \delta t$ D) $\sum_{k=1}^{n} \frac{\delta\bar{r}_i}{\delta t}\delta t$

5) Equation of motion of simple pendulum is given by......

A) $\ddot{\theta} = \frac{-g}{l}\sin\theta$ B) $\theta = \frac{-g}{l}\sin\theta$

C) $\ddot{\theta} = \frac{g}{l}\sin\theta$ D) $\theta = \frac{g}{l}\sin\theta$

Q.2) Attempt any three [15]

1)Explain how the generalized co-ordinates of a rigid body with N particles reduces to six for its description.

2)Explain Atwood machine and discuss it's motion.

3) Use Hamilton's Principle to find the equation of motion of a simple pendulum

4) Find the plain curve of fixed perimeter that encloses maximum area.

Q.3) Attempt any One [10]

1)Find the Euler-Lagranges differential equation satisfied by twice differentiable function y(x) which extremizes the functional $I(y(x)) = \int_{x_1}^{x_2} f(x, y, y')dx$

where y is prescribed at the end-points.

2) Derive Hamilton's principle for a non-conservative system from D'Alemberts Principle. Further, derive the Hamilton's principle for conservative system from it.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. II Semester-III Internal Examination: 2021-22
MATHEMATICS

Sub: Functional Analysis **Time: 02:00 pm -03: 00 pm**
Date: 06/01/2021 **Total Marks:30**

Q.1 . Choose correct Alternative for the following. **(5)**

1) Consider following two statements
I) Every normed linear space is a metric space. II) Every metric space is normed linear space.
A) Only II is true. B) I is true and II is false C)Only I is false D) II is true and I is false.

2) Consider following two statements;
I) Every normed linear space is a metric space.
II) Every metric space is normed linear space.
A) Only II is true. B) I is true and II is false
C) Only I is false D) II is true and I is false.

3) Every projection on a Banach space B is___.
A) linear, bounded, idempotent B) linear, idempotent, continuous
C) linear, norm preserving, nilpotent D) Both A and B

4)Every projection on a Banach space B is___
A)Linear, Bounded, Idempotent B) Linear , Idempotent, Continuous
C)Linear , Norm preserving , nilpotent D) Both A and B

5) Consider following two statements
I)Every Banach space is reflexive norm linear space
II)Every reflexive norm linear space is Banach Space
A) Only II is true. B) I is true and II is false C)Only I is false D) II is true and I is false.

Q2) Solve any THREE of the following. **(15)**

1)If N is a normed linear space and x_0 is non zero vector in N then show that there exist a functional f_0 in N^* such that $f_0(x_0) = ||x_0||$ and $||f_0|| = 1$

2)f N is a normed linear space and x_0 is non zero vector in N then show that there exist a functional f_0 in N^* such that $f_0(x_0) = ||x_0||$ and $||f_0|| = 1$

3)If N is Banach space and M is closed linear subspace of N then show that, quotient space N/M is Banach space.

4)If $\{T_n\}$ and $\{S_n\}$ are sequences in B(N) such that $T_n \to T$ and $S_n \to S$ as $n \to \infty$ then show that,

a) $T_n + S_n \to T+ S$ b) $kT_n \to kT$ for k in F c) $T_nS_n \to TS$ as $n \to \infty$

Q3) Solve any ONE of the following. **(10)**

1)Define normed linear space. If N and N' are normed linear spaces ,T is linear transformation from N into N' then show that following conditions are equivalent

a)T is continuous on N

b)T is continuous at origin

c)there exist a real number $k \geq 0$ with property $||T(x)|| \leq k||x||$ for all x in N

d)If s = {x in N such that $||x|| \leq 1$ } is closed unit sphere in N then T(S) is bounded in N'

2)Prove that a normed linear space N is finite dimensional if and only if S = {x in N /$||x|| \leq 1$} is compact.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III
Internal Examination(2021-22)
Advanced Discrete Mathematics

Time: 3:00PM–4:00PM **Total Marks: 30**

Date: 08/01/2021

Q.1) Choose the correct alternative for the following question. **[05]**

i) The adjacency matrix of graph is ----- matrix.

A) diagonal B) scalar C) symmetric D) skew-symmetric

ii) If G is a graph with n vertices, q edges and w(G) number of connected components, then G has at-least ----- number of edges.

A) n B) w(G) C) n + w(G) D) n − w(G)

iii) The particular solution to recurrence relation $a_r + 7a_{r-1} + 3a_{r-2} = 5$ is -----

A) 5 B) $\frac{11}{5}$ C) $\frac{5}{11}$ D) 11

iv) In a Boolean algebra an element can have ----- complement.

