Statistical Mechanics

SET e.g.

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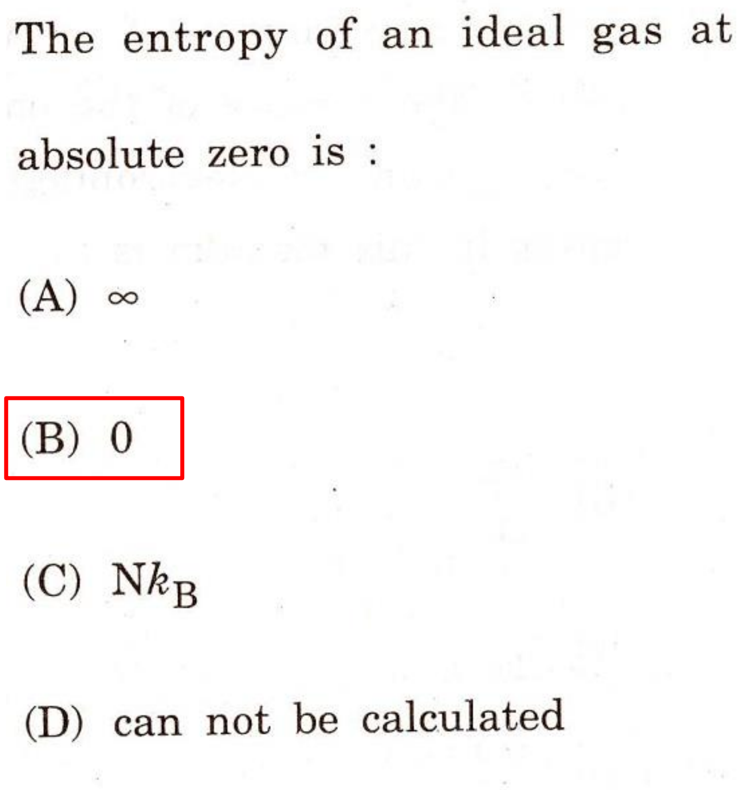
Aug 2011



The entropy of an ideal gas at absolute zero is :

(A) ∞

(B) 0

(C) Nk_B

(D) can not be calculated

This is also called as the **Nernst's Theorem**. The third law states that the entropy of any body vanishes at absolute zero of temperature, i.e. $S = 0$ **at** $T = 0$.

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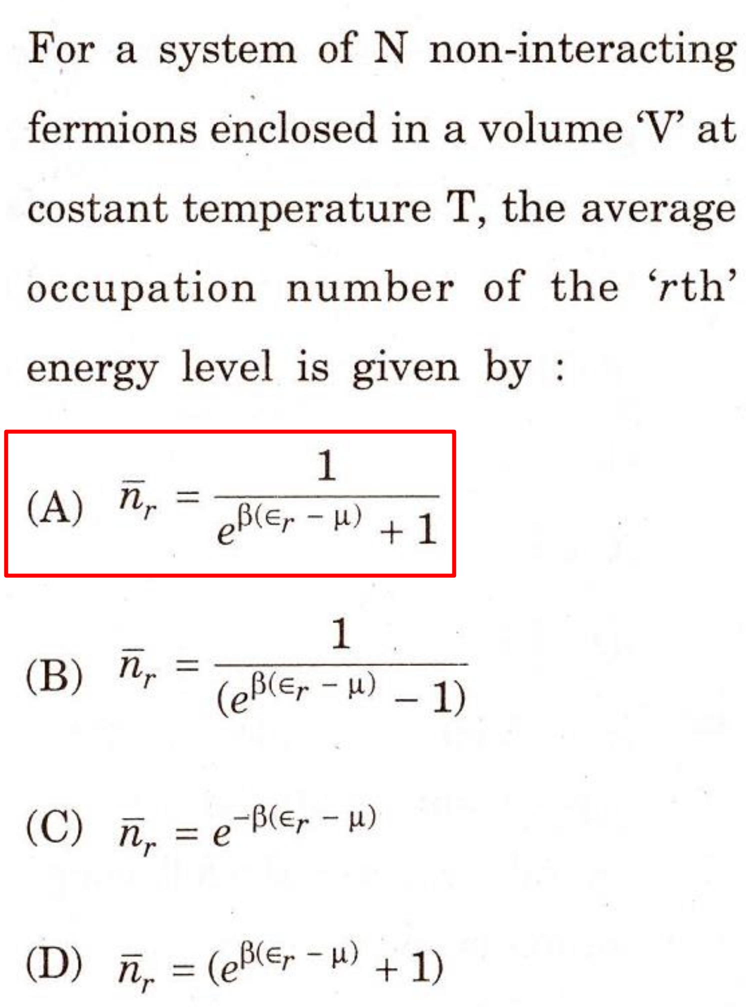

For a system of N non-interacting fermions enclosed in a volume 'V' at costant temperature T, the average occupation number of the 'rth' energy level is given by :

(A) $\bar{n}_r = \dfrac{1}{e^{\beta(\epsilon_r - \mu)} + 1}$

(B) $\bar{n}_r = \dfrac{1}{(e^{\beta(\epsilon_r - \mu)} - 1)}$

(C) $\bar{n}_r = e^{-\beta(\epsilon_r - \mu)}$

(D) $\bar{n}_r = (e^{\beta(\epsilon_r - \mu)} + 1)$

$$< n_s >= \sum_s \frac{1}{e^{-\beta(\mu-\epsilon_s)}+1} = \sum_s \frac{1}{e^{\beta(\epsilon_s-\mu)}+1}$$

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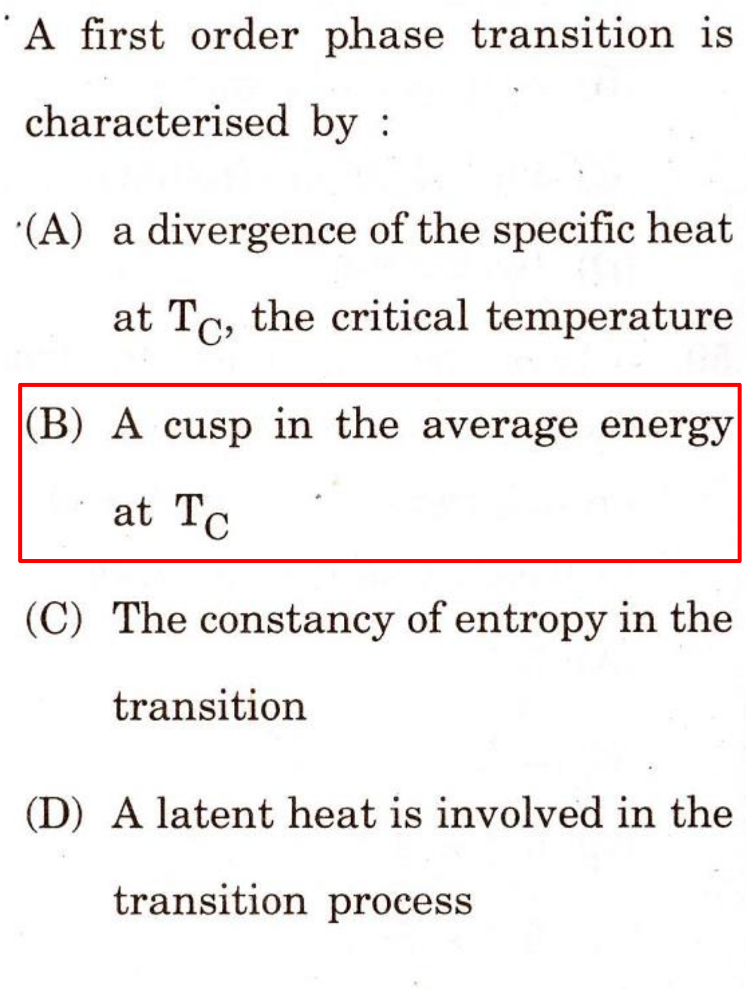

A first order phase transition is characterised by :

(A) a divergence of the specific heat at T_C, the critical temperature

(B) A cusp in the average energy at T_C

(C) The constancy of entropy in the transition

(D) A latent heat is involved in the transition process

First order -

- Discontinuous change of int. energy and sp. Volume
- Heat evolved / absorbed
  - e.g. Solid – Liquid, Liquid – Gas

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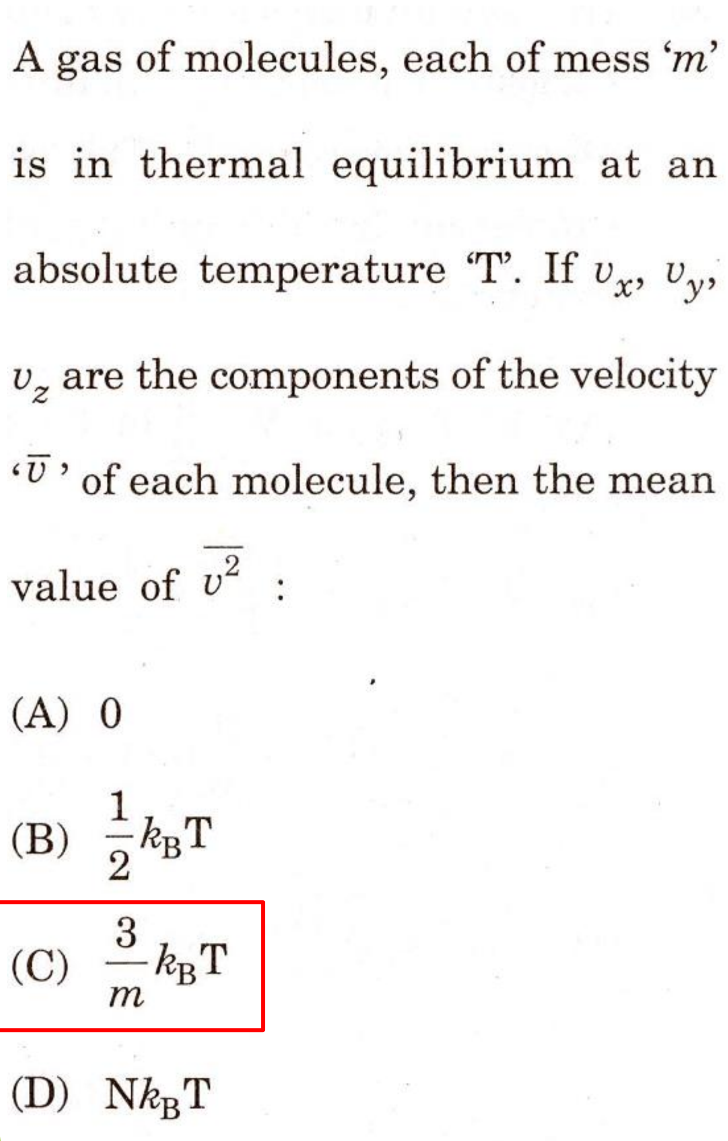


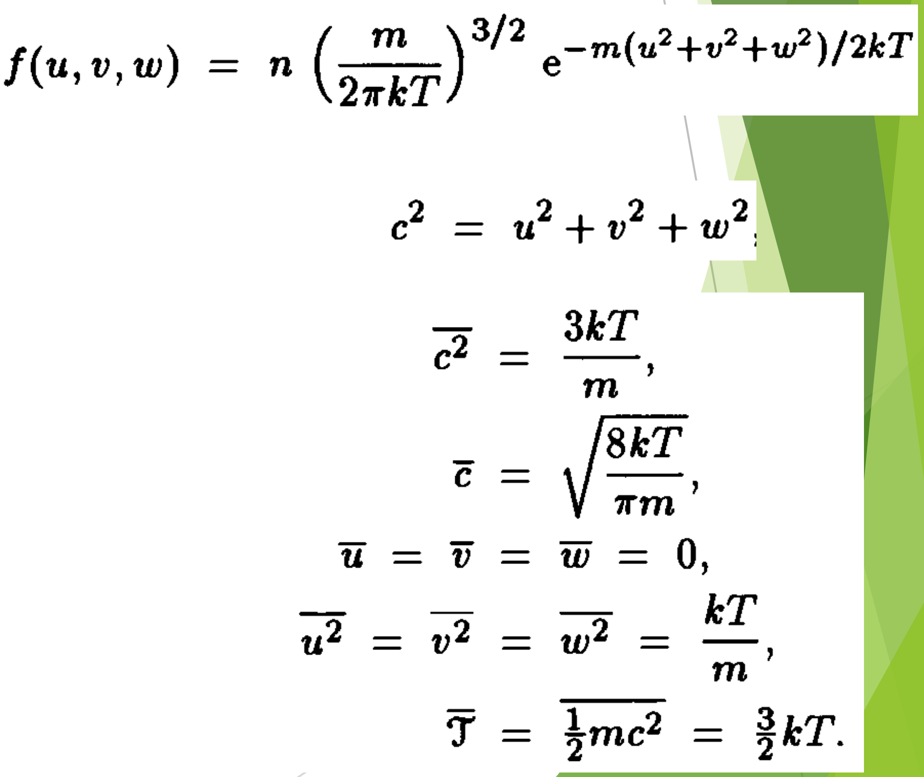
A gas of molecules, each of mess 'm' is in thermal equilibrium at an absolute temperature 'T'. If v_x, v_y, v_z are the components of the velocity '$\overline{v}$' of each molecule, then the mean value of $\overline{v^2}$:

(A) 0

(B) $\frac{1}{2}k_{\mathrm{B}}\mathrm{T}$

$\boxed{\text{(C)}\ \frac{3}{m}k_{\mathrm{B}}\mathrm{T}}$

(D) $\mathrm{N}k_{\mathrm{B}}\mathrm{T}$



$$f(u,v,w) = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-m(u^2+v^2+w^2)/2kT}$$

$$c^2 = u^2+v^2+w^2,$$

$$\overline{c^2} = \frac{3kT}{m},$$

$$\overline{c} = \sqrt{\frac{8kT}{\pi m}},$$

$$\overline{u} = \overline{v} = \overline{w} = 0,$$

$$\overline{u^2} = \overline{v^2} = \overline{w^2} = \frac{kT}{m},$$

$$\overline{\mathfrak{T}} = \overline{\tfrac{1}{2}mc^2} = \tfrac{3}{2}kT.$$

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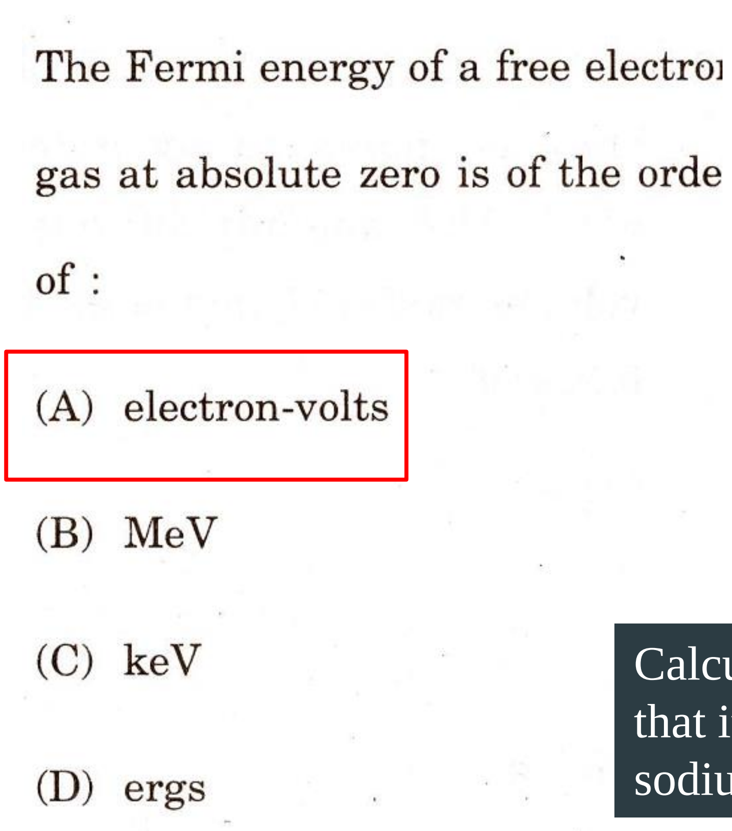

The Fermi energy of a free electro

gas at absolute zero is of the orde

of :

(A) electron-volts

(B) MeV

(C) keV

(D) ergs

Principles of Equilibrium Statistical Mechanics Debashish Chowdhury Dietrich Stauffer

$$\epsilon_F = \frac{\hbar^2 k_F^2}{2M} = \frac{\hbar^2}{2M}\left\{\frac{6\pi^2}{(2S+1)}\frac{N}{V}\right\}^{2/3}$$

Calculate the Fermi energy in eV for Sodium assuming that it has one free electron per atom. Given density of sodium = 0.97 g/cc, Atomic weight of sodium is 23.

