Oneness of Natural Number: Properties of Multiples of 3

4 authors, including:

Mehul Arun Jadhav
The New College Kolhapur

Sanjay Pandurang Thorat
Vivekanand College, Kolhapur

Impact Factor - 6.293 ISSN-2349-638x

Aayushi International Interdisciplinary Research Journal (AIIRJ)

PEER REVIEWED & INDEXED JOURNAL

February – 2020

Special Issue No. 68

Editors

Mr.Sarjerao Sadashiv Chile

Dr.Shreya Vinay Patil

Dr. Ashish Shivaji Sartape

Chief Editor

Pramod P. Tandale

IMPACT FACTOR

SJIF 6.293

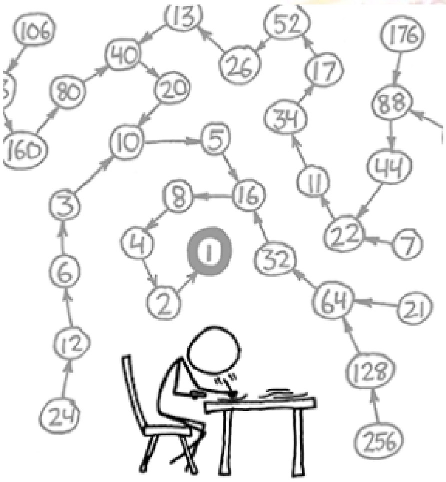
For details Visit our website

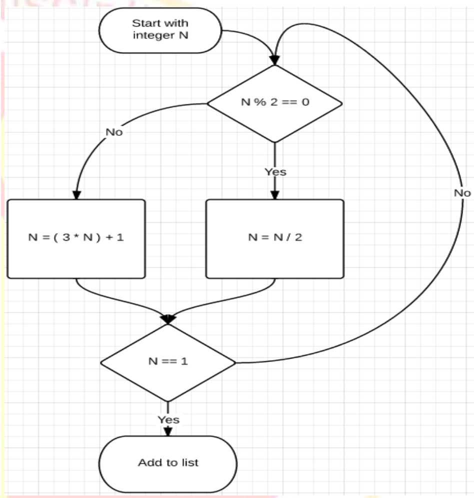
www.aiirjournal.com

Oneness of Natural Number: Properties of Multiples of 3

Mr. M. A. Jadhav[1], Mr. S. P. Patankar[2], Mr. S. P. Thorat[3] and Ms. S. A. Ingale[4]
[1]Research Student and Asst. Prof., Department of Computer Science(Entire), The New College, Kolhapur.
[2]Head and Asso. Prof., Department of Mathematics, Vivekanand College, Kolhapur.
[3] Head and Asso. Prof., Department of Mathematics, D. K. A. S. C. College, Ichalkaranji.
[4] Research Student, Department of Computer Science(Entire), The New College, Kolhapur.

Abstract:
The aim of this paper is to study the properties of natural numbers which are multiple of 3 using calculation of oneness factor, hit factor or convergent factor of every natural number with hailstone sequence to reach one.
Keywords: $3n + 1$ *conjecture, Collatz conjecture, Collatz function, Hailstone sequence, Hasse algorithm, hit factor, Kakutani's problem, Thwaites conjecture, Ulam conjecture.*





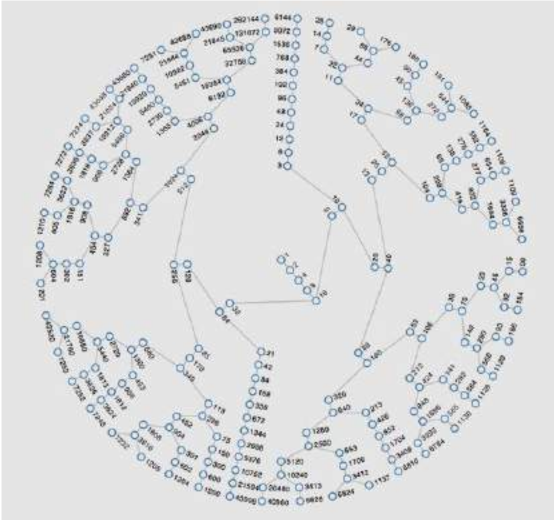
after Lothar Collatz, who first proposed it in 1937. The conjecture is also known as the 3n + 1 conjecture, the Ulam conjecture after Stanisław Ulam, Kakutani's problem after Shizuo Kakutani, the Thwaites conjecture after Sir Bryan Thwaites, Hasse's algorithm after Helmut Hasse, or the Syracuse problem; the sequence of numbers involved is referred to as the hailstone sequence or hailstone numbers because the values are usually subject to multiple descents and ascents like hailstones in a cloud, or as wondrous numbers. The eminent mathematician Paul Erdos suggested: "Mathematics is not ready for this kind of problem".