A) only one B) exactly two C) more than two D) zero

v) For a Boolean algebra B, if $a + b = 0$, then -----

A) $a = 0, b = 0$ B) $a = 0, b \neq 0$ C) $a \neq 0, b = 0$ D) $a \neq 0, b \neq 0$

Q.2) Attempt any three **[15]**

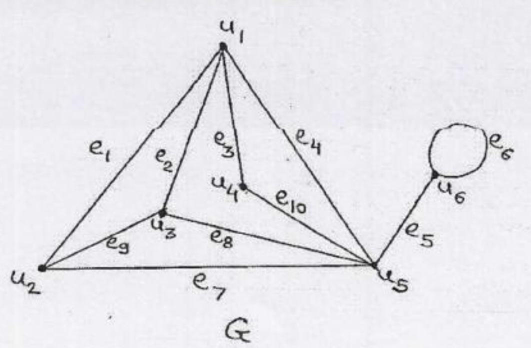
i) Prove that in any graph G, there is even number of odd vertices

ii) Define vertex disjoint subgraphs. Prove that join of two vertex disjoint complete graphs is a complete graph.

iii) Define complement of graph. Prove that if G is self-complementary graph of n-vertices, then 'n' is equal to either 4t or 4t+1, for some integer 't'.

iv) Define: a) Regular graph b) Path

Q.3) Attempt any One **[10]**



i) Find eccentricity of all vertices of graph G. Also find radius and diameter of G.

ii) If T is a tree with n-vertices, then prove that T has precisely (n-1) number of edges.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III Internal Examination: 2021-22
MATHEMATICS

Subject: Lattice Theory **Time: 02:00pm -03:00pm**
Date: 10/01/2021 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i. *Consider the following statements*
Statement $-$ 1) *Every ideal is heriditory subset.*
Statement $-$ 2) *Every heriditory subset is ideal.*
A) Only 1) *true* *B) Only* 2) *true* *C) Both* 1)&2) *true* *D) Both* 1)&2) *false.*

ii. *A chain is______*
A) Complemented lattice *B) Not complemented lattice*
C) may be complemented lattice *D) None of these*

iii. *In the poset* $< \mathbb{Z}^+ , | >$
where $\mathbb{Z}^+$ *is the set of positive integers* & | *is divides relation then* 3 & 18 *are______*
A) Comparable *B) Parellel* *C) both A)&B)* *D) neither a) nor b).*

iv. *L & M be two sublattices of lattice P then which of the following is also sublattice of P*
A) $L \cap M$ *B)* $L \cup M$
A) $L \times M$ *D) Both a) & c)*
Statement $-$ 1) $J(L)$ *is not ring of set.*
Statement $-$ 2) $H(J(L))$ *is ring of set.*
A) Only 1) *true* *B) Only* 2) *true* *C) Both* 1)&2) *true* *D) none of these*

v. *Consider the following statements*
Statement $-$ 1) *Every ideal is heriditory subset.*
Statement $-$ 2) *Every heriditory subset is ideal.*
A) Only 1) *true* *B) Only* 2) *true* *C) Both* 1)&2) *true* *D) Both* 1)&2) *false.*

Q.2. Attempt any three of the following: [15]

1) *Prove that every ideal I of distributive lattice is the intersection of all prime ideals containing it.*

2) *Show that in a finite lattice every element is join of join irreducible elements.*

3) *If* $H(P)$ *is collection of all heriditory subset of poset P then show that* $H(P)$ *is lattice.*

4) *In a lattice show that pseudo complement of an element is unique.*

Q.3. Attempt any one of the following: [10]

1) *In any lattice L prove the following conditions always holds.*

i) $(x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z)$

ii) $x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z) \quad \forall x, y, z \in L$

2) *Prove if* I &J *be ideals of distributive lattice L if* $I \cap J$ &$I \cup J$ *are principle then so* I & J

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III
Internal Examination(2021-22)
Number Theory

Time: 02:00PM to 03:00PM **Total Marks: 30**

Date:12/01/2021

Q.1) Choose the correct alternative for the following question. **[05]**

1) Sum of positive divisors of $n = 2^5$ are .....

A) 61 B) 62 C) 63 D) 64

2) Consider the following statements:

(I) Mobius μ-function is multiplicative

(II) The function τ and σ are both multiplicative, then...

A) Only (I) is true B) Only (II) is true

C) Both (I) and (II) are true D) Both (I) and (II) are false

3) If n is a prime then $\tau(n)$ is....

A) 1 B) 2 C) 3 D) 4

4) If n is a prime then $\sigma(n)$ is....

A) n+1 B) n+2 C) n+3 D) n+4

5) $\sigma(101) =$...

A) 100 B) 102 C) 201 D) 202

Q.2) Attempt any three **[15]**

1) State and Prove Euclid's theorem.

2) Solve the linear Diophantine equation $54x + 21y = 906$.

3) By using mathematical induction prove that $21/4^{n+1} + 5^{2n-1}$.

4)Prove that for given integers a and b not both zero there exists integers x and y such that $\gcd(a,b) = ax + by$.

Q.3) Attempt any One **[10]**

1) Prove that the linear Diophantine equation $ax + by = c$ has a solution iff d/c where $d = \gcd(a,b)$. If $(x_0, y_0)$is any particular solution of this equation then all other solutions are given by $x = x_0 + \frac{b}{d}t$ and $y = y_0 - \frac{a}{d}t$.

2)Prove that every positive integer $n > 1$ can be expressed as product of primes and this representation is unique apart from the order in which the factor occurs.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III
Internal Examination(2021-22)
Operational Research-I

Time: 02:00PM to 03:00 PM **Total Marks: 30**
Date: 13/01/2021

Q.1) Choose the correct alternative for the following question. [05]

i) The set of all convex combination of a finite number of points is called ....

A) Convex cone B) Convex polyhedron C) convex hull D) none of these

ii) In Big – M method, the coefficient of artificial variable in the objective function for maximization problem is....

A) +M B) -M C) Zero D) None of these

iii) The point at which $\nabla f(x) = 0$ are called ...

