ONENESS OF NATURAL NUMBER: SQUARE SUM OF ODD AND ALL TOKENS

Mehul Arun Jadhav
The New College Kolhapur

Progression in Science, Technology and Smart Computing

~ Executive Editor ~
Dr. Bhaskar V. Tamhankar

~ Editor ~
Dr. Neha Joshi

PRARUP PUBLICATIONS

ISBN – 978-81-948061-2-7

PROGRESSION IN SCIENCE, TECHNOLOGY AND SMART COMPUTING

Executive Editor

Dr. Bhaskar V. Tamhankar

Editors

Dr. Neha Joshi

Dr. Suresh Kumbhar

Dr. Narsing Padal

Shri. Amar Mane

Shri. Uttam Harale

Miss. Smruti Kulkarni

ISBN – 978-81-948061-2-7

PROGRESSION IN SCIENCE, TECHNOLOGY AND SMART COMPUTING

1st Edition 2021

Executive Editor:
Dr. Bhaskar V. Tamhankar
Principal,
Willingdon College, Sangli. (M.S.)

Publisher:
Prarup Publications
Kolhapur, Maharashtra.

ISBN 978-81-948061-2-7

Typeset and Printing
Mauli Computers & Printers, Kolhapur

ISBN – 978-81-948061-2-7

Progression in Science, Technology and Smart Computing

Oneness of Natural Number: Square Sum Of Odd and All Tokens

Mr. M. A. Jadhav
Research Student,
Department of Computer Science,
Shivaji University, Kolhapur

Mr. S. P. Patankar
Head and Asso. Prof.,
Department of Mathematics,
Vivekanand College, Kolhapur

Mr. H. R. Kulkarni
Research Student,
Department of Computer Science,
Shivaji University, Kolhapur

Ms. S. P. Darak
Research Student,
Department of Mathematics,
Vivekanand College, Kolhapur

ABSTRACT - The aim of this paper is to optimize algorithm for Collatz Sequence which is exponential in nature and has factor 2. To study the properties of Collatz sequence of odd natural numbers and all natural numbers. Every sequence can be converted into a series. To check the sum of odd sequence results in a perfect square or not, sum of complete sequence results in square or not and at last, to check whether odd sequence consist of all odd terms in a arithmetic sequence or not.

Keywords— $3n + 1$ ***conjecture, Collatz conjecture, Collatz function , Hailstone sequence, Hasse algorithm, hit factor, HOTPO, Kakutani's problem, Thwaites conjecture, Ulam conjecture, Collatz Square Numbers, Collatz Odd Square Numbers, Collatz Partial Odd Square Numbers., Collatz Complete Square Numbers, Collatz Complete Odd Square Numbers, Collatz Pair Square.***

INTRODUCTION:

The Collatz-conjecture isa conjecture in mathematics named after Lothar Collatz, who first proposed it in 1937. The conjecture is also known as the $3n +$ conjecture. The problem can be simply stated as, starts with any positive integer. If it is even number, halve it (which has been called "Half Or Triple Plus One", or HOTPO). Otherwise multiply it by 3 and add 1 to it. Take the result and repeat the process. Any such sequence seems to end up at one. The conjecture remain unanswered, although it has been proven that the process terminates for all values of n up to 5.764×10.

Let $f: N \rightarrow$ be Collatz function defined as: $f(x) = \begin{cases} \frac{x}{2}, & ifxisev \\ 3x + 1, ifxisoc \end{cases}$

ISBN – 978-81-948061-2-7

Collatz conjecture states that if $x \in$, then the sequence $x, f(x), f \circ f(x), f \circ f \circ f(x),$, reaches to .The number of iterations is not proportional to the magnitude of the starting number. And hence remain unsolved yet.

ONENESS OF NATURAL NUMBER, ONENESS FACTOR/HIT FACTOR:

DEFINITION 1:- (Oneness of Natural Number) The ability of a natural number reaching to 1 with collatz conjecture function f is called as oneness of natural number.