Data: $N_0 = 6.02 \times 10^{26}$ atoms per kg mole,

$\rho = 0.97$ g/cc $= 0.97 \times 10^3$ kg/m^3

$w = 23$

$h = 6.62\times10^{-34}$ joule.sec

$m = 9.1 \times10^{-31}$ kg

$$\in_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$$

$$\frac{N}{V} = \frac{N_0\rho}{w} = \frac{6.02\times10^{26}\times0.97\times10^{3}}{23} = 2.53\times10^{28}$$

$\in_F = 5.04265 \times 10^{-19}$ Joules

$= 3.147$ eV

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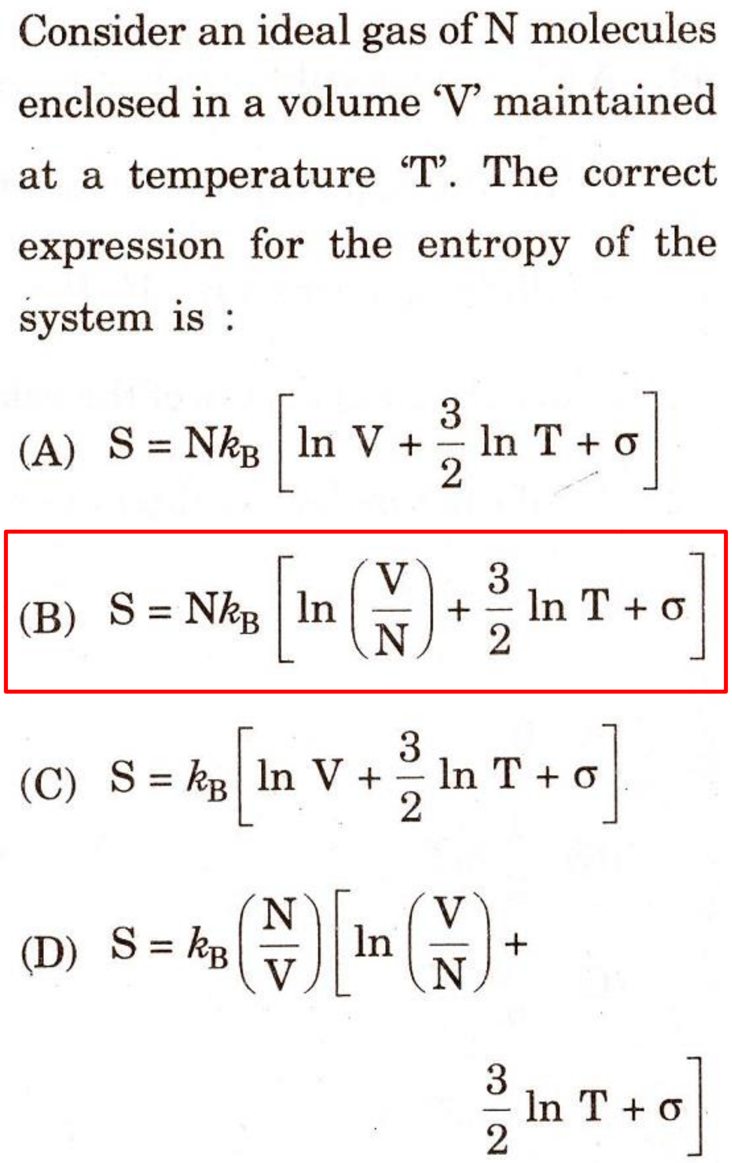

Consider an ideal gas of N molecules enclosed in a volume 'V' maintained at a temperature 'T'. The correct expression for the entropy of the system is :

(A) $S = Nk_B\left[\ln V + \frac{3}{2}\ln T + \sigma\right]$

(B) $\boxed{S = Nk_B\left[\ln\left(\frac{V}{N}\right) + \frac{3}{2}\ln T + \sigma\right]}$

(C) $S = k_B\left[\ln V + \frac{3}{2}\ln T + \sigma\right]$

(D) $S = k_B\left(\frac{N}{V}\right)\left[\ln\left(\frac{V}{N}\right) + \frac{3}{2}\ln T + \sigma\right]$

$$S = NkLn\left[V\left(\frac{E}{N}\right)^{3/2}\left(\frac{4\pi m}{3h^2}\right)^{3/2}\right] + \frac{3Nk}{2}$$

$$S = NkLn\left[\left(\frac{V}{N}\right)\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] + \frac{5}{2}Nk$$

$$S = Nk\left\{Ln\left[\left(\frac{V}{N}\right)\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] + \frac{5}{2}\right\}$$

$$S = Nk\left\{Ln\left[\left(\frac{V}{N}\right)\right] + Ln\left[\left(\frac{2\pi mk}{h^2}\right)^{3/2}\right] + Ln\left[T^{3/2}\right] + \frac{5}{2}\right\}$$

$$S = Nk\left\{Ln\left(\frac{V}{N}\right) + \frac{3}{2}LnT + \frac{3}{2}Ln\left(\frac{2\pi mk}{h^2}\right) + \frac{5}{2}\right\}$$

$$S = Nk\left\{Ln\left(\frac{V}{N}\right) + \frac{3}{2}LnT + \sigma\right\}$$

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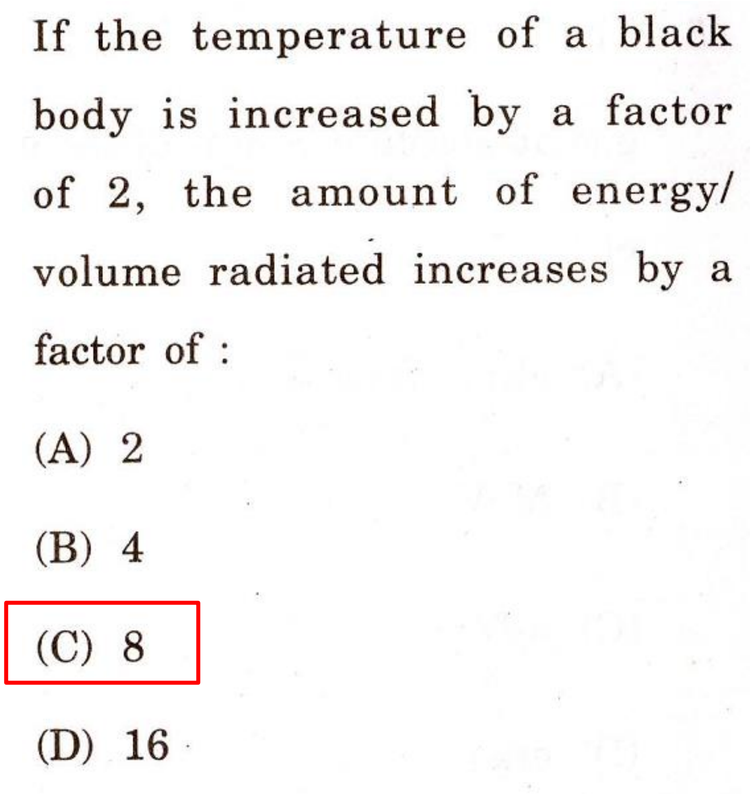


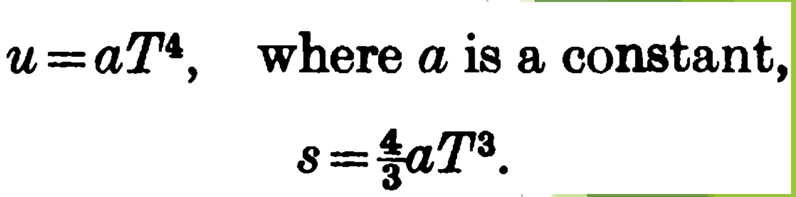
If the temperature of a black body is increased by a factor of 2, the amount of energy/ volume radiated increases by a factor of :

(A) 2

(B) 4

[✓ (C) 8]

(D) 16



$u = aT^4$, where a is a constant,

$$s = \tfrac{4}{3} a T^3.$$

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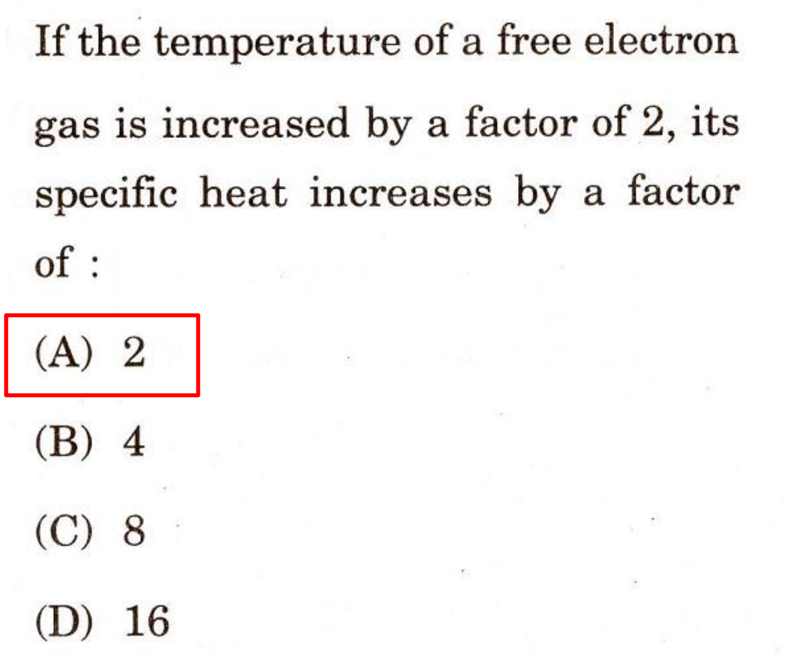


If the temperature of a free electron gas is increased by a factor of 2, its specific heat increases by a factor of :

[✓ (A) 2]

(B) 4

(C) 8

(D) 16

$$Cv = \frac{1}{2} Nk\pi^2 \left(\frac{kT}{\in_F} \right)$$

$$Cv = \frac{1}{2} Nk\pi^2 D$$

Nov 2011

The mean energy of a classical ideal gas having N monatomic particles at a temperature T will be :

(A) $\frac{1}{2}$ NkT

(B) NkT

(C) 2 NkT

$\boxed{\text{(D) } \frac{3}{2}\text{ NkT}}$

$$Q_N = \frac{V^N}{N!}\left(\frac{2\pi mkT}{h^2}\right)^{3N/2}$$

$$A(N,V,T) = -kT\ln Q_N \qquad A = -NkT\left[1+\ln\left\{\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right\}\right]$$

$$P = -\left(\frac{\partial A}{\partial V}\right)_{N,T} \qquad P = NkT\left(\frac{1}{V}\right)$$

$$S = -\left(\frac{\partial A}{\partial T}\right)_{N,V} \qquad S = Nk\left(\frac{5}{2}+\ln\left\{\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right\}\right)$$

$$U = A + TS \qquad \boxed{U = \frac{3}{2}NkT}$$

Nov 2011

The volume of a perfect gas is doubled, the number N of atoms and the energy being held constant. The change in entropy will be :

(A) Nk *ln* V

(B) 2 Nk *ln* V

[✓ (C) Nk *ln* 2]

(D) $\frac{1}{2}$ Nk *ln* (2 V)

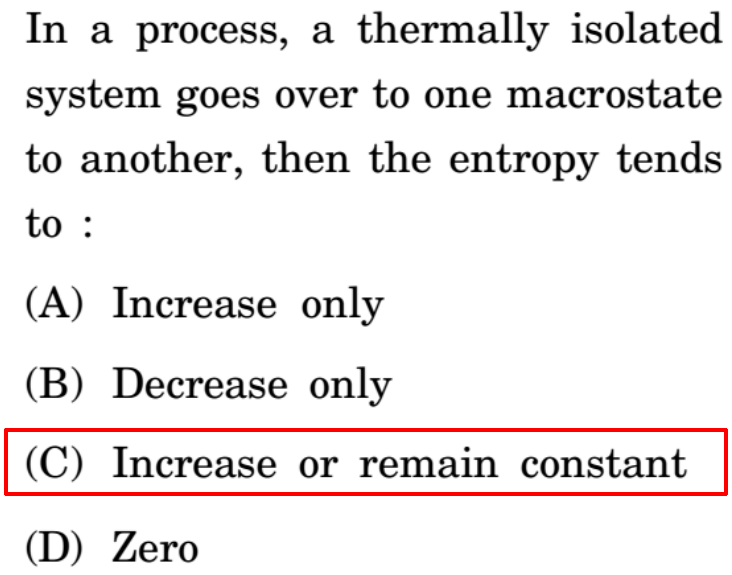
$$S = NkLn\left[V\left(\frac{E}{N}\right)^{3/2}\left(\frac{4\pi m}{3h^2}\right)^{3/2}\right]+\frac{3}{2}Nk$$

$$S' = NkLn\left[2V\left(\frac{E}{N}\right)^{3/2}\left(\frac{4\pi m}{3h^2}\right)^{3/2}\right]+\frac{3}{2}Nk$$

$$S' = NkLn2 + NkLn\left[V\left(\frac{E}{N}\right)^{3/2}\left(\frac{4\pi m}{3h^2}\right)^{3/2}\right]+\frac{3}{2}Nk$$

$$S' = NkLn2 + S$$

Nov 2011



In a process, a thermally isolated system goes over to one macrostate to another, then the entropy tends to :

(A) Increase only

(B) Decrease only

(C) Increase or remain constant

(D) Zero

Thus we can define the law of increase in entropy in the following way:

“If the entropy of a closed system does not have its maximum value at any time, then the entropy will increase or at least remain constant at a later time”.

Nov 2011

For the Fermi-Dirac distribution, the probability of occupation of a single particle energy level is equal to :

(A) the average occupancy of that level

(B) one

(C) $\frac{1}{2}$ the average occupancy of that level

(D) 0

Nov 2011

Consider degenerate Fermi gas at T = 0 with the Fermi energy E_F. The mean energy per particle will be :

(A) $\boxed{\frac{3}{5} E_F}$

(B) $\frac{1}{2} E_F$

(C) $\frac{2}{3} E_F$

(D) $\frac{5}{3} E_F$

Substituting equation (28) in equation (27) the total ground state energy of the gas is given by

$$U_0 = \frac{2\pi g V}{h^3}(2m)^{3/2}\int_0^{\in_F} \in^{3/2}\, d\in$$

$$U = \frac{2\pi g V}{h^3}(2m)^{3/2}\int_0^{\infty} \frac{\in^{3/2}\, d\in}{e^{(\in-\mu)/kT}+1}$$

$$<n_{\in}> \;= 1 \quad \text{for } \in < \in_F$$
$$= 0 \quad \text{for } \in > \in_F$$

(32)

$$= \frac{2\pi g V}{h^3}(2m)^{3/2}\,\frac{2}{5}\in_F^{5/2}$$

$$U_0 = \frac{4\pi g V}{h^3}(2m)^{3/2}\,\frac{1}{5}\in_F^{5/2}$$

From equation (29) and (32), we get

$$N = \frac{4\pi g V}{3h^3}(2m)^{3/2}\in_F^{3/2}$$

$$\frac{U_0}{N} = \frac{3}{5}\in_F$$

$$U_0 = \frac{4\pi g V}{3h^3}(2m)^{3/2}\in_F^{3/2}\,\frac{3}{5}\in_F = N\,\frac{3}{5}\in_F$$

(33)

Thus the average energy per particle of the Fermi gas at $T = 0$ is 3/5 times the Fermi energy $\in_F$.

Nov 2011

The equation of state of an ideal gas

in the non-relativistic state is given

by :

(A) $\boxed{PV = \frac{2}{3} U}$

(B) $PV = \frac{2}{5} U$

(C) $PV = \frac{1}{3} U$

(D) $PV = \frac{5}{2} U$

$$U = \frac{3}{2} NkT \left[\left(1 - \frac{1}{4\sqrt{2}} \frac{N}{gV} \left(\frac{h^2}{2\pi mkT} \right)^{3/2} \right) \right]$$

$$PV = NkT \left[\left(1 - \frac{1}{4\sqrt{2}} \frac{N}{gV} \left(\frac{h^2}{2\pi mkT} \right)^{3/2} \right) \right]$$

$$PV = \frac{2}{3} U$$

Feb 2013

Considering Hydrogen (H_2) and Helium (He) as classical ideal Maxwell-Boltzmann gas, the ratio of root mean square speeds of H_2 molecules to that of He atoms at the same temperature T is :

(A) 2

[✓ (B) $\sqrt{2}$]

(C) $\sqrt{\frac{1}{2}}$

(D) $\frac{1}{2}$

$$v_{RMS}(H_2) = \sqrt{\frac{3kT}{m_{H_2}}}$$

$$v_{RMS}(He) = \sqrt{\frac{3kT}{m_{He}}}$$

$$\frac{v_{RMS}(H_2)}{v_{RMS}(He)} = \frac{\sqrt{\frac{3kT}{m_{H_2}}}}{\sqrt{\frac{3kT}{m_{He}}}} = \sqrt{\frac{3kT}{m_{H_2}}\frac{m_{He}}{3kT}} = \sqrt{\frac{m_{He}}{m_{H_2}}} = \sqrt{\frac{4}{2}} = \sqrt{2}$$

$$f(u,v,w) = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-m(u^2+v^2+w^2)/2kT}$$

$$c^2 = u^2 + v^2 + w^2;$$

$$\overline{c^2} = \frac{3kT}{m},$$

$$\bar{c} = \sqrt{\frac{8kT}{\pi m}},$$

$$\bar{u} = \bar{v} = \bar{w} = 0,$$

$$\overline{u^2} = \overline{v^2} = \overline{w^2} = \frac{kT}{m},$$

$$\overline{\mathfrak{T}} = \overline{\tfrac{1}{2}mc^2} = \tfrac{3}{2}kT.$$

Feb 2013

The molar specific heat of conduction electrons at 300 K is :

(A) $>> \frac{3}{2}\text{R}$

(B) $\frac{3}{2}\text{R}$

$\boxed{\text{(C)}\ << \frac{3}{2}\text{R}}$

(D) R

Here R is the gas constant.

Ideal Fermi gas at T $\neq$ 0

$$c_v = \frac{\pi^2}{2} n k_B \frac{k_B T}{\epsilon_F}$$

$$\frac{3}{2} n k_B = \frac{3}{2} R$$

$$(c_v)_{electron} = \frac{\pi^2}{3}\frac{k_B T}{\epsilon_F}(c_v)_{classical}.$$

$<< 1$

Feb 2013

Two identical particles are to be distributed over 3 energy levels. Treating the particles as distinguishable Maxwell-Boltzmann particles, the number of ways in which the particles can be distributed is :

(A) 9

(B) 6

(C) 3

(D) 8

Find number of possible ways of distribution of 2 electrons in 3 states according to MB, BE and FD statistics.