One of the most tantalizing conjectures in number theory is the so called $3n + 1$ conjecture, stated by L. Collatz (1937). The problem can be simply stated as, starts with any positive integer. If it is even number, halve it (which has been called "Half Or Triple Plus One", or **HOTPO**). Otherwise multiply it by 3 and add 1 to it. Take the result and repeat the process. Any such sequence seems to end up at one. The conjecture remain unanswered, although it has been proven that the process terminates for all values of n up to 5.764×10^{18}.
Let $f: N \to N$ be Collatz function defined as:

$$f(x) = \begin{cases} \frac{x}{2}, & if\ x\ is\ even \\ 3x + 1, & if\ x\ is\ odd \end{cases}$$

Collatz conjecture states that if $x \in \mathbb{N}$, then the sequence $x, f(x), f \circ f(x), f \circ f \circ f(x), \cdots$, reaches to 1. If $x = 21$ then sequence produced is,

Step	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Value	21	64	32	16	8	4	2	1



The sequence has no obvious pattern, and no explanation that why the sequence should take 7 iterations to reach 1. When $x = 27$ it takes 111 steps. Hence the number of iterations is not proportional to the magnitude of the starting number. And hence remain unsolved yet.

Oneness Of Natural Number, Oneness Factor/Hit Factor:-

Definition 2.1:- (Oneness of Natural Number) The ability of a natural number reaching to 1 with collatz conjecture function f is called as oneness of natural number.

Definition 2.2:- (Oneness factor or Hit Factor or Convergent factor or Stoppage Time of Natural Number) The conjecture asserts that every natural number n has a well-defined Hit Factor. Total number of steps required by a natural number n to reach 1 using $f(x)$ is called as Total stoppage time, oneness factor of n.

Definition 2.3:- (Hit Factor Function) Let $T: \mathbb{N} \to \mathbb{N}$ be Hit Factor function defined over set of natural number as the total number steps needed to reach 1.

2.3.1 Hit Factor of Natural Number 2^n:

$T(2^n) = n.$

e.g. $T(1) = 0,\ T(2) = 1,\ T(64) = 6.$

2.3.2 Hit Factor of odd Natural Numbers having $n + 1$ hit factor :

Now we are familiar to the term than even number decreases, so we just emphasize on odd numbers. To find out odd number which transform to 2^n.

2.3.1 Result: If k be any natural number then $3 | 4^k - 1$.

2.3.2 Result: If a be a odd natural number and $r = 2k$ be the power of 2 then $3a + 1 = 2^r$ and $a = (4^{k-1} + 4^{k-2} + \cdots + 4 + 1)$.

2.3.3 Result: The sequence of odd numbers a such that $3a + 1 = 4^n$ are represented by recurrence relation, $O_n = 4O_{n-1} + 1$, with initial condition $O_1 = 1$.

2.3.4 Result: The sequence of odd numbers $a = 2k + 1, k \geq 0$ is an integer such that $6k + 4 = 4^n$ are represented by recurrence relation, $K_n = 4K_{n-1} + 2$, with initial condition $K_1 = 0$ and K_n is even.

2.3.5 Result: $K_{n+1} = 2 + 2^3 + 2^5 + \cdots + 2^{2n-1},\ K_{n+1} - K_n = 2^{2n-1}$, where $n \geq 1$.**2.3.6 Result:** $2\,O_{n+1} = K_{n+2}$ and $O_{n+1} - O_n = 4^n, n \geq 1$.