DEFINITION 2:- (Hit Factor) The conjecture asserts that every natural number has a well-defined Hit Factor. Total number of steps required by a natural number to reach using $f($ is called as Total stoppage time, oneness factor of .

DEFINITION 3:- (Hit Factor Function)Let $T: \mathbb{N} \rightarrow$ be Hit Factor function defined over set of natural number as the total number steps needed to reach 1.

A. *Hit Factor of Natural Number* 2^n

$T(2^n) =$.e.g. $T(1) = 0,\ T(2) = 1,\ T(64) =$.

B. *Hit Factor of odd Natural Numbers having* $n+1$ *hit factor*

Now we are familiar to the term than even number decreases, so we just emphasize on odd numbers. To find out odd number which transform to .

Result 1: If be any natural number then $3|4^k -$.

Result 2: If be a odd natural number and $r =$ be the power of then $3a + 1 =$ and $a = (4^{k-1} + 4^{k-2} + \cdots + 4 +$.

Result 3: The sequence of odd numbers such that $3a + 1 =$ are represented by recurrence relation, $\mathrm{O_n} = 4\mathrm{O_{n-1}} +$ with initial condition $\mathrm{O_1} =$.

Result 4: The sequence of odd numbers $a = 2k + 1, k \geq$ is an integer such that $6k + 4 =$ are represented by recurrence relation, $\mathrm{K_n} = 4\mathrm{K_{n-1}} +$ with initial condition $\mathrm{K_1} =$and $\mathrm{K_n}$ is ev.

Result 5:

$\mathrm{K_{n+1}} = 2 + 2^3 + 2^5 + \cdots + 2^{2n-1}, \mathrm{K_{n+1}} - \mathrm{K_n} = 2^{2n-1},$

where $n \geq$.

Result 6: $2\,\mathrm{O_{n+1}} = \mathrm{K_n}$ and $\mathrm{O_{n+1}} - \mathrm{O_n} = 4^n, n \geq$.

Result 7: Using recurrence relation in Result 3 every $3r$ transforms into $3m_2 +$ where $m_2 = 4r$, $3m_1 +$into $3m_2 +$ where $m_2 = 4m_1 +$ and $3m_1 +$ into $3r$ where $m_2 = 4m_1 +$.

From Result 2 it is clear that none of the odd number converts to odd power of, the odd number which converts to even power of 2 is of the form $a = (4^{k-1} + 4^{k-2} + \cdots + 4 + 1), k \in$

ISBN – 978-81-948061-2-7

TABLE I : Elements Of Odd Natural Numbers

Set	O_1	O_2	O_3	. . .
4^n	4^1	4^2	4^3	
O_n	1 5 21 . .			
Hit Factor	0 4 + 1 6 + 1			
K_n	0 2 10			
M_n	0 17			

TERNARY PARTITIONS OF ODD NUMBERS:

We define here 3 partitions of odd natural numbers as $\wp_0 = \{21,1365,87381,\ldots\}, \wp_1 = \{1,85,5461,\ldots\}, \wp_2 = \{5,341,21845\}$

with multiple of 3 partitions $\mathcal{M}_0 = \{7,455,29127,\ldots\}, \mathcal{M}_1 = \{0,28,1820,\ldots\}, \mathcal{M}_2 = \{1,113,7281,\ldots\}$

A. Partition $\wp_0$

Set of numbers divisible by 3.None of the odd natural number n converges to multiple of 3 natural number using Collatz function.

Result 8: As $3|4^n -$where $n \geq 1, p = (1 + 4 + 4^2 + \cdots + 4^{n-}$where $4^n - 1 =$.

Result 9: $21|16^n + 4^n +$where $n \geq$.

Result 10: If $3n + 1 = 4$ where $n \geq$ is an odd natural number and $k \geq$then 21.

Result 11: $21|(1 + 4 + 4^2 + 4^3 + \cdots + 4^{3k-}$where $k \geq$.

Result 12: The sequence of odd numbers such that $3a + 1 = 4$ are represented by recurrence relation, $O_n = 64O_{n-1} + 2$with initial condition $O_1 =$.