If $n_i = 2$: No. of electrons

& $g_i = 3$: No. of states

MB statistics:

$$= n!\prod_i \frac{g_i^{n_i}}{n_i}$$

$$= 2!\frac{3^2}{2}$$

$$= 2.1\frac{3\times 3}{2}$$

$$= 9$$

BE statistics:

$$= \frac{g_i(g_i+n_i-1)!}{g_i!n_i!}$$

$$= \frac{3(3+2-1)!}{3!2!}$$

$$= \frac{3\times 4\times 3\times 2\times 1}{3\times 2\times 1\times 2\times 1}$$

$$= 6$$

FD statistics:

$$= \frac{g_i!}{n_i!(g_i-n_i)!}$$

$$= \frac{g_i!}{n_i!(g_i-n_i)!} = \frac{3!}{2!\times 1!}$$

$$= \frac{3\times 2\times 1}{2\times 1\times 1}$$

$$= 3$$

Feb 2013

A system of N identical independent three dimensional harmonic oscillators vibrating with the same frequency ω. The system is contact with a heat reservoir at temperature T. Treating the oscillators as classical, the molar specific heat of the system is :

(A) $\frac{3}{2}$R

(B) 3NK

(C) $\sqrt{\frac{3}{2}}$R

(D) 3R

$$Q_N = \left[\frac{kT}{\hbar\omega}\right]^N \qquad F = U - TS = A$$

$$A(N,V,T) = -kT\ln Q_N \qquad A(N,V,T) = -NkT\ln\left[\frac{kT}{\hbar\omega}\right] = NkT\ln\left[\frac{\hbar\omega}{kT}\right]$$

$$P = -\left(\frac{\partial A}{\partial V}\right)_{N,T} \qquad P = -\left(\frac{\partial}{\partial V}\left\{-NkT\ln\left[\frac{kT}{\hbar\omega}\right]\right\}\right)_{N,T} = 0$$

$$S = -\left(\frac{\partial A}{\partial T}\right)_{N,V} \qquad S = Nk\left[1+\ln\left(\frac{kT}{\hbar\omega}\right)\right]$$

$$U = A + TS \qquad U = NkT$$

$$Cv = \frac{\partial U}{\partial T} = \frac{\partial}{\partial T}NkT = Nk = R$$

Feb 2013

A system has a relaxation time of the order of a millisecond. A quasistatic process on this system can be carried out on the time scale of the order of :

$$\Delta t < \tau$$

(A) 0.001 s

[✓ (B) 0.1 s]

(C) 0.0001 s

(D) 1 μs

Feb 2013

Let $\bar{E}$ be the mean kinetic energy and V be the volume of a classical ideal gas. The pressure of the gas is numerically equal to :

$\boxed{\text{(A)}\ \frac{2}{3}\frac{\bar{E}}{V}}$

(B) $\frac{1}{3}\frac{\bar{E}}{V}$

(C) $\frac{3}{2}\frac{\bar{E}}{V}$

(D) $\frac{1}{2}\frac{E}{V}$

$$Q_N = \frac{V^N}{N!}\left(\frac{2\pi mkT}{h^2}\right)^{3N/2}$$

$$A(N,V,T) = -kT\ln Q_N \qquad A = -NkT\left[1+\ln\left\{\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right\}\right]$$

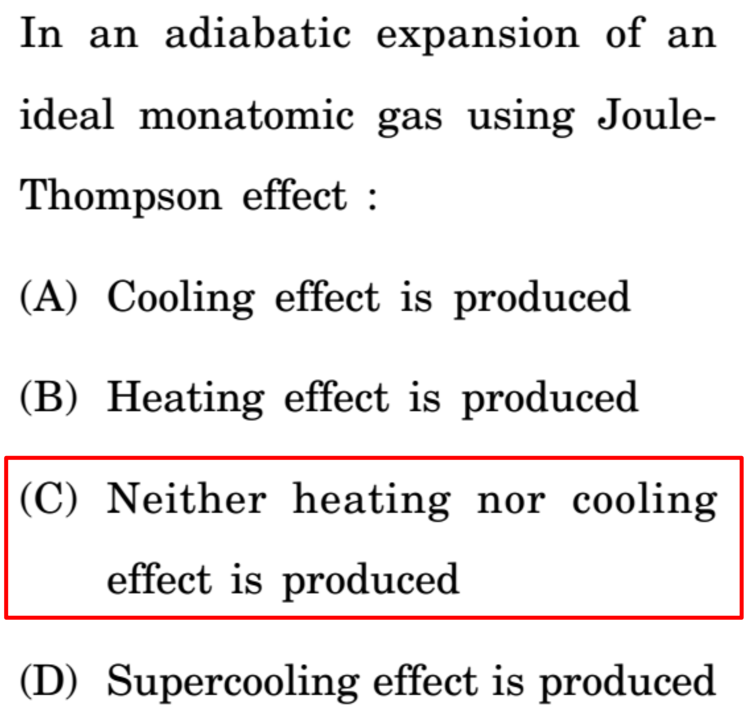
$$P = -\left(\frac{\partial A}{\partial V}\right)_{N,T} \qquad P = NkT\left(\frac{1}{V}\right)$$

$$S = -\left(\frac{\partial A}{\partial T}\right)_{N,V} \qquad S = Nk\left(\frac{5}{2}+\ln\left\{\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right\}\right)$$

$$U = A+TS \qquad \boxed{U = \frac{3}{2}NkT}$$

$$\frac{P}{U} = \frac{NkT\left(\frac{1}{V}\right)}{\frac{3}{2}NkT} \Rightarrow \frac{P}{U} = \frac{2}{3V} \Rightarrow P = \frac{2U}{3V} = \frac{2\bar{E}}{3V}$$

Feb 2013



In an adiabatic expansion of an ideal monatomic gas using Joule-Thompson effect :

(A) Cooling effect is produced

(B) Heating effect is produced

(C) Neither heating nor cooling effect is produced

(D) Supercooling effect is produced

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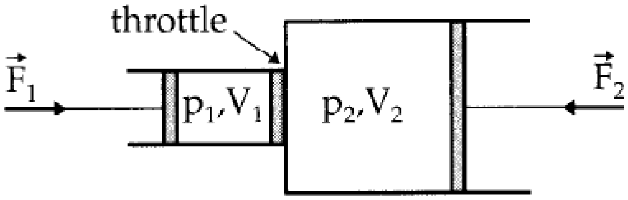
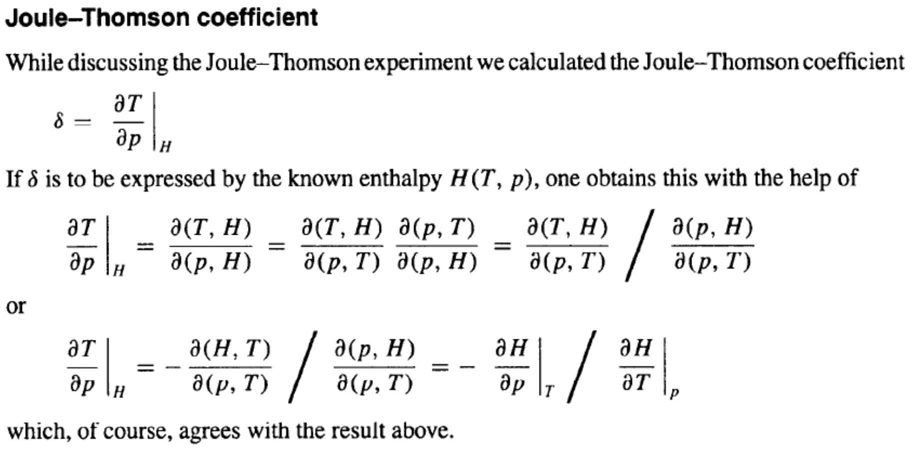


Figure 4.9. Joule–Thomson experiment.



Example 4.15: Joule–Thomson coefficient

While discussing the Joule–Thomson experiment we calculated the Joule–Thomson coefficient

$$\delta = \left.\frac{\partial T}{\partial p}\right|_H$$

If δ is to be expressed by the known enthalpy $H(T, p)$, one obtains this with the help of

$$\left.\frac{\partial T}{\partial p}\right|_H = \frac{\partial(T, H)}{\partial(p, H)} = \frac{\partial(T, H)}{\partial(p, T)}\,\frac{\partial(p, T)}{\partial(p, H)} = \frac{\partial(T, H)}{\partial(p, T)} \bigg/ \frac{\partial(p, H)}{\partial(p, T)}$$

or

$$\left.\frac{\partial T}{\partial p}\right|_H = -\frac{\partial(H, T)}{\partial(p, T)} \bigg/ \frac{\partial(p, H)}{\partial(p, T)} = -\left.\frac{\partial H}{\partial p}\right|_T \bigg/ \left.\frac{\partial H}{\partial T}\right|_p$$

which, of course, agrees with the result above.

Feb 2013

Which of the following thermodynamic relations is *incorrect* ?

(A) $P = -\left(\frac{\partial F}{\partial V}\right)_T$

$\boxed{\text{(B) } T = \left(\frac{\partial G}{\partial T}\right)_P}$

(C) $S = -\left(\frac{\partial F}{\partial T}\right)_V$

(D) $P = -\left(\frac{\partial U}{\partial V}\right)_S$

Here P, V, T are the pressure, volume and temperature, and F, G, S, U are the Helmholtz Free energy, Gibbs' free energy, entropy and average energy respectively.

$$T = \left.\frac{\partial U}{\partial S}\right|_{V,N,\ldots}, \quad \checkmark\ -p = \left.\frac{\partial U}{\partial V}\right|_{S,N,\ldots}, \quad \mu = \left.\frac{\partial U}{\partial N}\right|_{S,V,\ldots}$$

$$\checkmark\ -S = \left.\frac{\partial F}{\partial T}\right|_{V,N,\ldots}, \quad \checkmark\ -p = \left.\frac{\partial F}{\partial V}\right|_{T,N,\ldots}, \quad \mu = \left.\frac{\partial F}{\partial N}\right|_{T,V,\ldots},$$

$$-S = \left.\frac{\partial G}{\partial T}\right|_{p,N,\ldots}, \quad V = \left.\frac{\partial G}{\partial p}\right|_{T,N,\ldots}, \quad \mu = \left.\frac{\partial G}{\partial N}\right|_{T,p,\ldots}$$

Feb 2013

The equation of state of an ideal gas in the extreme relativistic approximation is given by :

(A) PV = $\frac{2}{3}$U

$\boxed{\text{(B) PV} = \frac{5}{2}\text{U}}$

(C) PV = $\frac{1}{3}$U

(D) PV = $\frac{2}{5}$U

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$$Z(T, V, N) = \frac{1}{N!}\left[4\pi V\left(\frac{mc}{h}\right)^3 \exp\{\beta mc^2\}\frac{K_2(\beta mc^2)}{\beta mc^2}\right]^N$$

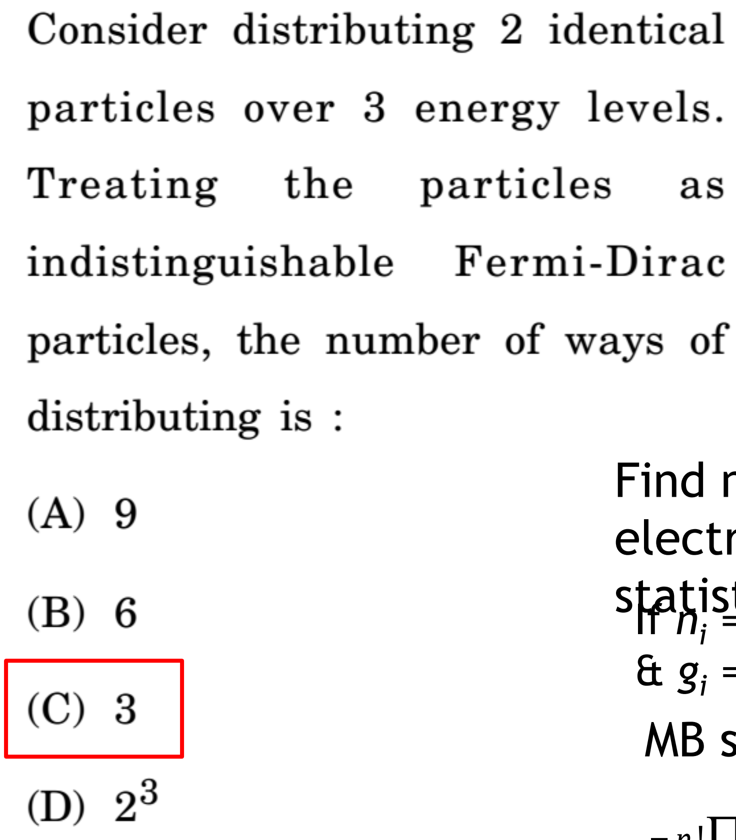
$$F\,(T, V, N) = -NkT\left[\ln\left\{4\pi\frac{V}{N}\left(\frac{mc}{h}\right)^3\frac{K_2(\beta mc^2)}{\beta mc^2}\right\} + 1\right] - Nmc^2$$

$$p(T, V, N) = -\left.\frac{\partial F}{\partial V}\right|_{T,N} = \frac{NkT}{V}$$

$$\mu(T, V, N) = \left.\frac{\partial F}{\partial N}\right|_{T,V} = -kT\ln\left\{4\pi\frac{V}{N}\left(\frac{mc}{h}\right)^3\frac{K_2(\beta mc^2)}{\beta mc^2}\right\} - mc^2$$

$$\begin{aligned} S(T, V, N) &= Nk\left[\ln\left\{4\pi\frac{V}{N}\left(\frac{mc}{h}\right)^3\frac{K_2(u)}{u}\right\} + 1\right] + Nk\left[u\frac{K_1(u)}{K_2(u)} + 3\right] \\ &= Nk\left[\ln\left\{4\pi\frac{V}{N}\left(\frac{mc}{h}\right)^3\frac{K_2(u)}{u}\right\} + 4 + u\frac{K_1(u)}{K_2(u)}\right] \end{aligned}$$

Feb 2013



Consider distributing 2 identical particles over 3 energy levels. Treating the particles as indistinguishable Fermi-Dirac particles, the number of ways of distributing is :

(A) 9

(B) 6

(C) 3

(D) 2^3

Find number of possible ways of distribution of 2 electrons in 3 states according to MB, BE and FD statistics.

If n_i = 2 : No. of electrons

& g_i = 3 : No. of states

MB statistics:

$$= n!\prod_i \frac{g_i^{n_i}}{n_i}$$

$$= 2!\frac{3^2}{2}$$

$$= 2.1\frac{3\times 3}{2}$$

$$= 9$$

BE statistics:

$$= \frac{g_i(g_i+n_i-1)!}{g_i!n_i!}$$

$$= \frac{3(3+2-1)!}{3!2!}$$

$$= \frac{3\times 4\times 3\times 2\times 1}{3\times 2\times 1\times 2\times 1}$$

$$= 6$$

FD statistics:

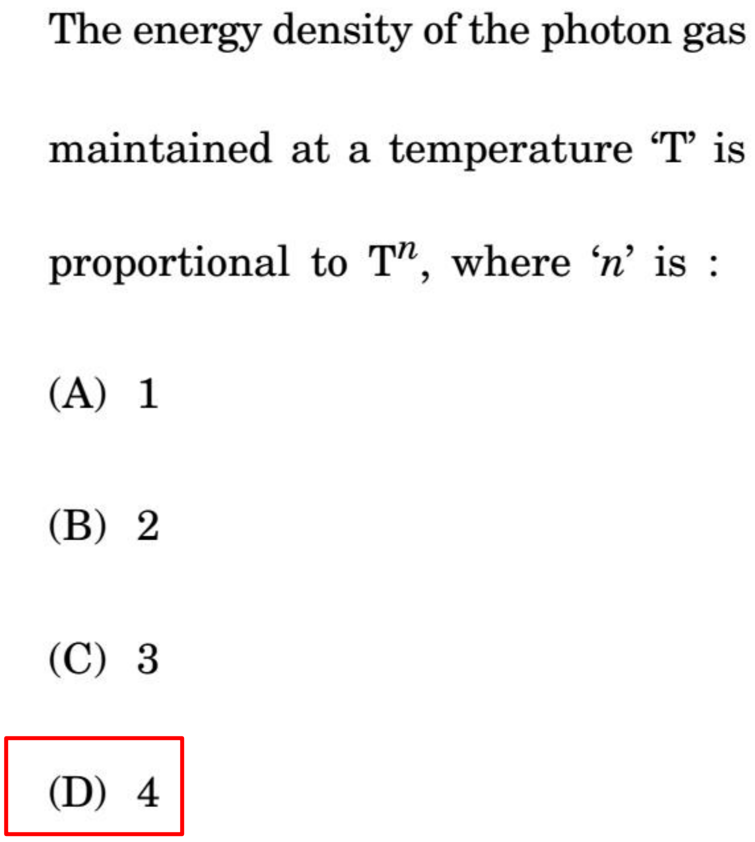
$$= \frac{g_i!}{n_i!(g_i-n_i)!}$$

$$= \frac{g_i!}{n_i!(g_i-n_i)!} = \frac{3!}{2!\times 1!}$$

$$= \frac{3\times 2\times 1}{2\times 1\times 1}$$

$$= 3$$

Feb 2013


The energy density of the photon gas maintained at a temperature ‘T’ is proportional to T^n, where ‘n’ is :

(A) 1

(B) 2

(C) 3

[✓ (D) 4]

The internal energy of the photon gas

$$U=\int_0^{\omega_D}\in_i n(\omega)d\omega$$

$$U=\frac{9NkT^4}{\theta_D^3}\int_0^{\frac{\theta_D}{T}}\frac{x^3}{e^x-1}dx$$

At low temperatures,

$$T<<\theta_D \qquad x=\frac{\hbar\omega_D}{kT}>>1 \qquad x\to\infty$$

$$U=\frac{3}{5}\frac{\pi^4Nk}{\theta_D^3}T^4$$

Feb 2013

The average value $\overline{v}$ (non-relativistic) of the velocity of a gas of molecules maintained at a temperature 'T' is given by :

(A) $\frac{1}{2}k_B\text{T}$

(B) zero

(C) $\frac{3}{2}k_B\text{T}$

(D) $\frac{k_B\text{T}}{m}$

$$f(u,v,w) = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-m(u^2+v^2+w^2)/2kT}$$

$$c^2 = u^2 + v^2 + w^2,$$

$$\overline{c^2} = \frac{3kT}{m},$$

$$\overline{c} = \sqrt{\frac{8kT}{\pi m}},$$

$$\boxed{\overline{u} = \overline{v} = \overline{w} = 0,}$$

$$\overline{u^2} = \overline{v^2} = \overline{w^2} = \frac{kT}{m},$$

$$\overline{\mathcal{T}} = \overline{\tfrac{1}{2}mc^2} = \tfrac{3}{2}kT.$$

Feb 2013

In the canonical ensembles the system :

(A) is not maintained at a fixed temperature

[✓ (B) can exchange energy with the surroundings]

(C) can exchange the number of particles with the surroundings

(D) is completely isolated from the surroundings

Feb 2013

The entropy of the universe in a reversible process is :

[✓ (A) is constant]