From Result 2.3.2 it is clear that none of the odd number converts to odd power of 2, the odd number which converts to even power of 2 is of the form$a = (4^{k-1} + 4^{k-2} + \cdots + 4 + 1), k \in \mathbb{N}$ and hence we populate these number as,

Set	O_1	O_2	O_3	O_4	O_5	O_6	O_7	. . .
Base Value	4^1	4^2	4^3	4^4	4^5	4^6	4^7	
First Value	1	5	21	85	341	1365	5461	. . .
Hit Factor	0	$4+1$	$6+1$	$8+1$	$10+1$	$12+1$	$14+1$	

Above elements are generated using recurrence relation in Result 2.2.3, and solution of this recurrence relation is $O_n = \sum_{k=0}^{n-1} 4^k$.

2.3.7 Result: From above table it is clear that every Natural number using Collatz function moves like a hailstone(i.e. increases, decreases) then first converges to 4^k and then onwards it decreases and reaches to 1.

3. Multiples of 3:

We state that none of the odd natural number n converges to multiple of 3 natural number using Collatz function.

3.1. Result: As $3|4^n - 1$where $n \geq 1, p = (1 + 4 + 4^2 + \cdots + 4^{n-1})$ where $4^n - 1 = 3p$.

3.2. Result: $3|16^n + 4^n + 1$where $n \geq 1$.

Proof:

We have

$16 = 15 + 1$ then $16^n = (15 + 1)^n = 15k_1 + 1$.

$4 = 3 + 1$ then $4^n = (3 + 1)^n = 3k_2 + 1$

Hence, $16^n + 4^n + 1 = 15k_1 + 1 + 3k_2 + 1 + 1 = 3(5k_1 + k_2 + 1) = 3k$

This show that result is true.

3.3. Result: If $3n + 1 = 4^{3k}$ where $n \geq 1$ is an odd natural number and $k \geq 1$ then $3|n$.

Proof:

We have

$3n + 1 = 4^{3k}$

$\Rightarrow 3n = 4^{3k} - 1 = \left(4^k\right)^3 - 1^3 = \left(4^k - 1\right)\left(4^{2k} + 4^k + 1\right)$

$\Rightarrow 3n = 3k_1.3k_2 \Rightarrow n = 3k_1.k_2 \Rightarrow 3|n$

3.4. Result: $21|\left(1 + 4 + 4^2 + 4^3 + \cdots + 4^{3k-1}\right)$ where $k \geq 1$.

Proof:

We know that

$21|21 = 1 + 4 + 4^2 = 21.4^0$

Similarly,

$21|4^3 + 4^4 + 4^5 = 21.4^3$

$21|4^{3k-3} + 4^{3k-2} + 4^{3k-1} = 21.4^{3k-3}$

Hence,

$21|\left(1 + 4 + 4^2 + 4^3 + \cdots + 4^{3k-1}\right)$

3.5. Result: $3\left|4^k - 1, 7\right|4^k - 1, 9|4^k - 1, 21|4^k - 1$ where $k \geq 1$.

3.6. Result: If n is any even natural number and $3|n$ then there will be no odd natural number which converges to n.

Proof:

We have given that, $3|n$

contrary we suppose that,

$\exists\ p = 2k + 1 \ \in N \ such\ that\ 3p + 1 = n$

$3(2k + 1) + 1 = n \Longrightarrow 6k + 4 = n$

Here $3 \nmid 6k + 4$.

hence our assumption is wrong.

hence the given statement is true..

3.7. Result: If $3n + 1 = 4^{3k}$ where $n \geq 1$ is an odd natural number and $k \geq 1$ then $21|n$.

3.8 Result: The sequence of odd numbers a such that $3a + 1 = 4^{3n}$ are represented by recurrence relation, $O_n = 64O_{n-1} + 21$, with initial condition $O_1 = 21$.

Proof:

If $a = 21$ then $3a + 1 = 4^3$.