A) boundary points B) interior points C) extreme points D) convex point

iv) A sufficient condition for a stationary point to be an extreme point is that the Hessian matrix H is evaluated at x_0 is ... when x_0 is minimum point

A) Positive definite B) Negative definite

C) Positive semidefinite D) Negative semidefinite

v) Which of the following is correct?

A) An extreme point is boundary point of set

B) An extreme point cannot be between any other two point of set

C) Both A and B

D)None of these

Q.2) Attempt any three [15]

i) Solve the following non – linear programming problem

Optimize Z = $x_1^2 + x_2^2 + x_3^2$,

$subject\ to\ x_1 + x_2 + 3x_3 = 2,\ 5x_1 + 2x_2 + x_3 = 5,\ \ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

ii) Show that the set S = $\{(x_1, x_2): 2x_1 - x_2 + x_3 \ \leq 4\}\ is\ convex\ set$

iii) Use dynamic programming to show that Z = $P_1 log P_1 + P_2 log P_2 + \ldots\ldots. P_n log P_n$

Subject to $P_1 + P_2 + \ldots\ldots. P_n = 1, P_i \geq 0$, i = 1, 2, ……n is minimum when

$P_1 = P_2 = \ldots\ldots. P_n$ =1/n.

iv) Explain the characteristics of standard form of Linear programming problem. Rewrite the following LPP in standard form. Min Z = $2x_1 + x_2 + 4x_3$,

$subject\ to\ -2x_1 + 4x_2 \leq 4,\ x_1 + 2x_2 + x_3\ \geq 5,\ 2x_1 + 3x_3 \leq 2,$

$x_1 \geq 0, x_2 \geq 0, x_3\ unrestricted\ in\ sign$

Q.3) Attempt any One [10]

i) Define quadratic programming problem. Solve the following quadratic programming problem by Beal's Method. Max $Z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2,$

$subject\ to\ x_1 + 2x_2 + x_3 = 10,\ x_1 + x_2 + x_4 = 9,$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0$

ii) Solve the following all integer programming problem using the branch and bound Method Max Z = $3x_1 + 5x_2\ \ subject\ to\ 2x_1 + 4x_2 \leq 25,\ 2x_2 \leq 10,$

$x_1 \leq 8\ and\ x_1 \geq 0, x_2 \geq 0\ and\ integers.$

Date : 25/04/2022

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
M. Sc. I Sem II and M.Sc. II Sem IV
Internal Examination 2021-22

All the students of M.Sc. I and M.Sc. II are hereby informed that their Internal Examination of Mathematics will be conducted on **as given below timetable.** The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus and timetable for examination will be as mentioned in following table.

Syllabus for M. Sc. I Sem. II

Sr.No.	Name of Paper	Topics
1	Linear Algebra (CP-1175B)	Unit I
2	Integral Equation (CP-1176B)	Unit I
3	General Topology (CP-1177B)	Unit I
4	Partial Differential Equations (CP-1178B)	Unit I
5	Numerical Analysis (CP-1179D)	Unit I

Syllabus for M. Sc. II Sem. IV

Sr. No.	Name of Paper	Topics
1	Field Theory (CP-1190D)	Unit I
2	Integral Equation (CP-1191D)	Unit I
3	Algebraic Number Theory (CP-1192D)	Unit I
4	Operational Research II(CP-1194D)	Unit I
5	Combinatorics (CP-1198D)	Unit I

Timetable

Date	Time	Class	Subject
04/05/2022	03:00 PM to 04: 00 PM	M.Sc. I	Linear Algebra (CP-1175B)
	03:00 PM to 04: 00 PM	M.Sc. II	Field Theory (CP-1190D)
05/05/2022	03:00 PM to 04: 00 PM	M.Sc. I	Integral Equation (CP-1176B)
	03:00 PM to 04: 00 PM	M.Sc. II	Integral Equation (CP-1191D)
06/05/2022	03:00 PM to 04: 00 PM	M.Sc. I	General Topology (CP-1177B)
	03:00 PM to 04: 00 PM	M.Sc. II	Algebraic Number Theory (CP-1192D)
07/05/2022	03:00 PM to 04: 00 PM	M.Sc. I	Partial Differential Equations (CP-1178B)
	03:00 PM to 04: 00 PM	M.Sc. II	Operational Research II(CP-1194D)
09/05/2022	03:00 PM to 04: 00 PM	M.Sc. I	Numerical Analysis (CP-1179B)
	03:00 PM to 04: 00 PM	M.Sc. II	Combinatorics (CP-1198D)

Nature of question paper

Time:-1 Hours **Total Marks: 30**

Q.1) Choose the correct alternative for the following question. [05]

Five questions

Q.2) Attempt any three [15]

Four questions

Q.3) Attempt any One [10]

Two questions

VIVEKANAND COLLEGE ESTD. JUNE 1964 KOLHAPUR-416 003

(Prof. S. P. Patankar)
HEAD
Department of Mathematics
Vivekanand College, Kolhapur

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II Internal Examination: 2021-22
MATHEMATICS

Time: 03: 00 PM-04:00pm **Subject: Linear Algebra**
Date: 04/05/2022 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i) Let V denote the vector space of n × n symmetric matrices, over R. Then dim V as a vector space over R is

A) n^2 B) $(n^2 + n)/2$ C) $n^2 + n$ D) $n^2 - n$

ii) T1, T2 and T3 are three maps defined on R^3,

$T1\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+1 \\ y \\ z \end{bmatrix}$, $T2\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} xy \\ z \end{bmatrix}$, $T3\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ y+z \\ z+x \end{bmatrix}$ as which of these maps are linear?