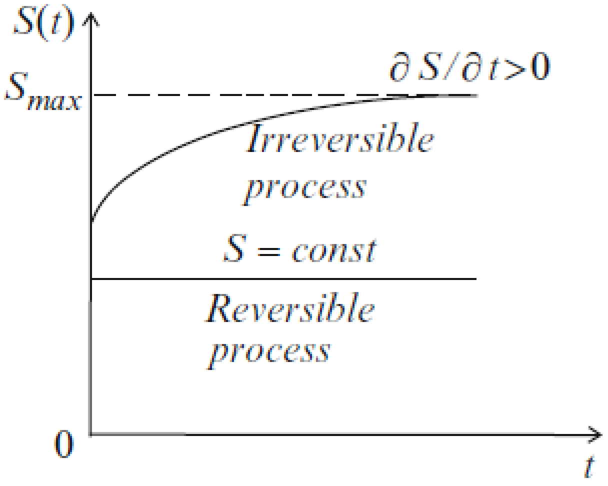
(B) is increasing

(C) is decreasing

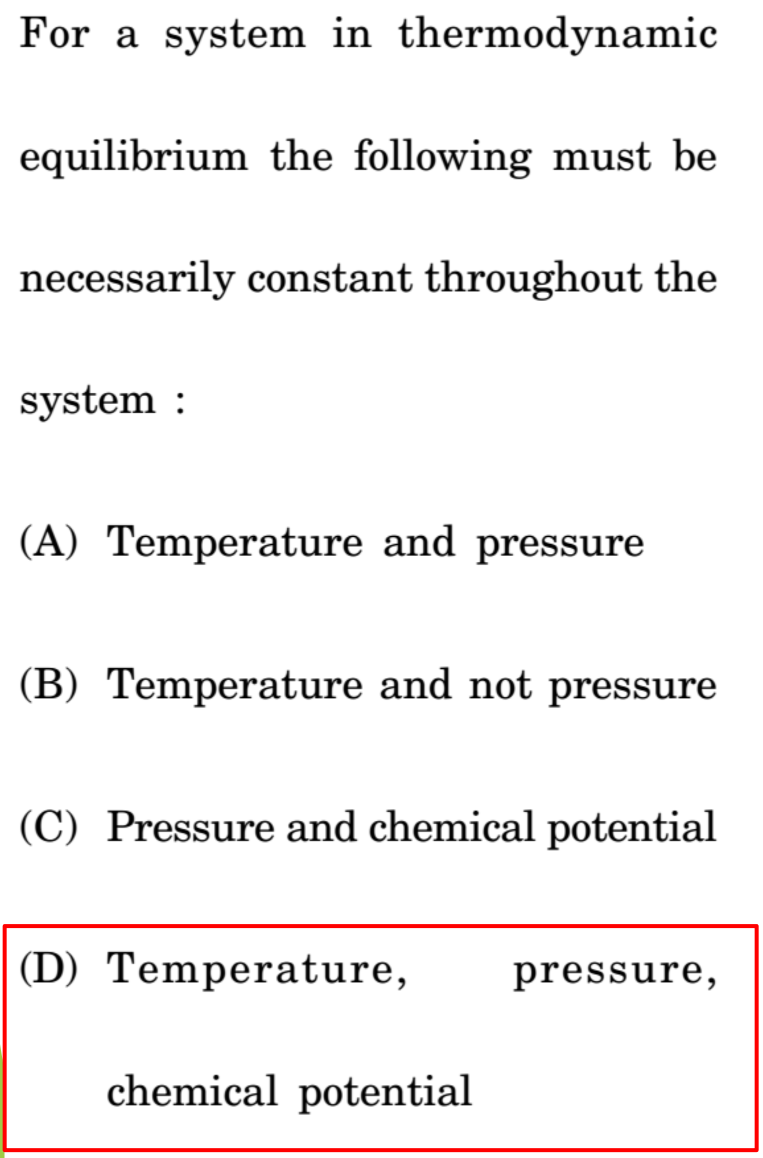
(D) becomes infinite

Thermodynamics, Gibbs Method and Statistical Physics of Electron Gases
Bahram M. Askerov

$$\mathrm{d}S/\mathrm{d}t > 0 \text{ – irreversible process}$$
$$\mathrm{d}S/\mathrm{d}t = 0 \text{ – reversible process.}$$



Feb 2013



For a system in thermodynamic equilibrium the following must be necessarily constant throughout the system :

(A) Temperature and pressure

(B) Temperature and not pressure

(C) Pressure and chemical potential

(D) Temperature, pressure, chemical potential

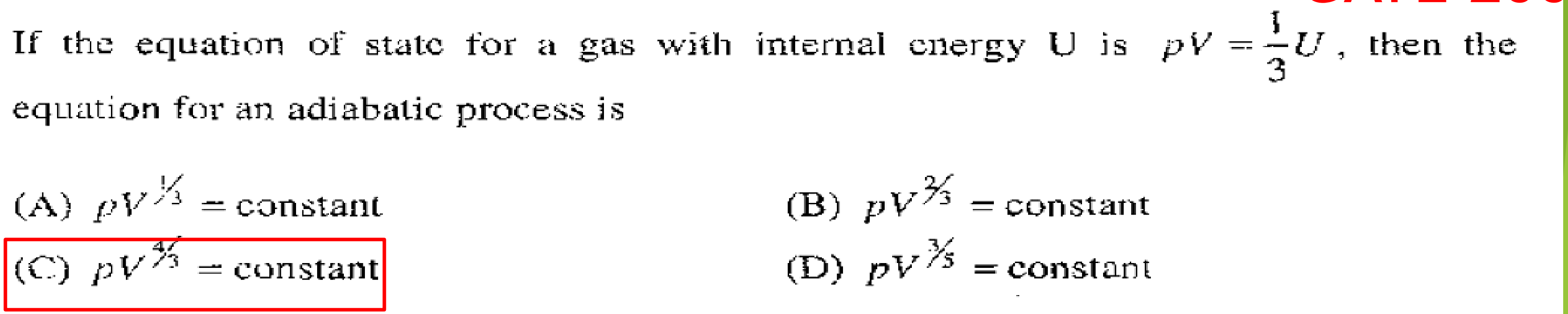
Therefore condition for phase equilibrium is

$T_1 = T_2$ (Thermal Equilibrium)

$P_1 = P_2$ (Mechanical Equilibrium)

$\mu_1 = \mu_2$ (Chemical Equilibrium)

GATE 2005



If the equation of state for a gas with internal energy U is $pV = \frac{1}{3}U$, then the equation for an adiabatic process is

(A) $pV^{1/3} = \text{constant}$

(B) $pV^{2/3} = \text{constant}$

$\boxed{\text{(C) } pV^{4/3} = \text{constant}}$

(D) $pV^{5/3} = \text{constant}$

Thermodynamics, Gibbs Method and Statistical Physics of Electron Gases
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Pg 298
Eq 7.300

$$P = \frac{\pi^2}{45c^3\hbar^3}(k_0T)^4 \sim T^4$$

$$VT^3 = \text{const}$$

$$PV^{4/3} = \text{const.}$$

GATE 2005

The pressure for a noninteracting Fermi gas with internal energy U at temperature T is

(A) $p = \frac{3}{2}\frac{U}{V}$

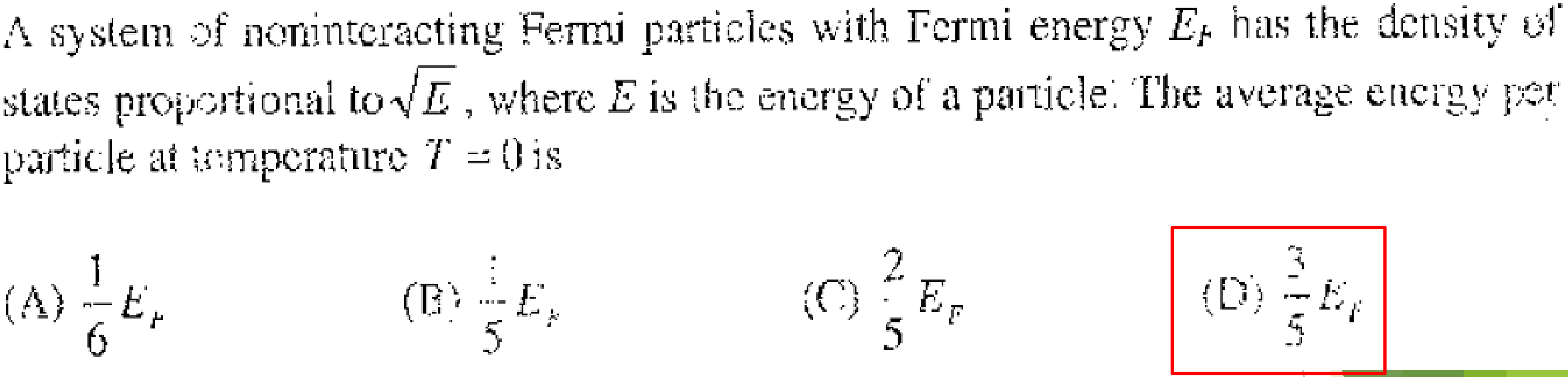
$\boxed{\text{(B)}\ p = \frac{2}{3}\frac{U}{V}}$

(C) $p = \frac{3}{5}\frac{U}{V}$

(D) $p = \frac{1}{2}\frac{U}{V}$

$$P_0 = \frac{2}{3}\left(\frac{U_0}{V}\right) = \frac{2}{3}\left(\frac{N}{V}\frac{3}{5}\in_F\right) = \frac{2}{5}\frac{N}{V}\in_F = \frac{2}{5}\rho\in_F$$

GATE 2005



A system of noninteracting Fermi particles with Fermi energy E_F has the density of states proportional to $\sqrt{E}$, where E is the energy of a particle. The average energy per particle at temperature $T = 0$ is

(A) $\frac{1}{6}E_F$ (B) $\frac{1}{5}E_F$ (C) $\frac{2}{5}E_F$ (D) $\boxed{\frac{3}{5}E_F}$

$$N = \frac{4\pi gV}{3h^3}(2m)^{3/2} \in_F^{3/2}$$

$$U_0 = \frac{4\pi gV}{3h^3}(2m)^{3/2} \in_F^{3/2} \frac{3}{5} \in_F = N\frac{3}{5} \in_F$$

$$\frac{U_0}{N} = \frac{3}{5} \in_F$$

GATE 2005

If the partition function of a harmonic oscillator with frequency ω at a temperature T is $\dfrac{kT}{\hbar\omega}$, then the free energy of N such independent oscillators is

(A) $\dfrac{3}{2}NkT$

(B) $kT\ln\dfrac{\hbar\omega}{kT}$

(C) $\boxed{NkT\ln\dfrac{\hbar\omega}{kT}}$

(D) $NkT\ln\dfrac{\hbar\omega}{2kT}$

$$Q_N = \left[\frac{kT}{\hbar\omega}\right]^N \qquad F = U - TS = A$$

$$A(N,V,T) = -kT\ln Q_N \qquad A(N,V,T) = -NkT\ln\left[\frac{kT}{\hbar\omega}\right] = NkT\ln\left[\frac{\hbar\omega}{kT}\right]$$

$$P = -\left(\frac{\partial A}{\partial V}\right)_{N,T} \qquad P = -\left(\frac{\partial}{\partial V}\left\{-NkT\ln\left[\frac{kT}{\hbar\omega}\right]\right\}\right)_{N,T} = 0$$

$$S = -\left(\frac{\partial A}{\partial T}\right)_{N,V} \qquad S = Nk\left[1+\ln\left(\frac{kT}{\hbar\omega}\right)\right]$$

$$U = A + TS \qquad U = NkT$$

GATE 2006

The free energy of a photon gas enclosed in a volume V is given by $F = -\frac{1}{3}aVT^4$, where a is a constant and T is the temperature of the gas. The chemical potential of the photon gas is

$\boxed{\text{(A)}\ \ 0}$

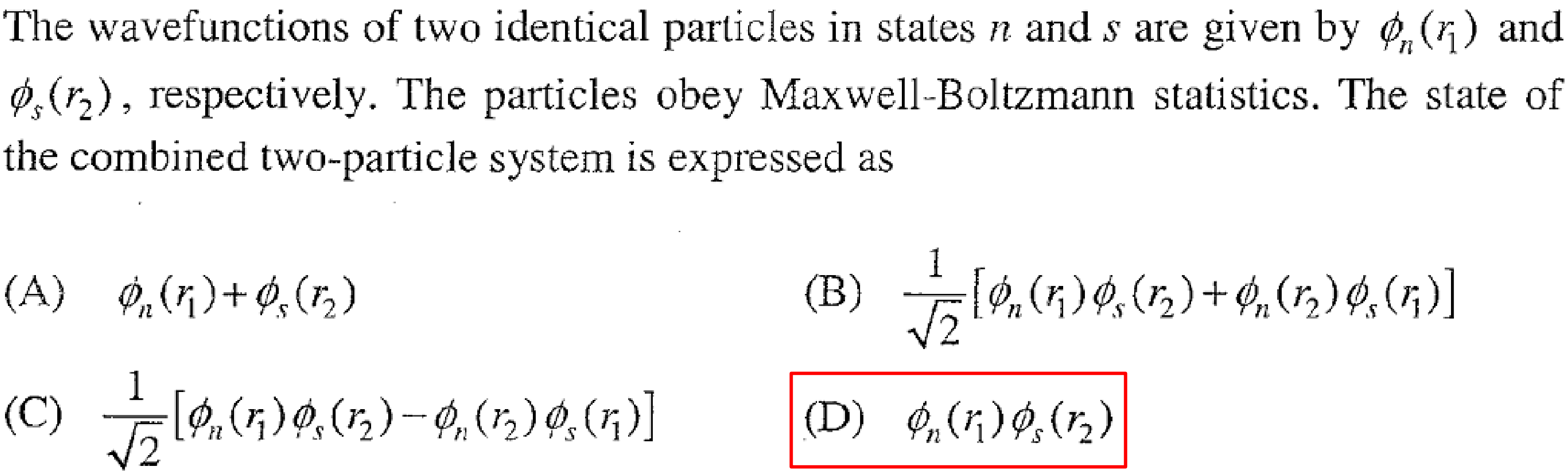
(B) $\frac{4}{3}aVT^3$

(C) $\frac{1}{3}aT^4$

(D) aVT^4

$$\mu = \left.\frac{\partial F}{\partial N}\right|_{T,V,\ldots},$$

GATE 2006



The wavefunctions of two identical particles in states n and s are given by $\phi_n(r_1)$ and $\phi_s(r_2)$, respectively. The particles obey Maxwell-Boltzmann statistics. The state of the combined two-particle system is expressed as

(A) $\phi_n(r_1)+\phi_s(r_2)$

(B) $\frac{1}{\sqrt{2}}[\phi_n(r_1)\phi_s(r_2)+\phi_n(r_2)\phi_s(r_1)]$

(C) $\frac{1}{\sqrt{2}}[\phi_n(r_1)\phi_s(r_2)-\phi_n(r_2)\phi_s(r_1)]$

(D) $\phi_n(r_1)\phi_s(r_2)$

When the particles are distinguishable there are two possibilities for occupancy of the states, as described by the wave functions

$$\psi_1 = \psi_a(1)\psi_b(2)$$
$$\psi_{11} = \psi_a(2)\psi_b(1)$$

If the particles are Bosons, the system is described by symmetric wave function

$$\psi_{Boson} = \frac{1}{\sqrt{2}}[\psi_a(1)\psi_b(2)+\psi_a(2)\psi_b(1)]$$

And if they are Fermions, the system is described by the antisymmetric wave function $\psi_{Fermion} = \frac{1}{\sqrt{2}}[\psi_a(1)\psi_b(2)-\psi_a(2)\psi_b(1)]$

GATE 2006

The mean internal energy of a one-dimensional classical harmonic oscillator in equilibrium with a heat bath of temperature T is

(A) $\frac{1}{2}k_BT$ $\boxed{\text{(B)}\quad k_BT}$ (C) $\frac{3}{2}k_BT$ (D) $3k_BT$

$$Q_N = \left[\frac{kT}{\hbar\omega}\right]^N \qquad F = U - TS = A$$

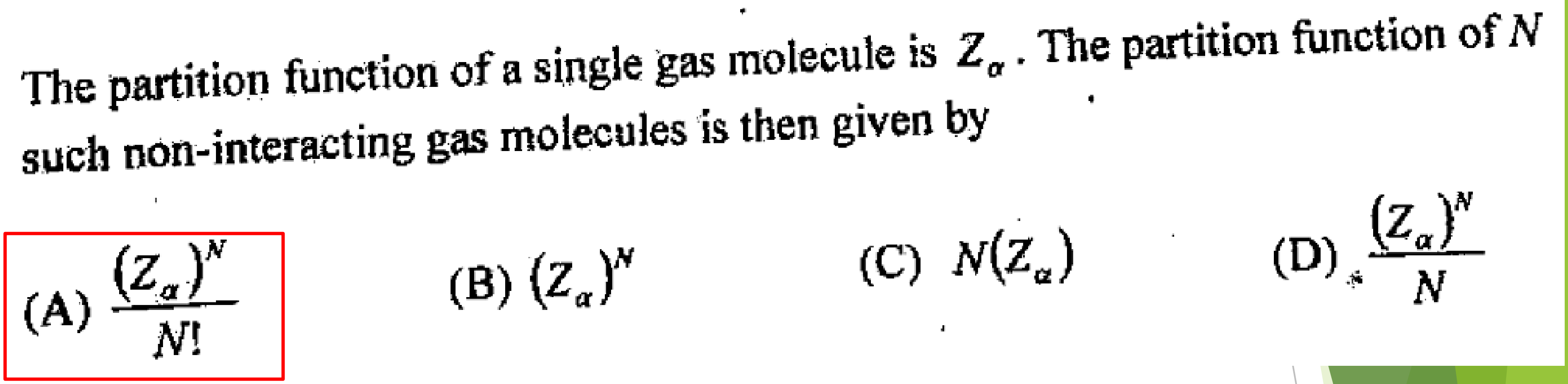
$$A(N,V,T) = -kT\ln Q_N \qquad A(N,V,T) = -NkT\ln\left[\frac{kT}{\hbar\omega}\right] = NkT\ln\left[\frac{\hbar\omega}{kT}\right]$$

$$P = -\left(\frac{\partial A}{\partial V}\right)_{N,T} \qquad P = -\left(\frac{\partial}{\partial V}\left\{-NkT\ln\left[\frac{kT}{\hbar\omega}\right]\right\}\right)_{N,T} = 0$$

$$S = -\left(\frac{\partial A}{\partial T}\right)_{N,V} \qquad S = Nk\left[1+\ln\left(\frac{kT}{\hbar\omega}\right)\right]$$

$$U = A + TS \qquad U = NkT$$

GATE 2007



The partition function of a single gas molecule is Z_α. The partition function of N such non-interacting gas molecules is then given by