Result 13: The sequence of odd numbers $a = 2k + 1, k \geq 0 s.t. 3a + 1 = 4$ is an integer such that $6k + 4 = 4$ are represented by recurrence relation, $K_n = 64K_{n-1} + 4$with initial condition $K_1 =$ and K_n is ev.

Result 14: $K_n = 2 + 2^3 + 2^5 + \cdots + 2^{3(2n-1)}, K_{n+1} - K_n = 2^{2n+1}.21,$

where $n \geq$.

Result 15: If elements of set are represented as 3 then the recurrence relation is $M_r = 64M_{r-1} +$, and $7|$ and $m = 7(1 + 64 + 64^2 + \cdots + 64^{r-}$

ISBN – 978-81-948061-2-7

TABLE II: Elements Of 3k Odd Natural Numbers

$Set\ O_1 O_2\ O_3$ . . .				
$Base\ Value$ $4^3 4^6 4^9$ ...				
O_n 21 1365				
K_n 10 682				
M_n	7	455	29127	...

B. Partition $\wp_1$

Set of numbers which leave remainder 1 when divisible by 3.

Result 16: If $p = 3n +$ is any odd number in δ such that $3p + 1 = 4^{3r}$ then 28 and $p = 1 + 4 + \cdots + 4$.

Result 17: The sequence of odd numbers such that $3a + 1 = 4^{3r}$ are represented by recurrence relation, $O_n = 64O_{n-1} + 2$ with initial condition $O_1 =$.

Result 18: The sequence of odd numbers $a = 2k + 1, k \geq 0 s.t. 3a + 1 = 4^{3r}$ is an integer such that $6k + 4 = 4^{3r}$ are represented by recurrence relation, $K_n = 64K_{n-1} + 4$ with initial condition $K_1 =$ and K_n is ev.

Result 19: If elements of set δ are represented as $3m +$ then the recurrence relation is $M_r = 64M_{r-1} +$.

TABLE III: Elements Of 3k+1 Odd Natural Numbers

$Set\ O_1\ O_2 O_3$ . .				
$Base\ Value$ $4^1 4^4\ 4^7$ ...				
O_n 1 85				
K_n 0 42				
M_n	0	28	1820	...

C. Partition $\wp_2$

Set of numbers which leave remainder 2 when divisible by 3.

Result 20: If $p = 3n +$ is any odd number in δ such that $3p + 1 = 4^{3r}$ then $n = 1 + 7 * 4 + 7 * 4^2 + \cdots + 7 * 4^{3r}$ and $p = 1 + 4 + \cdots + 4^{3r}$.

Result 21: The sequence of odd numbers such that $3a + 1 = 4^{3r}$ are represented by recurrence relation, $O_n = 64O_{n-1} + 2$ with initial condition $O_1 =$.

Result 22: The sequence of odd numbers $a = 2k + 1, k \geq 0 s.t. 3a + 1 = 4^{3r}$ is an integer such that $6k + 4 = 4^{3r}$ are represented by recurrence relation, $K_n = 64K_{n-1} + 4$ with initial condition $K_1 =$ and K_n is ev.

Result 23: If elements of set δ are represented as $3m +$ then the recurrence relation is $M_r = 64M_{r-1} +$.

TABLE IV: Elements Of 3k+2 Odd Natural Numbers

Set $O_1 O_2 O_3$ . . .				
Base Value $4^1 4^4 4^7$...				
O_n 5341				
K_n 2170				
M_n	1	113	7281	...

SQUARE SUM:

DEFINITION 4: (Collatz Square Number) If sum of all elements (odd and even both) in a given sequence for given natural number is result in a perfect square then that natural number is called as Collatz Square Number.

e.g. For natural number 3 the sum of sequence 3: 3+10+5+16+8+4+2+1=49, and for 5: 5+16+8+4+2+1=36.