A) T1, T2, T3 B) T1, T2 C) T3 D) T1, T3

iii) $W^{\perp\perp}$ = …….(with usual notation)

A) $W^{\perp}$ B) W C) F D) V

iv) $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ then Jordan form of A with minimal polynomial x^3 is……

A) diag[$J_3(0)$] B) diag[$J_2(0)$] C) diag[$J_3(1)$] D) diag[$J_1(0)$]

v] If α_1v1+α_2v2+ …+$\alpha_n$vn=0, where v1, v2, … ,vn are linearly independent vectors in a vector space V(F), then ________.

i) i=0 for all i=1,2, …, n
ii) i≠0 for all i=1,2, …, n
iii) i=0 for at least one i
iv) i≠0 for at least one i

Q.2. Attempt any three of the following: **[15]**

1.An non- empty subset A of vector space V (F) then linear span of A i.e L(A) is subspace of V.

2.S ⊆ T then i. L(S) ⊆ L(T) ii. L(L(S)) = L (S) iii. L(SUT) = L (S) + L (T)

3. $v_1 , v_2 , … v_n \in V$ are linearly independent then every element in their linear span has a unique representation in the form $a_1v_1 + a_2v_2 … + a_nv_n$, $a_i \in F$.

4. $v_1 , v_2 , … v_n \in V$ are either linearly independent or some v=k= is a linear combination of preceding once $v_1 , v_2 , … v_{k-1}$

5. If A and B are finite dimensional subspace of vector space V then

Dim(A + B) = dim(A) + dim(B) – dim(A∩B)

Q.3. Attempt any one of the following: **[10]**

1] State and prove Rank-nullity theorem

2] If V is internal direct sum of U 1 , U 2 ,…U n ,Then prove that V is isomorphic to external direct sum of U 1 , U 2 ,…U n

Vivekanand College, Kolhapur (Autonomous)

M.Sc. (Part-I) Semester-II

Internal Examination : 2021-22

Sub: Integral Equations **Date : 05/05/2022**

Total Marks: 30 **Time : 03:00pm-04:00pm**

Q.1) Choose the correct alternative for the following question. [05]

1) The homogeneous Fredholm integral equation has infinite number of solution, if ---
   a) $D(\lambda) = 0$ b) $D(\lambda) \neq 0$ c) $D(\lambda) > 0$ d) $D(\lambda) < 0$
2) The type of integral equation $g(s) = f(s) + \lambda \int K(s,t)g(t)dt\ s\ a$ is -----
   a) Fredholm integral equation of 1st kind
   b) Volterra integral equation of 1st kind
   c) Homogeneous Volterra integral equation of 2nd kind
   d) Non-homogeneous Volterra integral equation of 2nd kind
3) The eigen values of non-zero symmetric kernel are -----
   a) real b) zero c) only imaginary d) none of these
4)) Spectrum of symmetric kernel is always -----
   a) empty b) non-empty c) does not exist d) none of these
5) A symmetric kernel possesses ----- eigen value.
   a) only one b) at-least one c) at-most one d) none of these

Q.2) Attempt any three [15]

1) Convert the following boundary value problem to an integral equation.

   $y'' + \lambda y = 0,\ y(0) = 0,\ y(l) = 0,\ 0 \le x \le l$
2) Find the eigen values and eigen functions of the homogeneous integral equation $g(s) = \lambda \int_0^1 (6s - 2t)g(t)dt$
3) Convert the following boundary value problem to an integral equation. $y'' + xy = 1$, $y(0) = 0,\ y(1) = 1,\ 0 \le x \le 1$
4) Prove that eigen functions $g(s)$ and $\psi(s)$ corresponding to distinct eigen values λ_1 and λ_2 respectively of the homogeneous integral equation $g(s) = \lambda \int K(s,t)g(t)dt$ and its transpose are orthogonal.

Q.3) Attempt any One [10]

1) Describe the procedure of finding eigen values and eigen functions for the homogeneous Fredholm integral equation of 2nd kind with separable kernel.
2) Solve the integral equation $g(s) = f(s) + \lambda \int_0^{2\pi} \cos(s + t)\, g(t)dt$ by discussing all possible cases.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II
Internal Examination(2021-22)

Subject: General Topology **Total Marks: 30**
Date: 06/05/2022 **Time: 03:00 PM to 04:00**

Q.1) Choose the correct alternative for the following question. [05]

1) If (X, τ) is a topological space and $Y \subset X$ and (Y, τ_Y) is relative topology and $A \subseteq Y$. Then

a) $\text{int}_X(A) \subseteq \text{int}_Y(A)$ b) $\text{int}_X(A) \supseteq \text{int}_Y(A)$

c) $\text{int}_X(A) = \text{int}_Y(A)$ d) none of them

2) In Discrete topology , (X, D) is separable if and only if X is ……………..

a) uncountable b) countable c) infinite d) finite

3) Which of the following property is not hereditary property?

a) Discreteness b) indiscreetness c) separability d) T_1 space

4) In discrete topology, set of limit point of any subset A of X is………….

a) $\emptyset$ b) A c) $X - A$ d) none of them

5) If $\tau = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{2,3,4\}, X\}$ is topology on $X = \{1, 2, 3, 4\}$ and $f: X \to X$ be defined by