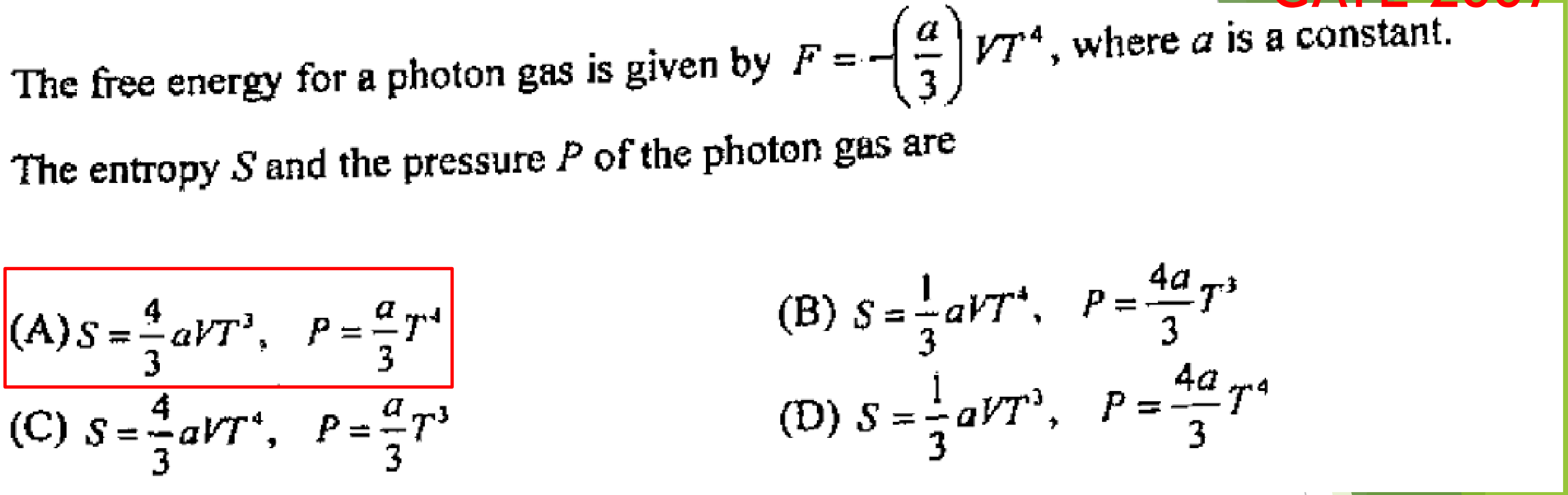
[✓ (A) $\dfrac{(Z_\alpha)^N}{N!}$]

(B) $(Z_\alpha)^N$

(C) $N(Z_\alpha)$

(D) $\dfrac{(Z_\alpha)^N}{N}$

GATE 2007



The free energy for a photon gas is given by $F = -\left(\frac{a}{3}\right) VT^4$, where a is a constant.

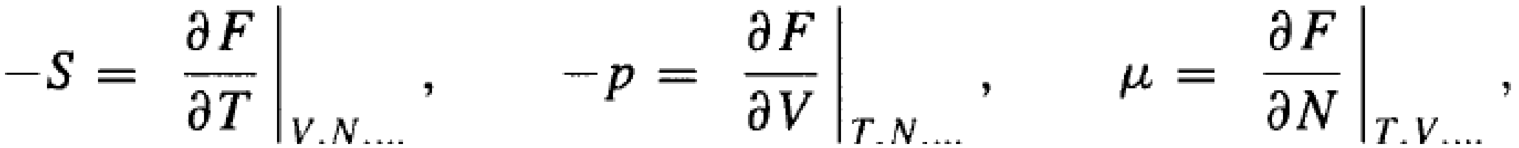
The entropy S and the pressure P of the photon gas are

$\boxed{\text{(A) } S = \frac{4}{3} aVT^3, \quad P = \frac{a}{3} T^4}$

(B) $S = \frac{1}{3} aVT^4, \quad P = \frac{4a}{3} T^3$

(C) $S = \frac{4}{3} aVT^4, \quad P = \frac{a}{3} T^3$

(D) $S = \frac{1}{3} aVT^3, \quad P = \frac{4a}{3} T^4$



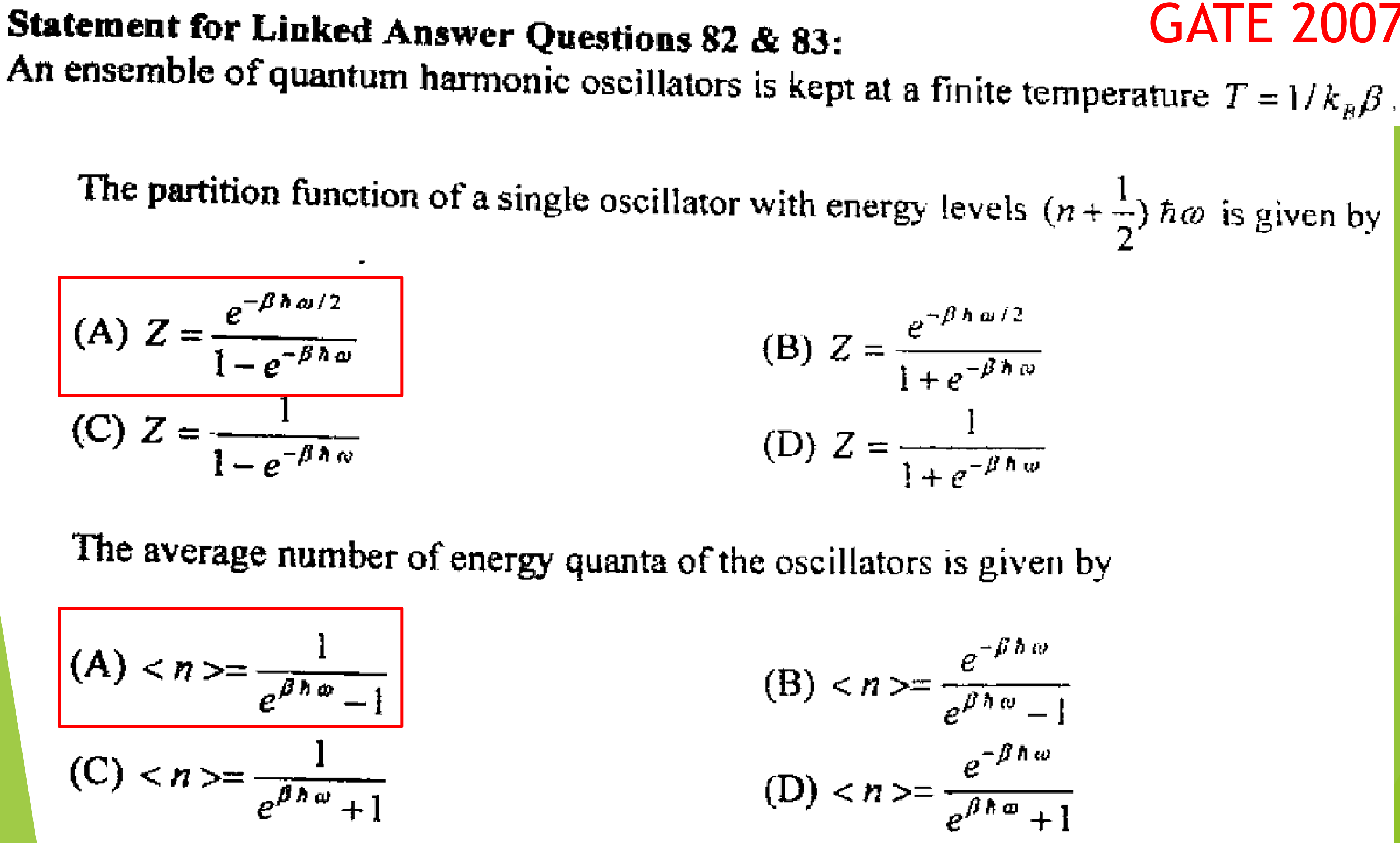
$$-S = \left.\frac{\partial F}{\partial T}\right|_{V,N,\ldots}, \qquad -p = \left.\frac{\partial F}{\partial V}\right|_{T,N,\ldots}, \qquad \mu = \left.\frac{\partial F}{\partial N}\right|_{T,V,\ldots},$$

$$F = -\frac{a}{3} VT^4$$

$$S = -\left.\frac{\partial F}{\partial T}\right|_{V,N} = \frac{4}{3} aVT^3$$

$$P = -\left.\frac{\partial F}{\partial V}\right|_{V,N} = \frac{a}{3} T^4$$

GATE 2007



Statement for Linked Answer Questions 82 & 83:

An ensemble of quantum harmonic oscillators is kept at a finite temperature $T = 1/k_B\beta$.

The partition function of a single oscillator with energy levels $(n+\frac{1}{2})\hbar\omega$ is given by

$\boxed{\text{(A) } Z = \dfrac{e^{-\beta\hbar\omega/2}}{1-e^{-\beta\hbar\omega}}}$ (B) $Z = \dfrac{e^{-\beta\hbar\omega/2}}{1+e^{-\beta\hbar\omega}}$

(C) $Z = \dfrac{1}{1-e^{-\beta\hbar\omega}}$ (D) $Z = \dfrac{1}{1+e^{-\beta\hbar\omega}}$

The average number of energy quanta of the oscillators is given by

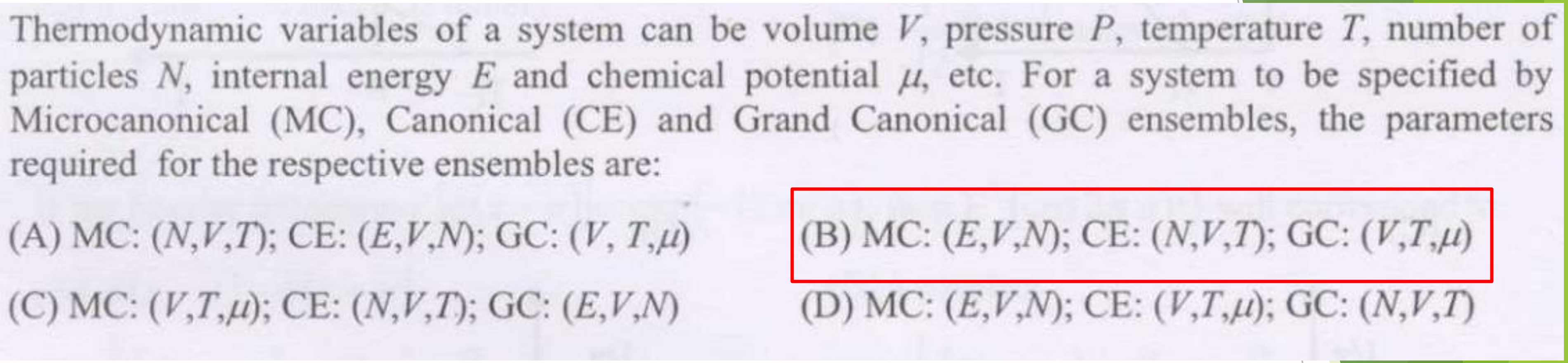
$\boxed{\text{(A) } <n> = \dfrac{1}{e^{\beta\hbar\omega}-1}}$ (B) $<n> = \dfrac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}-1}$

(C) $<n> = \dfrac{1}{e^{\beta\hbar\omega}+1}$ (D) $<n> = \dfrac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}+1}$

$$Z(T, V, N) = \left[\frac{\exp\{-\frac{1}{2}\beta\hbar\omega\}}{1-\exp\{-\beta\hbar\omega\}}\right] \qquad \langle n\rangle = \frac{1}{\exp\{\beta\hbar\omega\}-1}$$

Thermodynamics and Statistical Mechanics
2nd Ed Greiner-Neise-Stocker (pp 211)

GATE 2008



Thermodynamic variables of a system can be volume V, pressure P, temperature T, number of particles N, internal energy E and chemical potential μ, etc. For a system to be specified by Microcanonical (MC), Canonical (CE) and Grand Canonical (GC) ensembles, the parameters required for the respective ensembles are:

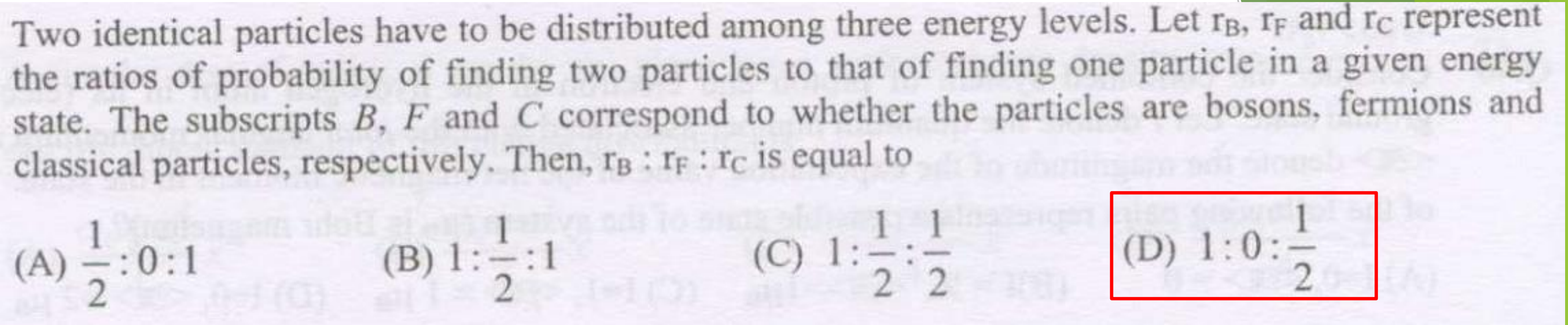
(A) MC: (N,V,T); CE: (E,V,N); GC: (V, T,μ)

(B) MC: (E,V,N); CE: (N,V,T); GC: (V,T,μ)

(C) MC: (V,T,μ); CE: (N,V,T); GC: (E,V,N)

(D) MC: (E,V,N); CE: (V,T,μ); GC: (N,V,T)

GATE 2008



Two identical particles have to be distributed among three energy levels. Let r_B, r_F and r_C represent the ratios of probability of finding two particles to that of finding one particle in a given energy state. The subscripts *B*, *F* and *C* correspond to whether the particles are bosons, fermions and classical particles, respectively. Then, $r_B : r_F : r_C$ is equal to

(A) $\frac{1}{2} : 0 : 1$

(B) $1 : \frac{1}{2} : 1$

(C) $1 : \frac{1}{2} : \frac{1}{2}$

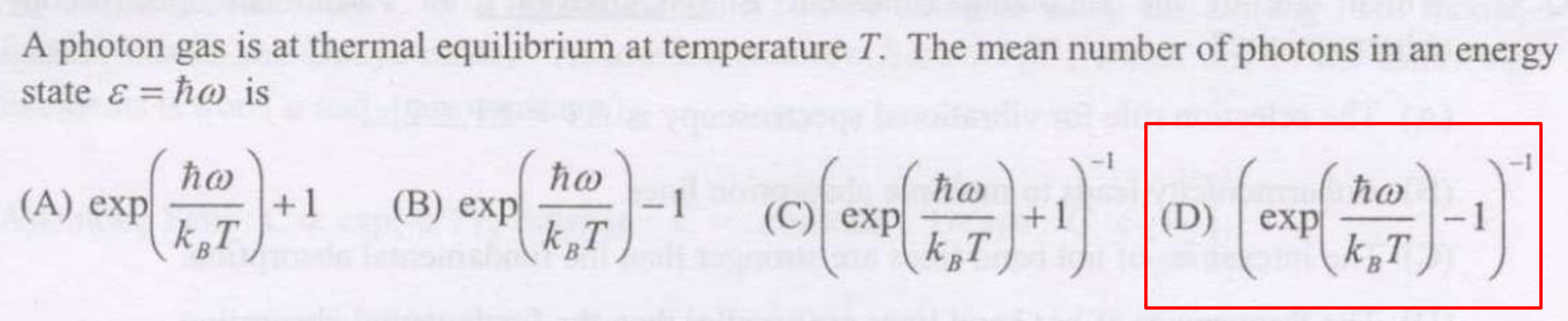
[✓ (D) $1 : 0 : \frac{1}{2}$]

$$r_{MB} = \frac{3}{6} = \frac{1}{2}$$

$$r_{BE} = \frac{3}{3} = 1$$

$$r_{FD} = \frac{0}{3} = 0$$

GATE 2008

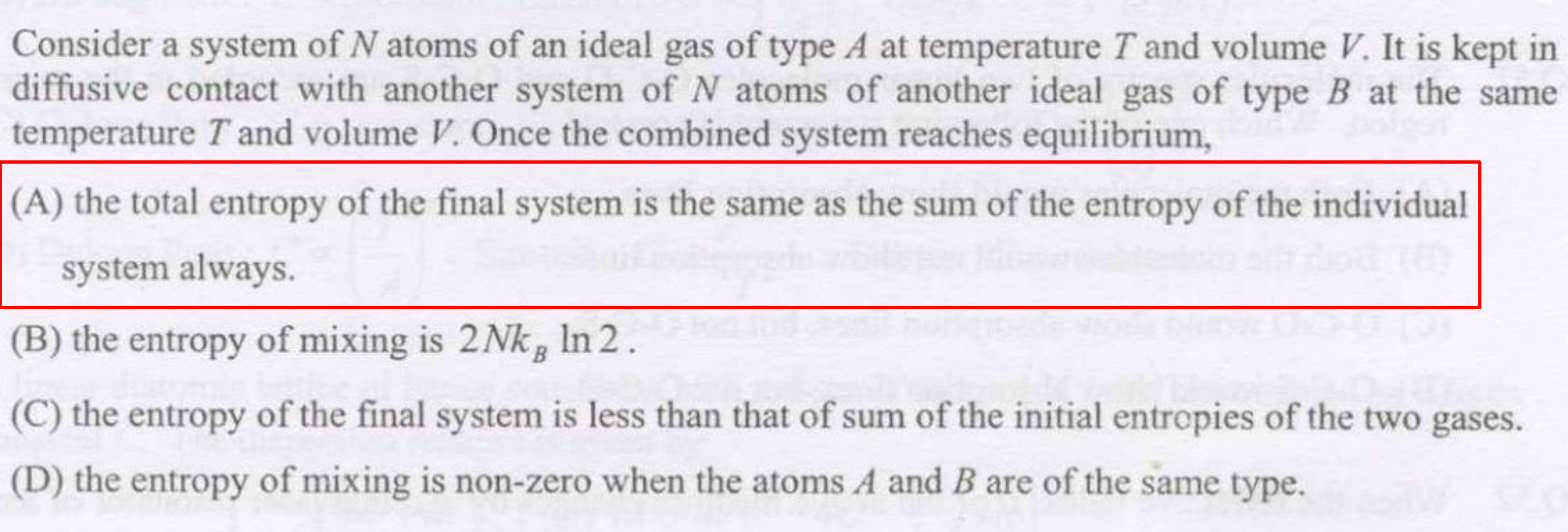


A photon gas is at thermal equilibrium at temperature T. The mean number of photons in an energy state $\varepsilon = \hbar\omega$ is

(A) $\exp\left(\dfrac{\hbar\omega}{k_B T}\right)+1$ (B) $\exp\left(\dfrac{\hbar\omega}{k_B T}\right)-1$ (C) $\left(\exp\left(\dfrac{\hbar\omega}{k_B T}\right)+1\right)^{-1}$ (D) $\boxed{\left(\exp\left(\dfrac{\hbar\omega}{k_B T}\right)-1\right)^{-1}}$

$$\langle n_\epsilon \rangle = \frac{1}{\exp\{\beta\epsilon\} - 1}$$

GATE 2008



Consider a system of N atoms of an ideal gas of type A at temperature T and volume V. It is kept in diffusive contact with another system of N atoms of another ideal gas of type B at the same temperature T and volume V. Once the combined system reaches equilibrium,

(A) the total entropy of the final system is the same as the sum of the entropy of the individual system always.