Let, $O_{n-1} = p, O_n = m$ are odd natural numbers such that $p < m$ and $3p + 1 = 4^{3n}, 3m + 1 = 4^{3n+3}$.

$$\frac{3m+1}{3p+1} = \frac{4^{3n+3}}{4^{3n}} \Rightarrow \frac{3m+1}{3p+1} = 4^3 \Rightarrow 3m + 1 = 4^3(3p + 1)$$

$\Rightarrow 3m + 1 = 3 \,.\, 4^3 p + 4^3 \Rightarrow 3m = 3 \,.\, 4^3 p + 4^3 - 1 \Rightarrow 3m = 3 \,.\, 4^3 p + 63$

$\Rightarrow m = 4^3 p + 21$

$\Rightarrow O_n = 64 O_{n-1} + 21$

Hence proved.

3.9 Result: The sequence of odd numbers $a = 2k + 1, k \geq 0 \; s.t. \, 3a + 1 = 4^{3n}$ is an integer such that $6k + 4 = 4^{3n}$ are represented by recurrence relation, $K_n = 64K_{n-1} + 42$, with initial condition $K_1 = 10$ and K_n is even.

Proof:

If $k = 10$ then $64 = 4^n \Rightarrow n = 3 \Rightarrow K_1 = 10$.

from Result 3.8 we have, $O_n = 64O_{n-1} + 21$

Since, O_n is odd natural number $O_n = 2K_n + 1$

$2K_n + 1 = 64(2K_{n-1} + 1) + 21 \Rightarrow K_n = 32(2K_{n-1} + 1) + 10$

$\Rightarrow K_n = 64K_{n-1} + 42$ and $2|K_n$

Hence proved.

3.10 Result: $K_n = 2 + 2^3 + 2^5 + \cdots + 2^{3(2n-1)}, K_{n+1} - K_n = 2^{2n+1}.21$, where $n \geq 1$.

Set	O_1	O_2	O_3	O_4	O_5	O_6	. . .
Base Value	4^3	4^6	4^9	4^{12}	4^{15}	4^{18}	
O_n	21	1365	87381	5592405	357913941	22906492245	. . .
K_n	10	682	43690	2796202	178956970	11453246122	

Conclusion:

It is clear that every Natural number using Collatz function moves like a hailstone(i.e. increases, decreases) then first converges to and then onwards it decreases and reaches to 1.

Here we conclude that no odd number will converges to multiple of 3 natural number.

If we try to find Hit factor for every natural number and if it is finite then we can say that Collatz conjecture is true for every natural number. If focus only on odd natural numbers then we can get results more fast.

Scope:

In graph theory we can describe it as "In every circuit free directed graph every node has path from node labeled with natural number to node labeled 1 which the root node.

References:

1) Chamberland, M.; A continuous extension of the 3x+1 problem to the real line. Dynamics of Continuous, Discrete and Impulsive Systems, 2:495-509, 1996.
2) David Applegate and Jeffrey C. Lagarias, The distribution of 3x + 1 trees, Experiment. Math. 4 (1995), no. 3, 193–209.
3) D.J. Bernstein, J.C. Lagarias, The 3x + 1 conjugacy map, Canadian Journal of Mathematics 48 (6) (1996) 1154-1169.
4) Guo-Gang Gao, On consecutive numbers of the same height in the Collatz problem, Discrete Math. 112 (1993), no. 1-3, 261–267.
5) Lagarias, J. C.; The 3x+1 problem and its generalizations. American Mathematical Monthly, 92:3-23,1985.
6) M.R. Feix, A. Muriel, J.L. Rouet, Statistical properties of an iterated arithmetic mapping, Journal of Statistical Physics 76 (1994) 725-741.

7) Motta, Francis C., Henrique R. De Oliveira, and Thiago A. Catalan. An Analysis of the Collatz Conjecture. Print.
8) Patankar S. P., Thorat S. P., Jadhav M. A.; Oneness Of Natural Number: Collatz Conjecture Journal Commerce, Business and Social Sciences (2015) Vol IV, 137-140
9) R.E. Crandall, On the 3x + 1 problem, Mathematics of Computation 32 (4) (1978) 1281-1292.
10) Riho Terras, A stopping time problem on the positive integers, Acta Arith. 30 (1976), no. 3, 241–252.