$f(1) = 2$, $f(2) = 4$, $f(3) = 2$ and $f(4) = 3$, then

a) f is continuous at 3 b) f is discontinuous at 4
c) f is continuous at 1 d) f is continuous at 2

Q.2) Attempt any three [15]

1) If X be an infinite set and $\tau = \{\emptyset\} \cup \{A \subseteq X | A^C \textit{ is } \text{countable}\}$ then show that τ is topology on X.
2) Define the following terms:
   a) Limit point b) closure of set c) interior set d) neighbourhood
3) Show that $A \cup D(A)$ is closed set
4) Show that a mapping f of a space X onto another space Y is continuous if and only if $\overline{f^{-1}(B)} \subseteq f^{-1}(\overline{B})$

Q.3) Attempt any One [10]

1) Prove that if (X, τ) and (Y, V) are topological space and f is bijective mapping from $X \to Y$ thenf is homeomorphism if and only if f is continuous and closed.
2) Consider the topology $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ on $X = \{a, b, c\}$ and $V = \{\emptyset, \{r\}, \{p\}, \{q\}, Y\}$ on $Y = \{p, q, r\}$.
   Which of the following mapping is a) continuous b) open c) closed d)homeomorphism
   $i)\ f(a) \to r, f(b) \to r, f(c) \to r$
   $ii)\ g(a) \to p, g(b) \to q, g(c) \to r$
   $iii)\ h(a) \to r, h(b) \to p, h(c) \to q$

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II
Internal Examination(2021-22)
Partial Differential Equations

Time:3:00PMto4:00PM **Total Marks: 30**
Date :07/05/2022

Q.1) Choose the correct alternative for the following question. [05]

1) *The normals to the two surfaces represented by the equations* $Pdx+Qdy+Rdz=0$ &$Pp+Qq=R$ *are…*
   a) *collinear* b) *Orthogonal* c) *Parellel* d) *intersects at acute angle*
2) *The equation* $(x^2+z^2)p-xyq = z^3x$ *is*
   a) *Linear* b) *semilinear* c) *Quasilinear* d) *Nonlinear*
3) *The complete integral of* $z=px+qy+\sqrt{pq}$ *is*
   a) $z=a+b+ab$ b) $z=ax+by+\sqrt{pq}$ c) $z=c$ d) *none of these*
4) *The equation…represents the set of all right circular cones with x−axis as the axis of symmetry.*

   a) $(x^2 + y^2) = (z - c)^2tan^2(\alpha)$ b) $(x^2 - y^2) = (z - c)^2tan^2(\alpha)$

   c) $(z^2 + y^2) = (x - c)^2tan^2(\alpha)$ d) $(x^2 + z^2) = (y - c)^2tan^2(\alpha)$
5) *The complete integral of* $z=px+qy+pq$ *is*
   a) $z=a+b+ab$ b) $z=ax+by+ab$ c) $z=c$ d) *none of these*

Q.2) Attempt any three [15]

1) *Find the general solution of* $z(xp - yq) = y^2 - x^2$.

2) *Obtaine pde by eliminating a,b from* $z = ax^2 + by^2 + c$

3) *Find the general solution of* $p + q = 2\sqrt{z}$.

4) *Find the general integral of* $(x^2 + y^2)p+2xyq=(x+y)z$.

Q.3) Attempt any One [10]

1) *Solve Pfaffian differntial equation*

$$(6x + yz)dx + (xz - 2y)dy + (xy + 2z)dz = 0$$

2) *A tightly stretched string with fixed end point* $x=0$, $x=1$ *initially in a position given by*

$y(x,0)=x(1-x)$ *it released from rest position find the displacement* $y(x,t)$*at any time*

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II
Internal Examination(2021-22)

Subject : Numerical Analysis **Total Marks: 30**
Date: 09/05/2022 **Time: 03:00PM to 04:00PM**

Q.1) Choose the correct alternative for the following question. **[05]**

1) If f(x) is continuous function in the interval $[a, b]$ & f(a).f(b) < a , then the equation f(x) = 0 has at least one real root or an odd number of real roots in (a,b) is known as ------
A) Bisection Method B) Iterative Method
C) Direct Method D) Intermediate Value Theorem

2) If $\{x_k\}$ is convergent sequence i.e. $\lim_{k\to\infty}\{x_k\} = x^*$ is root of f(x) = 0 and x_k is called ------- of f(x).
A) Order B) Approximate root C) Zero D) Convergence

3) If eigen values are not same $\left(i.e\ \lambda_j \neq \lambda_i\right)$ then the corresponding eigen vectors are -----
A) Parallel B) Perpendicular C) Symmetric D) Distinct

4)The largest eigen value in modulus of a square matrix A cannot exceed , the largest sum of the module of the elements along any row and column ,is known as -----
A) Brauer Theorem B) Gerschgorin Theorem
C) Intermediate Value Theorem D) None of these

5) The value of integral using Gauss – Legendre one point formula
$I = \int_0^2 \frac{dx}{3+4x}$ is ----
$A)\ \frac{2}{7}\quad B)\ \frac{3}{7}\quad C)\ \frac{2}{5}\quad D)\ \frac{7}{2}$

Q.2) Attempt any three **[15]**

1) Determine the rate of convergence of Regula Falsi method.
2) Explain order conditions of fourth order Runge-Kutta method by using rooted tree.
3) Estimate the eigen value of the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ by using Gerschgorin theorem and Brauer theorem.
4) Explain General form of linear multistep method

Q.3) Attempt any One **[10]**

1) Explain Adams Moulton method of second order
2) Describe third order Runge- Kutta method

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV Internal Examination: 2021-22
MATHEMATICS

Subject : Field Theory | **Time: 03: 00 PM-04:00pm**
Date: 04/05/2022 | **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i. e and π are ……… elements over Q

A) Transcendental B) Algebraic C) Irreducible D) Reducible

ii. Polynomial of degree one is always ……

A) Inseparable B) Separable C) Monic D) Simple

iii. If $f(x)$ is of degree 3 , Then $f(x)$ has …. Root.