The list of such numbers less than 1,000,000 are : 1,3,5,33,60,245,304,372,1265,1568,1756,1799,1856,2409,2532,2976,3100,3281,3376,3394,3813,5637,5972,6147,6538,7213,7299,7896,7966,8371,10419,11526,13411,13856,14168,15024,15283,15709,16506,16577,16916,19212,19829,21372,22049,23052,23542,24165,25408,25488,26016,29034,30101,30347,32005,32896,33006,33608,35297,36000,39432,41058,42799,44810,53270,54329,55723,56705,63696,64348,66508,69155,74253,75216,83082,83200,86080,86650,89392,91264,93165,94122,96847,99651,105816,106980,107264,108171,110741,114052,120612,122074,124614,126503,126736,127456,128028,128884,131389,137039,137409,140892,141102,142438,150163,152601,159040,160270,162048,165955,167280,167656,169164,171053,171603,182432,183322,185770,189789,191721,198333,202044,202745,213160,217097,219230,219588,221664,227361,234852,241252,258579,262442,274988,282561,282606,284120,284808,289293,300628,301688,308528,316162,323596,328168,329322,333198,333480,336120,338099,340962,344666,357849,365099,365249,384640,387729,387769,399265,403088,404949,409715,425500,428002,432736,433037,455031,458696,463398,477081,486196,501520,505408,505649,506956,507337,514987,543648,550001,552209,568160,574157,576890,582122,582804,584630,586337,591826,594956,609540,609722,635530,640045,640271,641079,647191,648208,649220,650347,650557,652776,660317,666468,673760,684344,691974,698944,700267,700322,702964,710155,710709,710897,718232,723570,727309,729504,735547,740464,749070,765712,770612,771173,776440,776576,778459,784596,818866,819500,823221,832023,832116,834709,851441,862445,867757,871570,873177,880692,884160,903124,904514,911568,916815,926604,932429,946778,949452,954726,970632,971266,977760,982177,986018,990752,991158,996941,998977.

DEFINITION 5 (Collatz Odd Square Number)If sum of odd elements in a given sequence is result in a perfect square for odd natural number then that odd natural number is called as Collatz odd Square Number.

e.g. For natural number 3 the sum of sequence 3: 3+5+1=9, and for 17: 17+13+5+1=36.

The list of such numbers less than 1000000 are : 1,3,17,45,101,219,519,547,743,777,1281,2749,4667,5669,7637,8793,9673,11049,11229,12665,14155,14961,16595,16769,17859,18163,20619,21269,22491,23059,24309,25749,29841,32059,32165,33973,35401,36495,37035,37125,38817,41433,46295,46411,47009,47521,47651,54019,54213,55363,55749,62193,64933,65557,70371,70563,72157,77219,77697,82861,82895,92325,93651,97849,98803,99715,100653,102449,111787,114801,115785,118291,118725,124929,129821,135057,135797,147187,148185,148565,151967,165597,167919,168199,187251,188503,199669,201285,212087,219669,222017,233101,242229,243677,261627,261675,264757,265387,273843,276371,307683,318947,319429,326091,333611,339861,341269,342953,349533,359609,372509,375469,386857,395057,400273,445379,446385,449173,454679,469559,490773,492677,496751,502101,512393,515985,526989,536853,539377,541485,546627,547133,571349,571363,595279,604403,607917,617769,646263,647567,650129,652759,653887,667579,670869,679269,684333,702621,704889,708161,714073,719859,720035,735787,737051,751217,773325,780459,793961,826181,833431,843191,848587,854133,860801,863885,865075,870257,884027,897409,899315,899711,909867,914967,918575,921975,930929,939487,940329,943853,945313,951945,957659,963941,979953,986485,992403,994821.

DEFINITION 6: (Collatz Complete Square Number)If a natural number is both i.e. Collatz Square number as well as Collatz Odd Square Number, then it is also called as Collatz Complete Square Number.

e.g. For natural number 3 the sum of sequence 3: 3+5+1=9, and 3: 3+10+5+16+8+4+2+1=49

Even in Odd Square numbers we may categorized other two types also.

DEFINITION 7: (Collatz Complete Odd Square Number)If a sequence contains all odd numbers in arithmetic progression with common difference 2 for a given square number. Then it is called as Collatz Complete Odd Square Number.

e.g. For natural number 3 the sum of sequence is 9 but 9 = 1+3+5 sum of odd numbers and for 3 the sequence contains 3,5,1 i.e. same element.