(B) the entropy of mixing is $2Nk_B \ln 2$.

(C) the entropy of the final system is less than that of sum of the initial entropies of the two gases.

(D) the entropy of mixing is non-zero when the atoms A and B are of the same type.

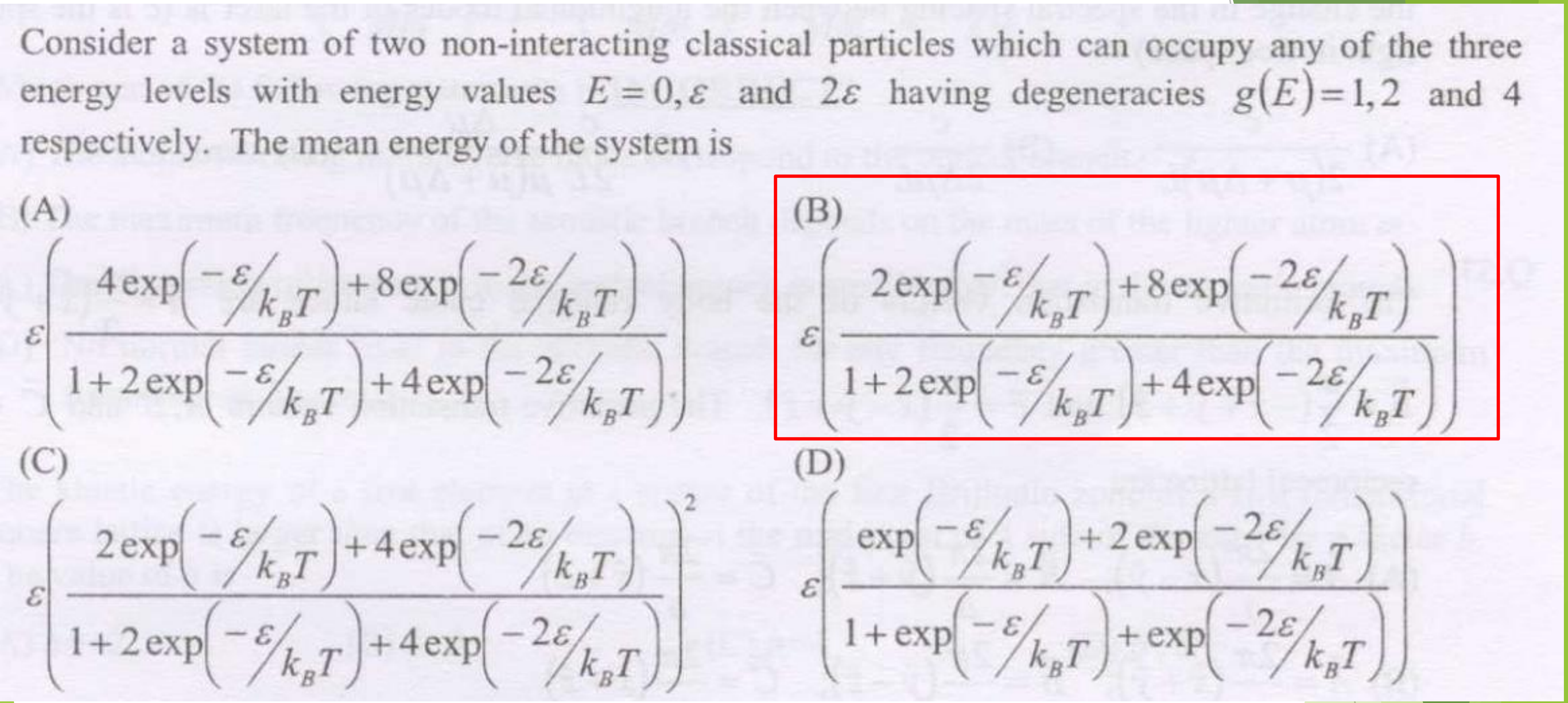
$$S = NkLn\left[V\left(\frac{E}{N}\right)^{3/2}\left(\frac{4\pi m}{3h^2}\right)^{3/2}\right]+\frac{3Nk}{2} = NkLn\left[V\left(\frac{3}{2}\frac{NkT}{N}\right)^{3/2}\left(\frac{4\pi m}{3h^2}\right)^{3/2}\right]+\frac{3Nk}{2}$$

$$S = NkLn\left[V\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right]+\frac{3Nk}{2}$$

$$S' = 2NkLn\left[2V\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right]+\frac{3(2N)k}{2}$$

$$S' = 2NkLn2 + 2\left\{NkLn\left[V\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right]+\frac{3Nk}{2}\right\} = 2NkLn2 + 2S$$

GATE 2008



Consider a system of two non-interacting classical particles which can occupy any of the three energy levels with energy values $E=0,\varepsilon$ and 2ε having degeneracies $g(E)=1,2$ and 4 respectively. The mean energy of the system is

(A) $$\varepsilon\left(\frac{4\exp\left(-\varepsilon/k_BT\right)+8\exp\left(-2\varepsilon/k_BT\right)}{1+2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}\right)$$

(B) $$\varepsilon\left(\frac{2\exp\left(-\varepsilon/k_BT\right)+8\exp\left(-2\varepsilon/k_BT\right)}{1+2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}\right)$$

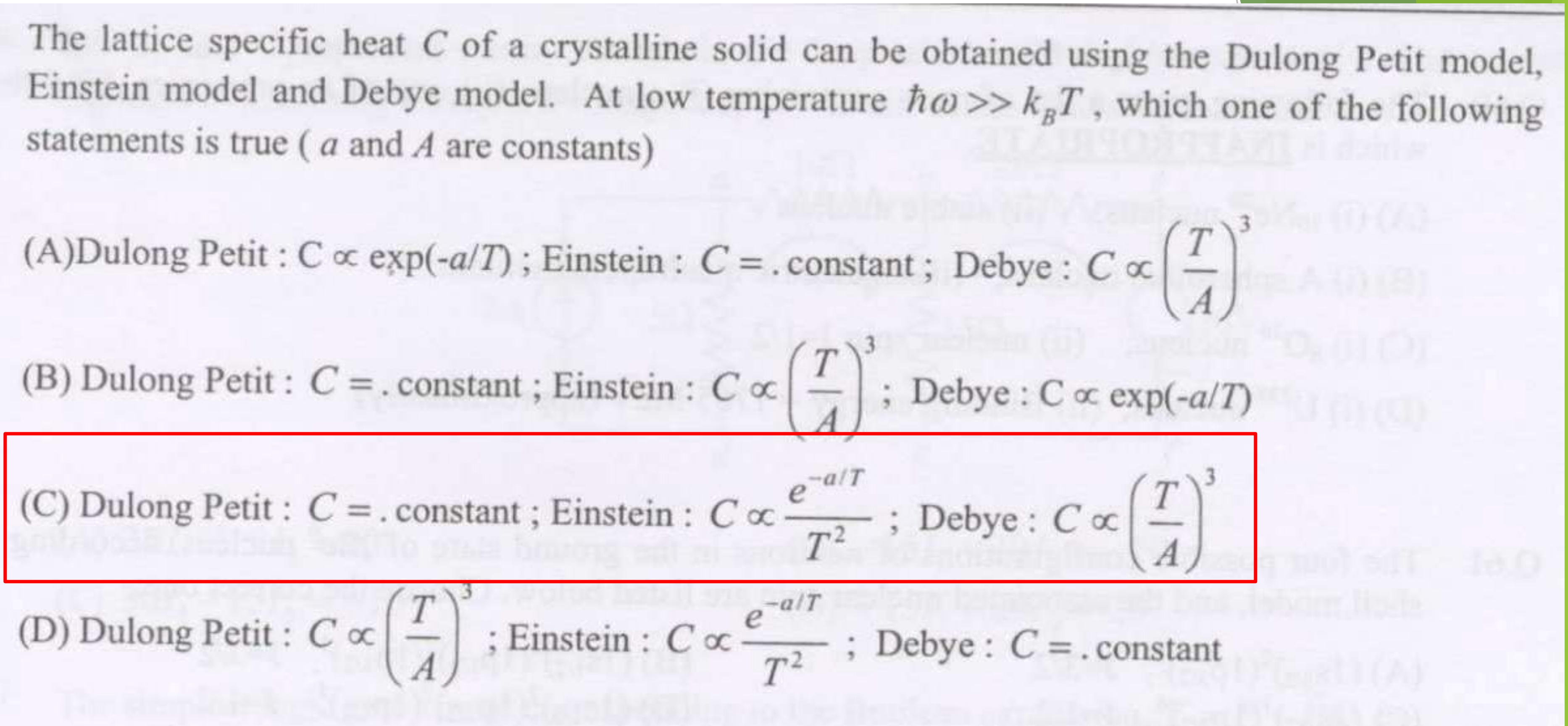
(C) $$\varepsilon\left(\frac{2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}{1+2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}\right)^2$$

(D) $$\varepsilon\left(\frac{\exp\left(-\varepsilon/k_BT\right)+2\exp\left(-2\varepsilon/k_BT\right)}{1+\exp\left(-\varepsilon/k_BT\right)+\exp\left(-2\varepsilon/k_BT\right)}\right)$$

$$<f>=\frac{\sum_n f_n g_n e^{-E/kT}}{\sum_n g_n e^{-E/kT}}$$

$$U=\frac{(1)(0)e^{-0/kT}+(2)(\varepsilon)e^{-\varepsilon/kT}+(4)(2\varepsilon)e^{-2\varepsilon/kT}}{(1)e^{-0/kT}+(2)e^{-\varepsilon/kT}+(4)e^{-2\varepsilon/kT}}=\varepsilon\frac{2e^{-\varepsilon/kT}+8e^{-2\varepsilon/kT}}{1+2e^{-\varepsilon/kT}+4e^{-2\varepsilon/kT}}$$

GATE 2008



The lattice specific heat C of a crystalline solid can be obtained using the Dulong Petit model, Einstein model and Debye model. At low temperature $\hbar\omega >> k_B T$, which one of the following statements is true (a and A are constants)

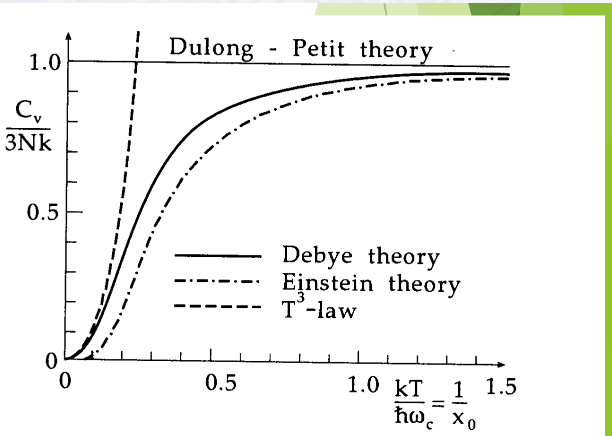
(A) Dulong Petit : $C \propto \exp(-a/T)$; Einstein : $C =$. constant ; Debye : $C \propto \left(\frac{T}{A}\right)^3$

(B) Dulong Petit : $C =$. constant ; Einstein : $C \propto \left(\frac{T}{A}\right)^3$; Debye : $C \propto \exp(-a/T)$

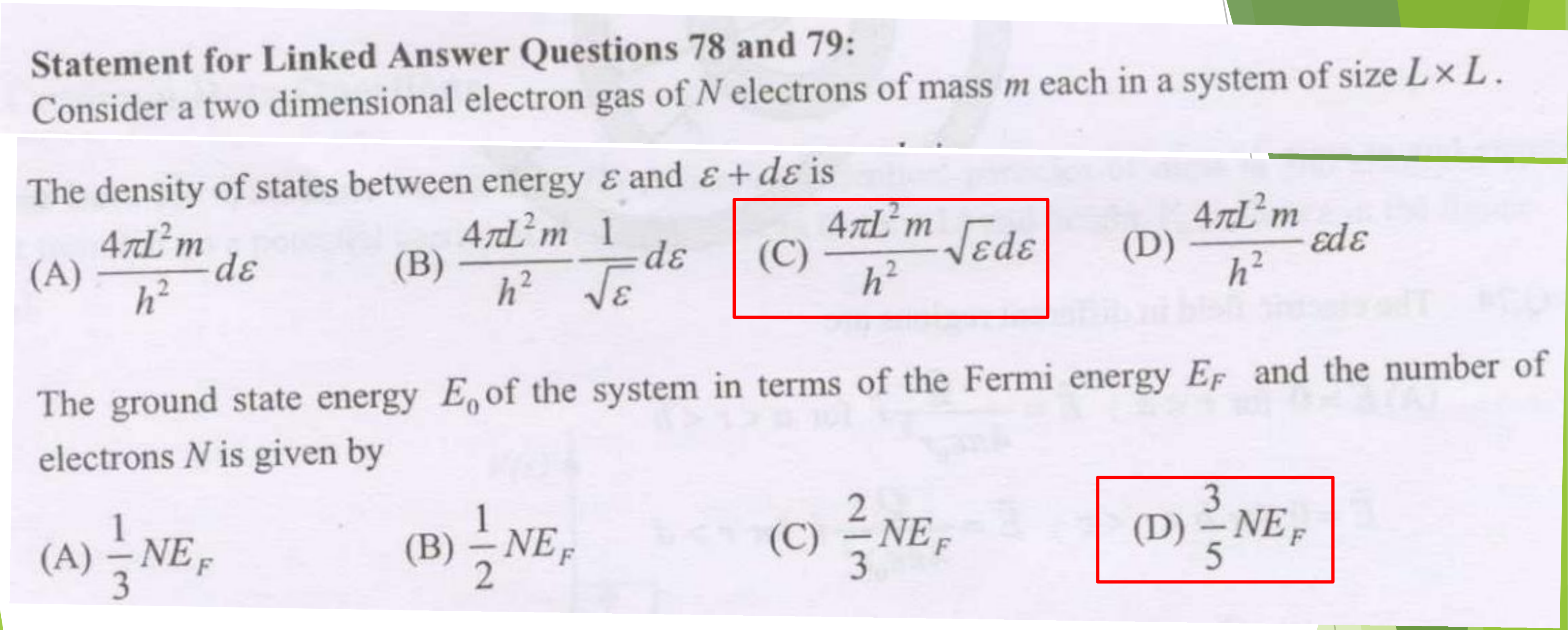
(C) Dulong Petit : $C =$. constant ; Einstein : $C \propto \frac{e^{-a/T}}{T^2}$; Debye : $C \propto \left(\frac{T}{A}\right)^3$

(D) Dulong Petit : $C \propto \left(\frac{T}{A}\right)^3$; Einstein : $C \propto \frac{e^{-a/T}}{T^2}$; Debye : $C =$. constant

$$Cv = \left(\frac{\partial U}{\partial T}\right)_V = \frac{12}{5}\frac{\pi^4 Nk}{\theta_D^3}T^3 = \frac{12}{5}\pi^4 Nk\left(\frac{T}{\theta_D}\right)^3$$



GATE 2008



Statement for Linked Answer Questions 78 and 79:
Consider a two dimensional electron gas of N electrons of mass m each in a system of size $L \times L$.