A) Complex B) Unique C) Distinct D) Real

iv. Any subgroup and any quotient group of a ……… group is solvable.

A)Solvable B) Normal C) Separable D) None

v. If $F \subseteq K \subseteq L$ are fields. If $a \in L$ be algebraic over K and K is an algebraic extension of F. Then, a is …….

A) Algebraic over K B) Algebraic Over F C) Algebraic D) Separable

Q.2. Attempt any three of the following: **[15]**

1. If f(x) be a nonconstant polynomial over a field F and If E and K be two splitting fields of f(x) over F. Then prove that, there exist an isomorphism $\sigma : E \rightarrow K$ which is identity on F
2. Find the splitting field of $x^3 - 2$ over the field Q of rational number and its degree over Q.
3. If $f(x) = x^2 + 3$ and $g(x) = x^2 + x + 1$ be polynomial over Q. Prove that their splitting fields are equal and find it's degree over Q .
4. For any prime p, find the splitting field of $x^p - 1$ over Q and it's degree over Q.

Q.3. Attempt any one of the following: **[10]**

1. Prove that Following are equivalent to each other for any field K
   - i. K is algebraically closed
   - ii. Any non constant polynomial over K can be factored completely into linear factors in K [x]
   - iii. If f(x) is a non constant polynomial over K, then all the roots of f(x) belongs to K.
   - iv. Every nonconstant polynomial over K has atleast one root in K.
2. Prove that Any field has an algebraically closed extension.

Vivekanand College, Kolhapur (Autonomous)

M.Sc. (Part-II) Semester-IV

Internal Examination : 2021-22

Sub: Integral Equations **Date : 05/05/2022**

Total Marks: 30 **Time : 03:00pm-04:00pm**

Q.1) Choose the correct alternative for the following question. [05]

1) The homogeneous Fredholm integral equation has infinite number of solution, if ---
   a) $D(\lambda) = 0$ b) $D(\lambda) \neq 0$ c) $D(\lambda) > 0$ d) $D(\lambda) < 0$
2) The type of integral equation $g(s) = f(s) + \lambda \int K(s,t)g(t)dt\ s\ a$ is -----
   a) Fredholm integral equation of 1st kind
   b) Volterra integral equation of 1st kind
   c) Homogeneous Volterra integral equation of 2nd kind
   d) Non-homogeneous Volterra integral equation of 2nd kind
3) The eigen values of non-zero symmetric kernel are -----
   a) real b) zero c) only imaginary d) none of these
4)) Spectrum of symmetric kernel is always -----
   a) empty b) non-empty c) does not exist d) none of these
5) A symmetric kernel possesses ----- eigen value.
   a) only one b) at-least one c) at-most one d) none of these

Q.2) Attempt any three **[15]**

1) Convert the following boundary value problem to an integral equation.

   $y'' + \lambda y = 0,\ y(0) = 0,\ y(l) = 0,\ 0 \leq x \leq l$
2) Find the eigen values and eigen functions of the homogeneous integral equation $g(s) = \lambda \int_0^1 (6s - 2t)g(t)dt$
3) Convert the following boundary value problem to an integral equation. $y'' + xy = 1$, $y(0) = 0,\ y(1) = 1,\ 0 \leq x \leq 1$
4) Prove that eigen functions $g(s)$ and $\psi(s)$ corresponding to distinct eigen values λ_1 and λ_2 respectively of the homogeneous integral equation $g(s) = \lambda \int K(s,t)g(t)dt$ and its transpose are orthogonal.

Q.3) Attempt any One **[10]**

1) Describe the procedure of finding eigen values and eigen functions for the homogeneous Fredholm integral equation of 2nd kind with separable kernel.
2) Solve the integral equation $g(s) = f(s) + \lambda \int_0^{2\pi} \cos(s + t)\, g(t)dt$ by discussing all possible cases.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV
Internal Examination(2021-22)
Algebraic Number Theory

Time: 3:00PM–4:00PM **Total Marks: 30**
Date: 06/05/2022

Q.1) Choose the correct alternative for the following question. [05]

1) Let R be a ring. The associate of additive identity element 0 is ....
   (a) $\{0\}$ (b) $R - \{0\}$
   (c) R (d) None of the above

2) Let D be a integral domain.
   P : Cancellation law does not hold in D.
   Q : $x \in D$ is a unit if and only if $1 \mid x$.

   (a) P is true and Q is false (b) P is false and Q is true
   (c) P and Q are false (d) P and Q are true

3) Let $x, y \in D$. x and y are associates then which of the following conditions satisfies
   (a) $x = yu$ for $u \in D$ (b) $xy = u$ for u is a unit in D
   (c) $x \mid y$ and $y \mid x$ (d) All of the above

4) Let $x \in D$ and D is a integral domain.
   P : x is irreducible if and only if every divisor of x is an associate of x or a unit Q : an associate of an irreducible is irreducible.