DEFINITION 8: (Collatz Partial Odd Square Number)If a sequence contains odd numbers but not in arithmetic progression for a given square number. Then it is called as Collatz Partial Odd Square Number.

e.g. For natural number 17 the sum of sequence 17: 17+13+5+1=36. But for 36 = 1+3+5+7+9+11.

DEFINITION 9: (Collatz Pair Square) There are some Collatz square numbers and Collatz odd square numbers which have identical sum.

e.g. Collatz Odd Square Numbers: Up to 1000000

(743,14155):2302, (38817,77219):5182, (187251,199669):8332,(188503,276371):11432,(82895,751217):11632,(515985,607917):16082

Collatz Square Numbers: Up to 1000000

(13411,33608):4952, (66508,344666):14892, (274988,990752):15502,

CONJECTURES:

STATEMENT 1: There are infinitely many Collatz Square Numbers, Collatz Odd Square Numbers and Collatz Partial Odd Square Numbers.

STATEMENT 2: There are only two Collatz Complete Square Numbers and Collatz Complete Odd Square Numbers. Those are 1 and 3. None of the odd natural number greater than 3 fit into this category.

STATEMENT 3: There are infinitely many Collatz Pair Square.

STATEMENT 4: (Loose) There are no such ordered triplets or of higher degree terms containing identical sum.

ACKNOWLEDGMENT:

It is clear that every Natural number using Collatzfunction moves like a hailstone (i.e. increases, decreases) then first converges to and then onwards it decreases and reaches to 1. First odd numbers have 3 partitions and every element in one partition transforms to other element in partition hence we found one to one correspondence between them. Conjecture above mentioned above are always hold. But the last loose conjecture may be false for large numbers but to generate sequence, calculate sum and to validate it is square or not is very tedious job beyond 1,000,000,000. To optimize this algorithm we have to find relationship between natural number and its square sum.

REFERENCES:

1) Chamberland, M.(1996); A continuous extension of the 3x+1 problem to the real line. Dynamics of Continuous, Discrete and Impulsive Systems, 2:495-509.
2) David Applegate and Jeffrey C. Lagarias, (1995) The distribution of 3x + 1 trees, Experiment. Math. 4, no. 3, 193–209.
3) D.J. Bernstein, J.C. Lagarias, (1996) The 3x + 1 conjugacy map, Canadian Journal of Mathematics 48 (6) 1154-1169.
4) Guo-Gang Gao, (1993) On consecutive numbers of the same height in the Collatz problem, Discrete Math. 112, no. 1-3, 261–267.
5) Jadhav M. A. ,Patankar S. P. , Thorat S. P. andIngale S. A.(2020) Oneness of Natural Number: Properties of Multiples of 3, AayushiInternational Interdisciplinary Research Journal (ISSN 2349-638x) (Special Issue No.68), 885-889.

ISBN – 978-81-948061-2-7

6) M.R. Feix, A. Muriel, J.L. Rouet,(1994) Statistical properties of an iterated arithmetic mapping, Journal of Statistical Physics 76 725-741.
7) Motta, Francis C., Henrique R. De Oliveira, and Thiago A. Catalan. An Analysis of the Collatz Conjecture. Print.
8) Patankar S. P., Thorat S. P., Jadhav M. A.; (2015) Oneness Of Natural Number: Collatz Conjecture, Journal Commerce, Business and Social Sciences Vol IV, 137-140
9) Patankar S. P., Thorat S. P., Jadhav M. A., Choudhary R. P. ; (2015) Oneness Of Natural Number: Ternary Partitions of Odd Natural Numbers, International Journal of Multidisciplinary Research Vol IV Issue 6(I), ISSN: 2277-9302 26-29, 137-140
10) R.E. Crandall, (1978) On the 3x + 1 problem, Mathematics of Computation 32 (4) 1281-1292.
11) RihoTerras, (1976) A stopping time problem on the positive integers, Acta Arith. 30, no. 3, 241–252.