The density of states between energy ε and $\varepsilon + d\varepsilon$ is

(A) $\frac{4\pi L^2 m}{h^2} d\varepsilon$ (B) $\frac{4\pi L^2 m}{h^2} \frac{1}{\sqrt{\varepsilon}} d\varepsilon$ [✓ (C) $\frac{4\pi L^2 m}{h^2} \sqrt{\varepsilon} d\varepsilon$] (D) $\frac{4\pi L^2 m}{h^2} \varepsilon d\varepsilon$

The ground state energy E_0 of the system in terms of the Fermi energy E_F and the number of electrons N is given by

(A) $\frac{1}{3} NE_F$ (B) $\frac{1}{2} NE_F$ (C) $\frac{2}{3} NE_F$ [✓ (D) $\frac{3}{5} NE_F$]

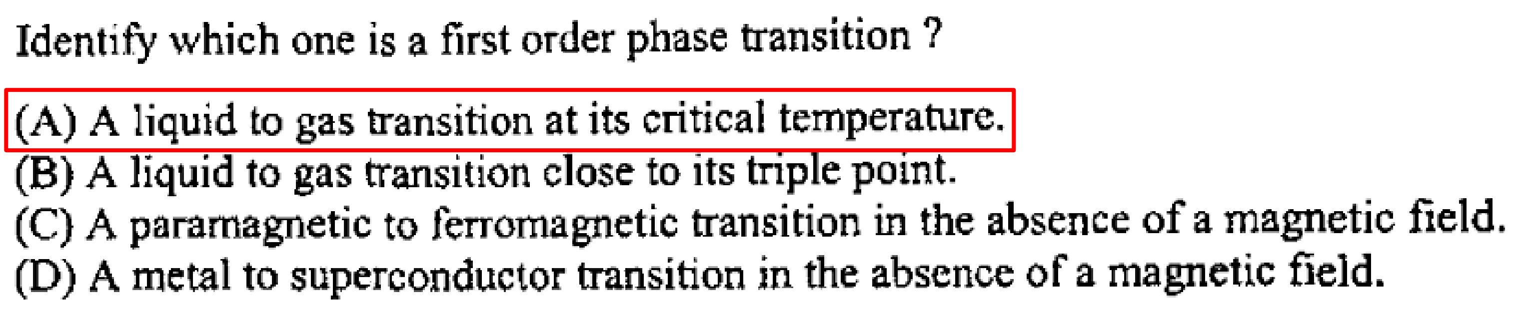
$$N = \frac{4\pi gV}{3h^3} (2m)^{3/2} \in_F^{3/2}$$

$$g(\in) d \in = \frac{2\pi gV}{h^3} (2m)^{3/2} \in^{1/2} d \in$$

$$U_0 = \frac{4\pi gV}{3h^3} (2m)^{3/2} \in_F^{3/2} \frac{3}{5} \in_F = N \frac{3}{5} \in_F$$

$$\frac{U_0}{N} = \frac{3}{5} \in_F$$

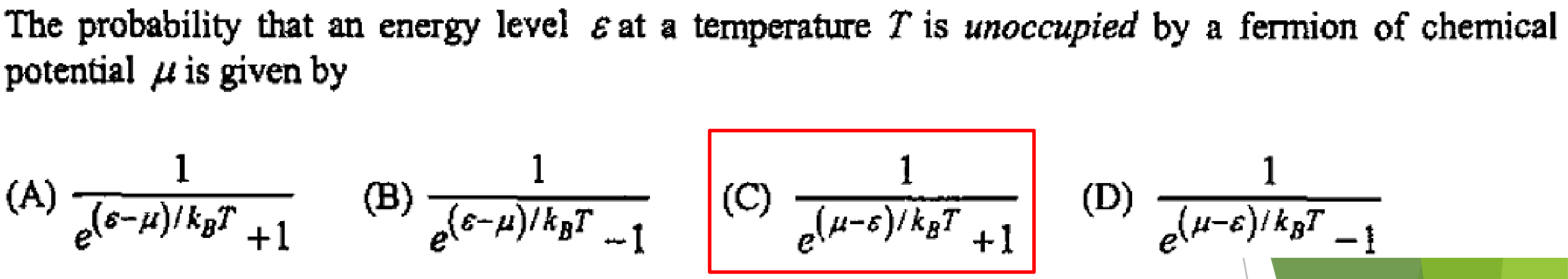
GATE 2009



Identify which one is a first order phase transition ?

(A) A liquid to gas transition at its critical temperature.
(B) A liquid to gas transition close to its triple point.
(C) A paramagnetic to ferromagnetic transition in the absence of a magnetic field.
(D) A metal to superconductor transition in the absence of a magnetic field.

GATE 2009



The probability that an energy level ε at a temperature T is *unoccupied* by a fermion of chemical potential μ is given by

(A) $\frac{1}{e^{(\varepsilon-\mu)/k_BT}+1}$ (B) $\frac{1}{e^{(\varepsilon-\mu)/k_BT}-1}$ (C) $\boxed{\frac{1}{e^{(\mu-\varepsilon)/k_BT}+1}}$ (D) $\frac{1}{e^{(\mu-\varepsilon)/k_BT}-1}$

$$P_{occ}=\frac{1}{\exp\left(\frac{\varepsilon-\mu}{k_BT}\right)+1}$$

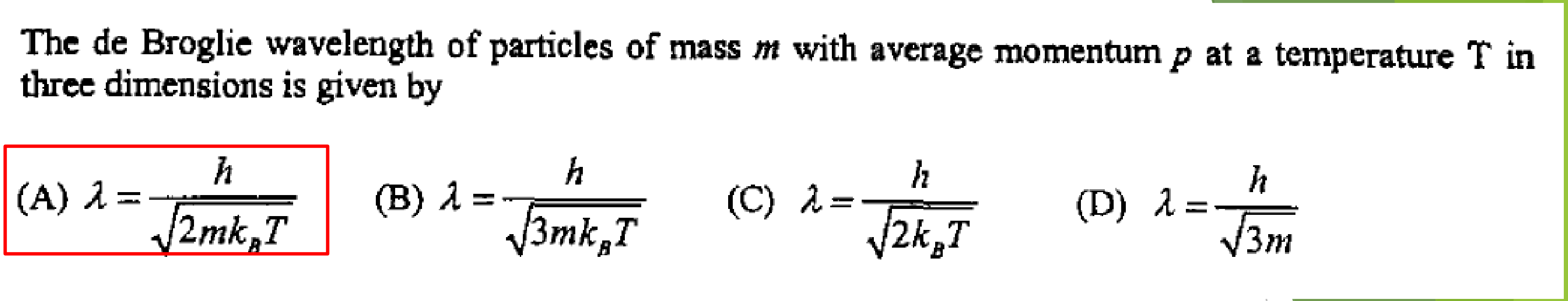
$$P_{un-occ}=1-\frac{1}{\exp\left(\frac{\varepsilon-\mu}{k_BT}\right)+1}=\frac{\exp\left(\frac{\varepsilon-\mu}{k_BT}\right)+1-1}{\exp\left(\frac{\varepsilon-\mu}{k_BT}\right)+1}=\frac{\exp\left(\frac{\varepsilon-\mu}{k_BT}\right)}{\exp\left(\frac{\varepsilon-\mu}{k_BT}\right)+1}=\frac{1}{1+\exp\left(\frac{-\varepsilon+\mu}{k_BT}\right)}$$

GATE 2009

For a Fermi gas of N particles in three dimensions at T = 0 K, the Fermi energy, E_F is proportional to

[✓ (A) $N^{2/3}$] (B) $N^{3/2}$ (C) N^3 (D) N^2

$$\in_F = \left(\frac{3N}{4\pi g V}\right)^{2/3}\left(\frac{h^2}{2m}\right)$$



The de Broglie wavelength of particles of mass *m* with average momentum *p* at a temperature T in three dimensions is given by

(A) $\lambda = \frac{h}{\sqrt{2mk_BT}}$ (B) $\lambda = \frac{h}{\sqrt{3mk_BT}}$ (C) $\lambda = \frac{h}{\sqrt{2k_BT}}$ (D) $\lambda = \frac{h}{\sqrt{3m}}$

$$\lambda_T = \sqrt{\frac{h^2}{2\pi mkT}} = \frac{h}{(2\pi mkT)^{1/2}}$$

GATE 2009

Consider a two level quantum system with energies $\varepsilon_1 = 0$ and $\varepsilon_2 = \varepsilon$.

Q.51 The Helmholtz free energy of the system is given by

$\boxed{\text{(A)}\ -k_BT\ln\left(1+e^{-\varepsilon/k_BT}\right)}$

(B) $k_BT\ln\left(1+e^{-\varepsilon/k_BT}\right)$

(C) $\frac{3}{2}k_BT$

(D) $\varepsilon - k_BT$

Q.52 The specific heat of the system is given by

(A) $\frac{\varepsilon}{k_BT}\frac{e^{-\varepsilon/k_BT}}{\left(1+e^{-\varepsilon/k_BT}\right)^2}$

(B) $\frac{\varepsilon^2}{k_BT^2}\frac{e^{-\varepsilon/k_BT}}{\left(1+e^{-\varepsilon/k_BT}\right)}$

(C) $-\frac{\varepsilon^2 e^{-\varepsilon/k_BT}}{\left(1+e^{-\varepsilon/k_BT}\right)^2}$

$\boxed{\text{(D)}\ \frac{\varepsilon^2}{k_BT^2}\frac{e^{-\varepsilon/k_BT}}{\left(1+e^{-\varepsilon/k_BT}\right)^2}}$

$$Q_N = \sum_n e^{-E/kT}$$

$$Q_N = e^{0/kT} + e^{-\varepsilon/kT} = 1 + e^{-\varepsilon/kT}$$

$$A = -kT \ln Q_N$$

$$A = -kT \ln(1 + e^{-\varepsilon/kT})$$

$$S = -\left(\frac{\partial A}{\partial T}\right)_{N,V} = -\frac{\partial}{\partial T}\left[-kT \ln(1 + e^{-\varepsilon/kT})\right]$$

$$S = -(-k)\left[T \frac{1}{(1 + e^{-\varepsilon/kT})} e^{-\varepsilon/kT}\left(\frac{\varepsilon}{kT^2}\right) + \ln(1 + e^{-\varepsilon/kT}\right]$$

$$S = \left[\left(\frac{\varepsilon}{T}\right)\frac{e^{-\varepsilon/kT}}{(1 + e^{-\varepsilon/kT})} + k \ln(1 + e^{-\varepsilon/kT}\right]$$

GATE 2009

$$U = A + TS$$

$$U = -kT\ln(1+e^{-\varepsilon/kT}) + T\left[\left(\frac{\varepsilon}{T}\right)\frac{e^{-\varepsilon/kT}}{(1+e^{-\varepsilon/kT})} + k\ln(1+e^{-\varepsilon/kT})\right]$$

$$U = -kT\ln(1+e^{-\varepsilon/kT}) + kT\ln(1+e^{-\varepsilon/kT}) + \varepsilon\frac{e^{-\varepsilon/kT}}{(1+e^{-\varepsilon/kT})} = \varepsilon\frac{e^{-\varepsilon/kT}}{(1+e^{-\varepsilon/kT})}$$

$$U = \varepsilon\frac{e^{-\varepsilon/kT}}{(1+e^{-\varepsilon/kT})}$$

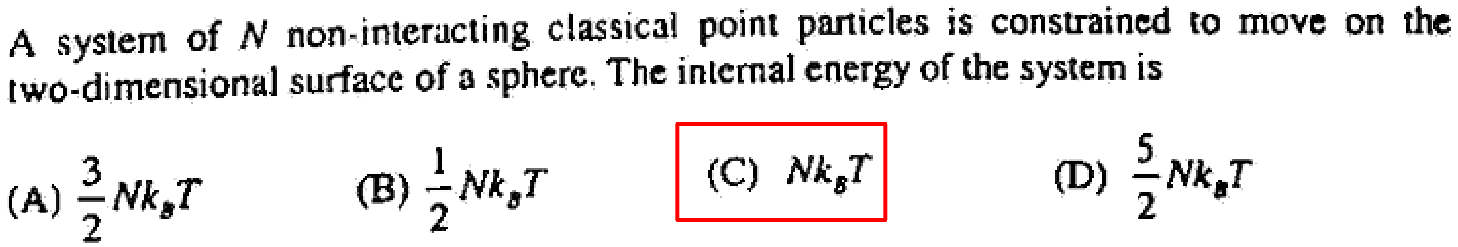
$$Cv = \frac{\partial U}{\partial T} = \varepsilon\left[\frac{(1+e^{-\varepsilon/kT})e^{-\varepsilon/kT}\left(\varepsilon/kT^{2}\right) - e^{-\varepsilon/kT}\left(0 + e^{-\varepsilon/kT}\left(\varepsilon/kT^{2}\right)\right)}{(1+e^{-\varepsilon/kT})^{2}}\right]$$

$$Cv = \frac{\varepsilon^{2}}{kT^{2}}\left[\frac{(1+e^{-\varepsilon/kT})e^{-\varepsilon/kT} - e^{-2\varepsilon/kT}}{(1+e^{-\varepsilon/kT})^{2}}\right]$$

$$Cv = \frac{\varepsilon^{2}}{kT^{2}}\left[\frac{e^{-\varepsilon/kT} + e^{-2\varepsilon/kT} - e^{-2\varepsilon/kT}}{(1+e^{-\varepsilon/kT})^{2}}\right]$$

$$Cv = \frac{\varepsilon^{2}}{kT^{2}}\left[\frac{e^{-\varepsilon/kT}}{(1+e^{-\varepsilon/kT})^{2}}\right]$$

GATE 2010



A system of N non-interacting classical point particles is constrained to move on the two-dimensional surface of a sphere. The internal energy of the system is

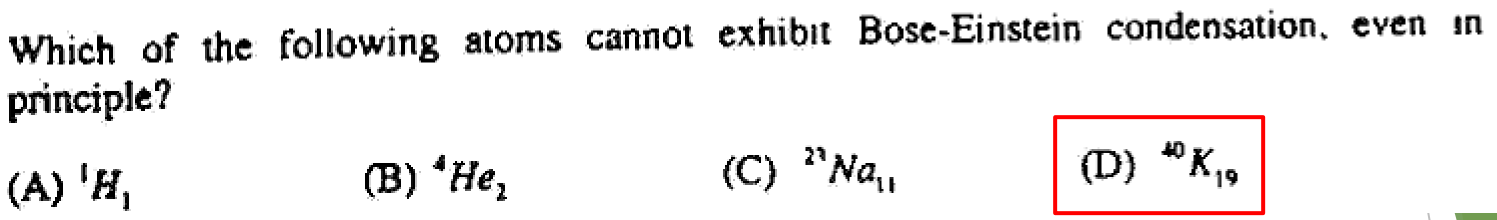
(A) $\frac{3}{2}Nk_BT$

(B) $\frac{1}{2}Nk_BT$

[✓ (C) Nk_BT]

(D) $\frac{5}{2}Nk_BT$

GATE 2010



Which of the following atoms cannot exhibit Bose-Einstein condensation, even in principle?

(A) $^1H_1$ (B) $^4He_2$ (C) $^{23}Na_{11}$ (D) $^{40}K_{19}$

The composite objects that have even number of Fermions are Bosons and those containing an odd number of Fermions are themselves Fermions.

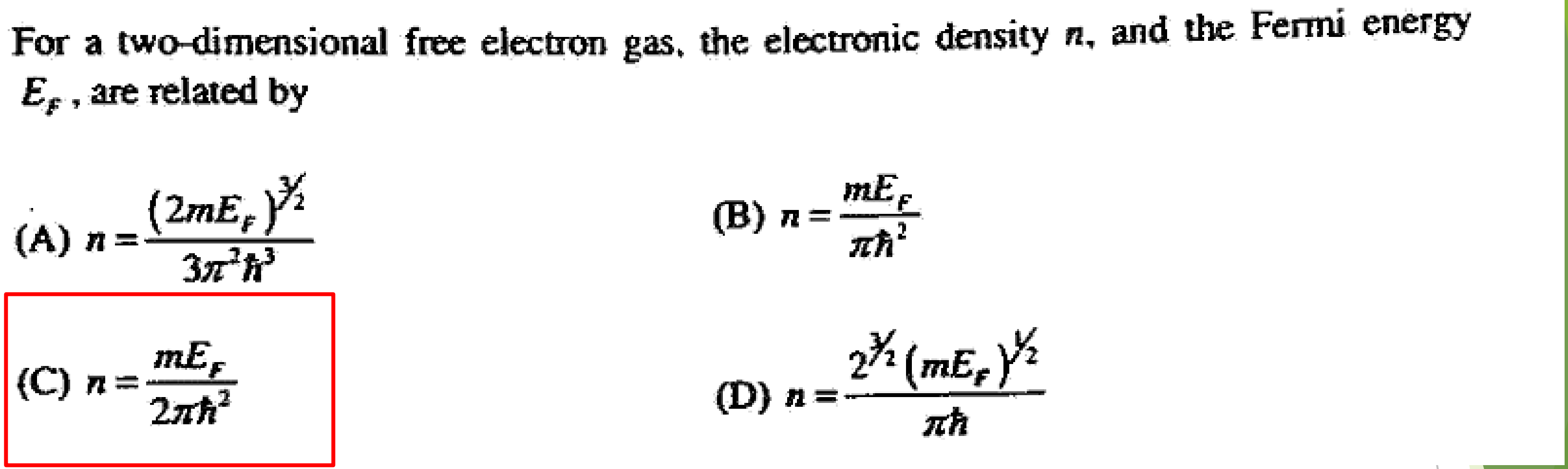
(an atom of ^{3}He = 2 electrons + 2 protons + 1 neutron ⇒ hence ^{3}He atom is a Fermion)

(an atom of ^{4}He = 2 electrons + 2 protons + 2 neutron ⇒ hence ^{4}He atom is a Boson)

In general, if a neutral atom contains an odd number of neutrons then it is a Fermion, and if it contains en even number of neutrons then it is a Boson.