   (a) P is true and Q is false (b) P is false and Q is true
   (c) P and Q are false (d) P and Q are true

5) Let $e_1 = (1,0)$ and $e_2 = (0,1)$ are two elements of the additive group $(R^2,+)$. Find the lattice of dimension 2 generated by e_1 and e_2?
   (a) $\{(0,0)\}$ (b) $\{e_1, e_2\}$ (c) $Z \times Z$ (d) R^2

Q.2) Attempt any three [15]

1) If A and B are non-zero ideals of O then show that N(AB)=N(A)N(B)
2) With usual notations prove that The Field polynomial f_α is a power of minimal polynomial p_α
3) Suppose $\alpha 1, \alpha 2, \dots \alpha n \in O$ for a Q basis for K if $\Delta[\alpha 1, \alpha 2, \dots \alpha n]$ is square free then show that $\{\alpha 1, \alpha 2, \dots \alpha n\}$ is an integral basis.
4) Show that the coefficient of the field polynomial are rational numbers so that

   $f_\alpha(t) \in Q(t)$.

Q.3) Attempt any One **[10]**

1) Show that the ring of integers O of $Q(\zeta)$ is $Z(\zeta)$

2) Let d be a square free rational integer then the integer if $Q(\sqrt{d})$ are

a) $Z(\sqrt{d})$ $if\ d \not\equiv 1(mod4)$

b) $Z\left(\frac{1}{2}+\frac{1}{2}\sqrt{d}\right)$ $if\ d \equiv 1(mod4)$.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV
Internal Examination(2021-22)
Operational Research-II

Time: 03:00PM to 04:00 PM **Total Marks: 30**
Date :07/05/2022

Q.1) Choose the correct alternative for the following question. [05]

i) The present worth factor of one rupee spent in n years with r interest rate is given by.......

A) $\frac{1}{1+r}$ B) $\frac{1}{(1+r)^n}$ C) $\frac{1}{(1+r)^{-n}}$ D) None of these

ii) A dummy activity is used in network diagram, when...

A) two parallel activities have the same tail and head event

B) the chain of activities may have a common event, yet be independent by themselves

C) both A and B

D) None of these

iii) In .... customers are moving from one queue to another queue to receive service more quickly

A) Balking B) Reneging C) Jockeying D) None of these

iv) In ....inventory the rate of consumption is the same as the rate of production so that the items are produced in large quantity than they are required

A) Fluctuating B) Cycle C) Transportation D) Anticipation

v) In model I(b), minimum average inventory cost is...........

A) $\sqrt{\frac{2C_1C_3q}{T}}$ B) $\sqrt{\frac{2C_1C_3D}{T}}$ C) $\sqrt{2C_1C_3D}$ D) None of these

Q.2) Attempt any three **[15]**

i) Let the value of money be assumed to be 10% per year and suppose that machine A is replaced after every 3 years whereas machine B is replaced after every six years. The yearly costs of both the machine are given as under:

Year	1	2	3	4	5	6
Machine A	1000	200	400	1000	200	400
Machine B	1700	100	200	300	400	500

Determine which machine should be purchased.

ii) You have to supply your customers 100 units of a certain product every Monday (and only then). You obtain the product from a local supplier at Rs. 60 per unit. The costs of ordering and transportation from the suppliers are Rs. 150 per order. The costs of carrying inventory is estimated at 15% per year of the cost of the product carried

a) Find the lot size which will minimize the cost of the system

b) Determine the optimal cost

iii) Explain the concept of EOQ.

iv) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate the following

i) the mean queue size

ii) the probability that the queue size exceeds 10

if the input of trains increase to an to an average 33 per day, what will be the change in i) and ii)

Q.3) Attempt any One **[10]**

i) Derive the optimal economic lot size formula q = $\sqrt{\frac{2C_3RK}{C_1(K-R)}}$ in the usual notations when the rate of replenishment is finite. Also, derive the minimum cost formula.

ii) Explain the costs involved in inventory problems in detail.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV Internal Examination: 2021-22
MATHEMATICS

Subject: Combinatorics **Time: 03: 00-04:00pm**
Date: 09/05/2022 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

I. *The coefficient of* x^2 *in the expansion of* $(1-x)^{-2}$ *is*

a) 1 b) 2 c) 3 d) 4

II. *The weight of permutation* $(1,3,2,4) \in S_4$ *is*

a) 2 b) 4 c) 6 d) 8

III. *The number of derangements of* $(1,2,3)$ *is/are* _____

a) 1 b) 2 c) 3 d) none of these

IV. *The number of circular permutation of* 6 *objects is*

a) 120 b) 24 c) 6 d) 5

V. *The Ramsey Number* $R(3,3) =$ ______

a) 6 b) 5 c) 4 d) 0

Q.2. Attempt any three of the following: **[15]**

i) *State & prove principle of inclusion and exclusion for n finite sets.*

ii) *For every positive integer n prove that* $\sum_{r=0}^{\infty} \binom{n}{r}^2 = \binom{2n}{n}$

iii) *Solve* $a_r = 10a_{r-1} - 9a_{r-2}$ *with initial conditions* $a_0 = 3 \& a_1 = 11$

iv) *With usual notations show that* $C_n = \frac{1}{n}\binom{2n-2}{n-1}$

Q.3. Attempt any one of the following: **[10]**

i) *Find a cycle index of dihedral group on symmetries of square*

ii) *Find a cycle index of dihedral group on symmetries of triangle*

Shri Swami Vivekanand Shikshan Sanstha`s

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

2130, E Ward, Tarabai Park, Kolhapur, Maharashtra 416003

Subject Wise Student Blank Marks Entry

Session: JAN-FEB 2022 **Subject:** NUMBER THEORY (CBP-1183C)

Stream: M.Sc.(Maths)

Standard: M.SC. (MATHS) SEM 3 **Sub-Subject:** CIE

Semester: **Max Marks:** 30

Print Date : 10-03-2022

SrNo	PRN	SeatNo	GRNos	RollNo	StudentName	Marks
1	2020121201	931001	2830718	2201	AUTADE PRAGATI PRABHAKAR	24
2	2020121202	931002	2836021	2202	BATE SONALI SHANKAR	24
3	2020121203	931003	2830924	2203	BHOSALE SAKSHI VIJAY	26
4	2020121205	931004	2838313	2204	CHOUGULE ASMITA ADINATH	28
5	2020121206	931005	2826835	2205	DEOKARE VIPUL VIJAY	19
6	2020121207	931006	2826658	2206	DURUGALE SHARAYU DINKAR	26
7	2020121208	931007	2828976	2207	GOLIWADEKAR MRUDULA GURUNATH	28
8	2020121209	931008	2830745	2208	INGALE AAKANKSHA AJIT	28
9	2020121210	931009	2827130	2209	JADHAV ASHWINI ASHOK	12
10	2020121211	931010	2858518	2210	JAMBONI SHIVARATNA SUNIL	22
11	2020121212	931011	2826631	2211	KADAM VEDIKA SANJAY	26
12	2020121213	931012	2827643	2212	KAMBLE MANISHA BHIMRAO	27
13	2020121215	931013	2827029	2213	KAMBLE SHUBHAM TANAJI	12
14	2020121216	931014	2827024	2214	KHATKAR DIGVIJAY ASHOK	19
15	2020121217	931015	2827198	2215	KHOCHAGE SHRUTI SUNIL	26
16	2020121218	931016	2830830	2216	KOLEKAR SHIVANI TANAJI	20
17	2020121220	931017	2830712	2217	MANE PRATIBHA NARAYAN	21
18	2020121221	931018	2830440	2218	NEMISHTE RUTURAJ BHARAT	19
19	2020121224	931019	2830808	2219	PATIL KAJAL AMAR	26
20	2020121225	931020	2830446	2220	PATIL PRUTHVIRAJ VIKAS	26
21	2020121226	931021	2825790	2221	RUTUJA TANAJI PATIL	26
22	2020121227	931022	2835604	2222	PATIL SHARAD DHANAJI	12
23	2020121229	931023	2828866	2223	PATIL VIDULA MILIND	26
24	2020121230	931024	2830434	2224	PATIL VIKAS MARUTI	19
25	2020121231	931025	2857922	2225	PISHTE REVATI SHRIDHAR	26
26	2020121232	931026	2830922	2226	REGADE POONAM PUNDALIK	25
27	2020121233	931027	2826375	2227	SANKPAL SONALI SARJERAO	27
28	2020121234	931028	2828989	2228	SATHE ANKITA MAHIPATI	28
29	2020121235	931029	2858522	2229	SHELAKE ABHIJEET BHAGAVAN	17
30	2020121236	931030	2836026	2230	SOLAPURE MRUNALI MAHADEV	26
31	2020121237	931031	2825584	2231	SUTAR SHIVANI ANIL	28
32		931032	2921033	2232	TAMBE ABHISHEK APPASAHEB	17

Shri Swami Vivekanand Shikshan Sanstha`s

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

2130, E Ward, Tarabai Park, Kolhapur, Maharashtra 416003

Subject Wise Student Blank Marks Entry

Session: JAN-FEB 2022 **Subject:** LATTICE THEORY (CBP-1182C)

Stream: M.Sc.(Maths)

Standard: M.SC. (MATHS) SEM 3 **Sub-Subject:** CIE

Semester: **Max Marks:** 30

Print Date : 10-03-2022 **Page No :**Page 1 of 1

SrNo	PRN	SeatNo	GRNos	RollNo	StudentName	Marks
1	2020121201	931001	2830718	2201	AUTADE PRAGATI PRABHAKAR	15
2	2020121202	931002	2836021	2202	BATE SONALI SHANKAR	24
3	2020121203	931003	2830924	2203	BHOSALE SAKSHI VIJAY	24
4	2020121205	931004	2838313	2204	CHOUGULE ASMITA ADINATH	15
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23	2020121229	931023	2828866	2223	PATIL VIDULA MILIND	27
24	2020121230	931024	2830434	2224	PATIL VIKAS MARUTI	24
25	2020121231	931025	2857922	2225	PISHTE REVATI SHRIDHAR	15
26	2020121232	931026	2830922	2226	REGADE POONAM PUNDALIK	18
27	2020121233	931027	2826375	2227	SANKPAL SONALI SARJERAO	30
28	2020121234	931028	2828989	2228	SATHE ANKITA MAHIPATI	30
29	2020121235	931029	2858522	2229	SHELAKE ABHIJEET BHAGAVAN	21
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VIVEKANAND COLLEGE ESTD. JUNE 1964 KOLHAPUR-416003

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