Atom	Electrons	Protons	Neutrons	Total
$^1H_1$	1	1	0	2
$^4He_2$	2	2	2	6
$^{23}Na_{11}$	11	11	12	34
$^{40}K_{19}$	19	19	21	59

GATE 2010



For a two-dimensional free electron gas, the electronic density n, and the Fermi energy E_F, are related by

(A) $n = \frac{(2mE_F)^{3/2}}{3\pi^2\hbar^3}$

(B) $n = \frac{mE_F}{\pi\hbar^2}$

(C) $\boxed{n = \frac{mE_F}{2\pi\hbar^2}}$

(D) $n = \frac{2^{3/2}(mE_F)^{1/2}}{\pi\hbar}$

$$\in_F = \left(\frac{3\rho}{4\pi g}\right)^{2/3}\left(\frac{h^2}{2m}\right) = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$

3D

GATE 2010

Which among the following sets of Maxwell relations is correct? (U - internal energy, H - enthalpy, A - Helmholtz free energy and G - Gibbs free energy)

(A) $T = \left(\frac{\partial U}{\partial V}\right)_S$ and $P = \left(\frac{\partial U}{\partial S}\right)_V$

$\boxed{\text{(B) } V = \left(\frac{\partial H}{\partial P}\right)_S \text{ and } T = \left(\frac{\partial H}{\partial S}\right)_P}$

(C) $P = -\left(\frac{\partial G}{\partial V}\right)_T$ and $V = \left(\frac{\partial G}{\partial P}\right)_S$

(D) $P = -\left(\frac{\partial A}{\partial S}\right)_T$ and $S = -\left(\frac{\partial A}{\partial P}\right)_V$

$$T = \left.\frac{\partial U}{\partial S}\right|_{V,N,\ldots}, \qquad -p = \left.\frac{\partial U}{\partial V}\right|_{S,N,\ldots}, \qquad \mu = \left.\frac{\partial U}{\partial N}\right|_{S,V,\ldots}$$

$$-S = \left.\frac{\partial F}{\partial T}\right|_{V,N,\ldots}, \qquad -p = \left.\frac{\partial F}{\partial V}\right|_{T,N,\ldots}, \qquad \mu = \left.\frac{\partial F}{\partial N}\right|_{T,V,\ldots},$$

$$-S = \left.\frac{\partial G}{\partial T}\right|_{p,N,\ldots}, \qquad V = \left.\frac{\partial G}{\partial p}\right|_{T,N,\ldots}, \qquad \mu = \left.\frac{\partial G}{\partial N}\right|_{T,p,\ldots}$$

we have the following **thermodynamic potentials:**

(i) The **Helmholtz free energy**,

$$U[T] = F(T, V, N) = U - TS, \tag{3.51}$$

where the variable S has been replaced by the temperature T.

(ii) The **enthalpy**,

$$U[p] = H(S, p, N) = U + pV, \tag{3.52}$$

where the volume V has been replaced by the pressure p.

(iii) The function

$$U[\mu] = f_1(S, V, \mu) = U - \mu N, \tag{3.53}$$

where N has been replaced by the chemical potential μ.

(iv) The **Gibbs free energy**,

$$U[T, p] = G(T, p, N) = U - TS + pV, \tag{3.54}$$

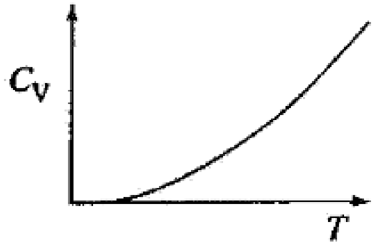
GATE 2010

The partition function for a gas of photons is given by

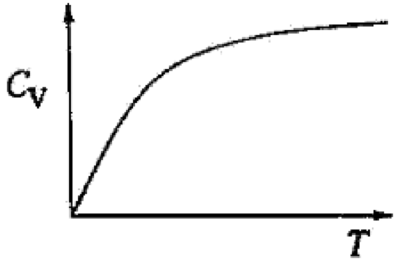
$$\ln Z = \frac{\pi^2}{45}\frac{V(k_B T)^3}{\hbar^3 C^3}.$$

The specific heat of the photon gas varies with temperature as

(A)

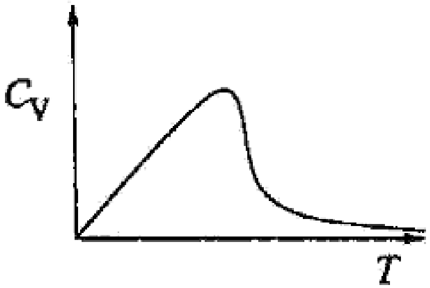


(B)



(C)

(D)



$$U = kT^2\left(\frac{\partial \ln \Xi}{\partial T}\right)_{Z',V}$$

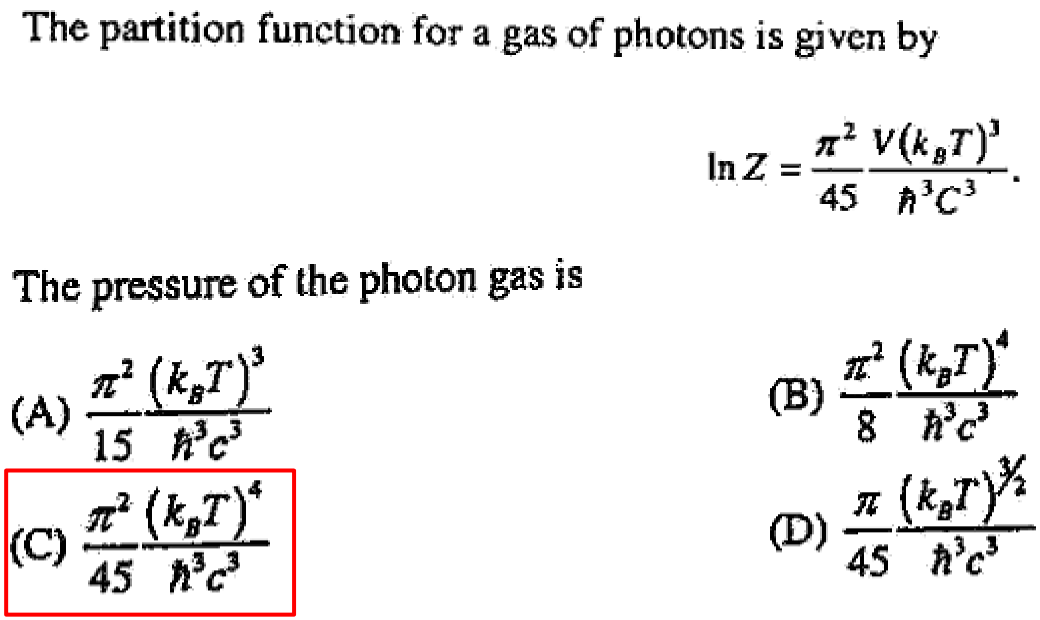
$$\ln Z = \frac{\pi^2}{45}\frac{V(kT)^3}{\hbar^3 C^3}$$

$$U = kT^2\frac{\partial}{\partial T}\left(\frac{\pi^2}{45}\frac{V(kT)^3}{\hbar^3 C^3}\right)$$

$$U = \frac{\pi^2}{15}\frac{Vk^4T^4}{\hbar^3 C^3}$$

$$Cv = \frac{\partial U}{\partial T} = \frac{4\pi^2}{15}\frac{Vk^4T^3}{\hbar^3 C^3}$$

GATE 2010



The partition function for a gas of photons is given by

$$\ln Z = \frac{\pi^2}{45}\frac{V(k_BT)^3}{\hbar^3C^3}.$$

The pressure of the photon gas is

(A) $\frac{\pi^2}{15}\frac{(k_BT)^3}{\hbar^3c^3}$

(B) $\frac{\pi^2}{8}\frac{(k_BT)^4}{\hbar^3c^3}$

(C) $\boxed{\frac{\pi^2}{45}\frac{(k_BT)^4}{\hbar^3c^3}}$

(D) $\frac{\pi}{45}\frac{(k_BT)^{3/2}}{\hbar^3c^3}$

$$P = -\left(\frac{\partial\Omega}{\partial V}\right)_{S,\mu}$$

$$P = -\left[\frac{\partial}{\partial V}\left(-kT\ln\Xi\right)\right]_{S,\mu}$$

$$P = kT\left(\frac{\partial\ln\Xi}{\partial V}\right)_{S,\mu}$$

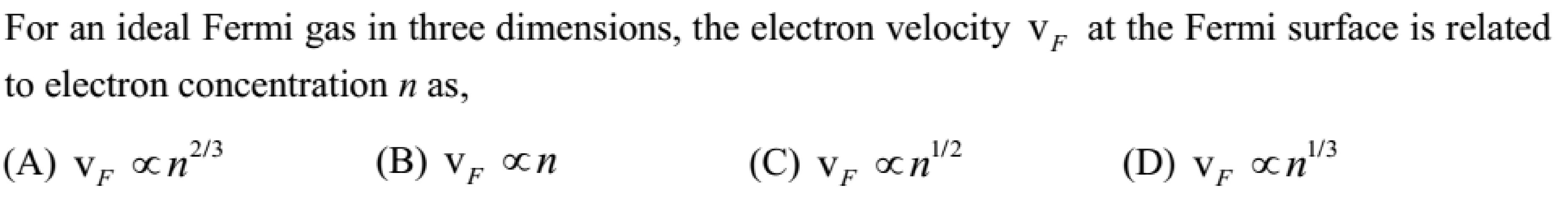
$$\ln Z = \frac{\pi^2}{45}\frac{V(kT)^3}{\hbar^3C^3}$$

$$P = kT\left(\frac{\partial\ln Z}{\partial V}\right)$$

$$P = kT\frac{\partial}{\partial V}\frac{\pi^2}{45}\frac{V(kT)^3}{\hbar^3C^3}$$

$$P = \frac{\pi^2}{45}\frac{(kT)^4}{\hbar^3C^3}$$

GATE 2012



For an ideal Fermi gas in three dimensions, the electron velocity v_F at the Fermi surface is related to electron concentration n as,

(A) $v_F \propto n^{2/3}$ (B) $v_F \propto n$ (C) $v_F \propto n^{1/2}$ (D) $v_F \propto n^{1/3}$

GATE 2012

The total energy, E of an ideal non-relativistic Fermi gas in three dimensions is given by $E \propto \frac{N^{5/3}}{V^{2/3}}$ where N is the number of particles and V is the volume of the gas.

Identify the CORRECT equation of state (P being the pressure),

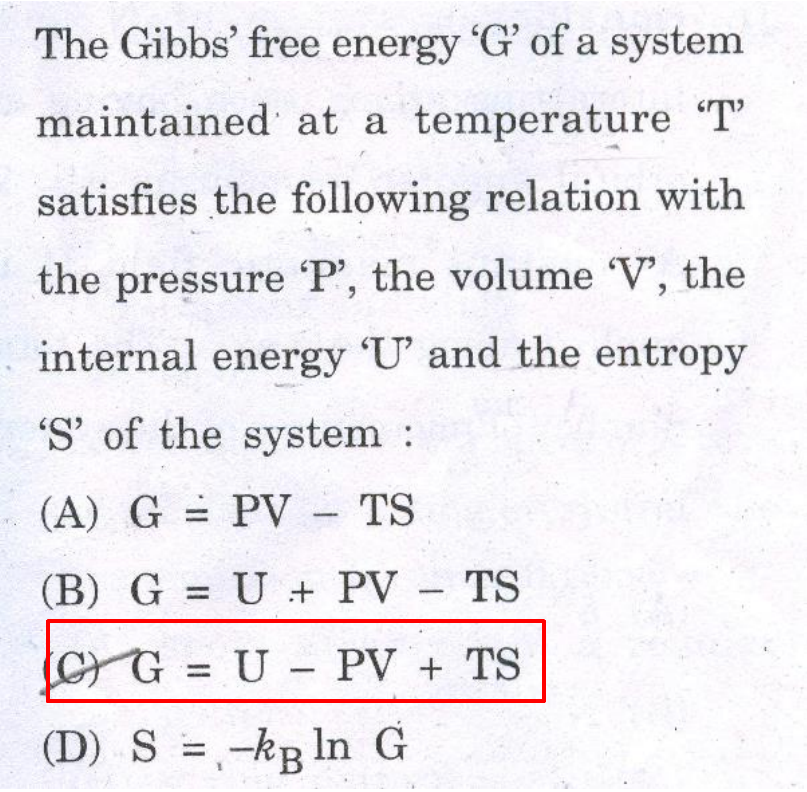
(A) $PV = \frac{1}{3}E$

$\boxed{\text{(B) } PV = \frac{2}{3}E}$

(C) $PV = E$

(D) $PV = \frac{5}{3}E$

SET 2013



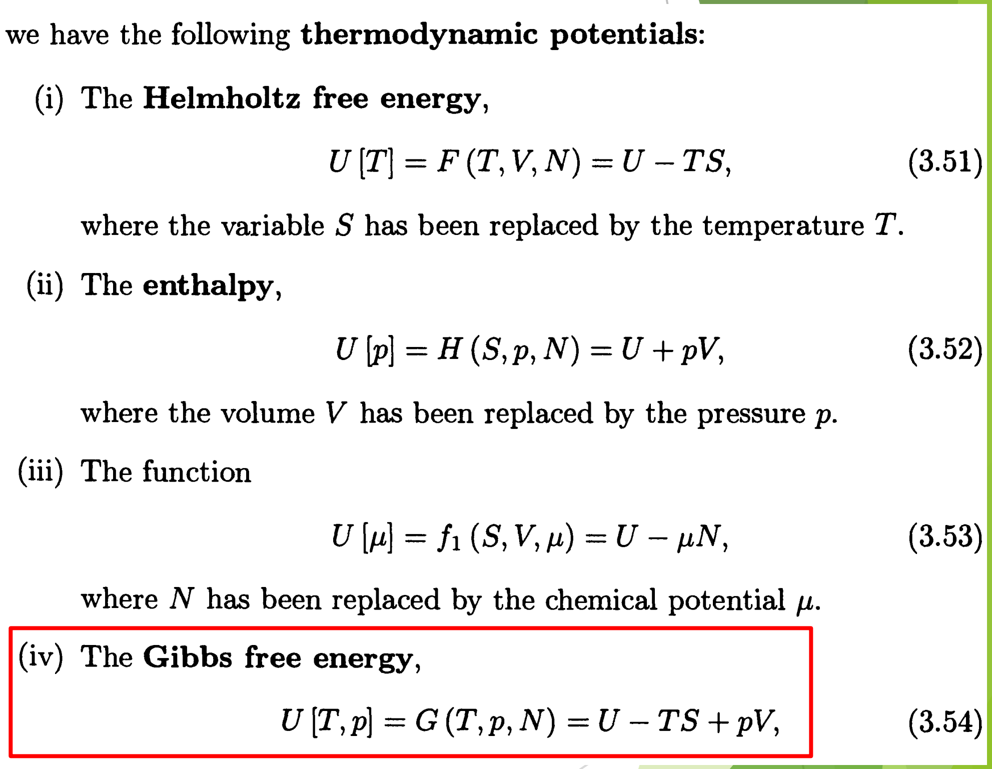
The Gibbs' free energy 'G' of a system maintained at a temperature 'T' satisfies the following relation with the pressure 'P', the volume 'V', the internal energy 'U' and the entropy 'S' of the system :

(A) $G = PV - TS$

(B) $G = U + PV - TS$

(C) $G = U - PV + TS$

(D) $S = -k_B \ln G$



we have the following **thermodynamic potentials**:

(i) The **Helmholtz free energy**,

$$U[T] = F(T, V, N) = U - TS, \tag{3.51}$$

where the variable S has been replaced by the temperature T.

(ii) The **enthalpy**,

$$U[p] = H(S, p, N) = U + pV, \tag{3.52}$$

where the volume V has been replaced by the pressure p.

(iii) The function

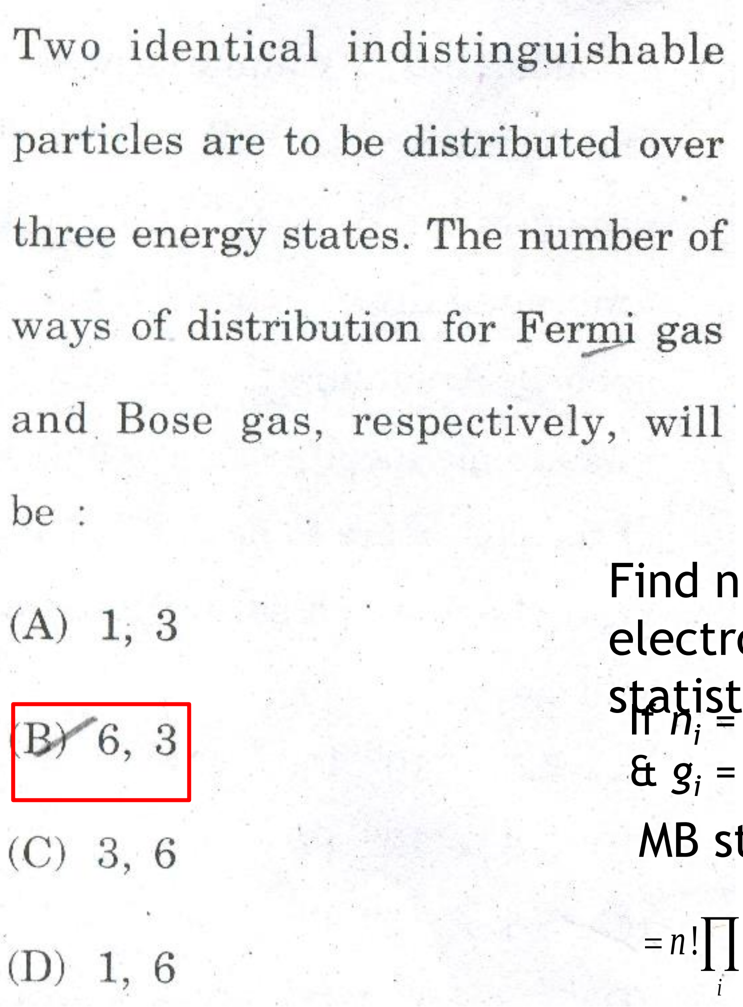
$$U[\mu] = f_1(S, V, \mu) = U - \mu N, \tag{3.53}$$

where N has been replaced by the chemical potential μ.

(iv) The **Gibbs free energy**,

$$U[T, p] = G(T, p, N) = U - TS + pV, \tag{3.54}$$

SET 2013



Two identical indistinguishable particles are to be distributed over three energy states. The number of ways of distribution for Fermi gas and Bose gas, respectively, will be :

(A) 1, 3

(B) 6, 3

(C) 3, 6

(D) 1, 6

Find number of possible ways of distribution of 2 electrons in 3 states according to MB, BE and FD statistics.

If n_i = 2 : No. of electrons

& g_i = 3 : No. of states

MB statistics:

$$= n!\prod_i \frac{g_i^{n_i}}{n_i}$$

$$= 2!\frac{3^2}{2}$$

$$= 2.1\frac{3\times 3}{2}$$

$$= 9$$

BE statistics:

$$= \frac{g_i(g_i+n_i-1)!}{g_i!n_i!}$$

$$= \frac{3(3+2-1)!}{3!2!}$$

$$= \frac{3\times 4\times 3\times 2\times 1}{3\times 2\times 1\times 2\times 1}$$

$$= 6$$

FD statistics:

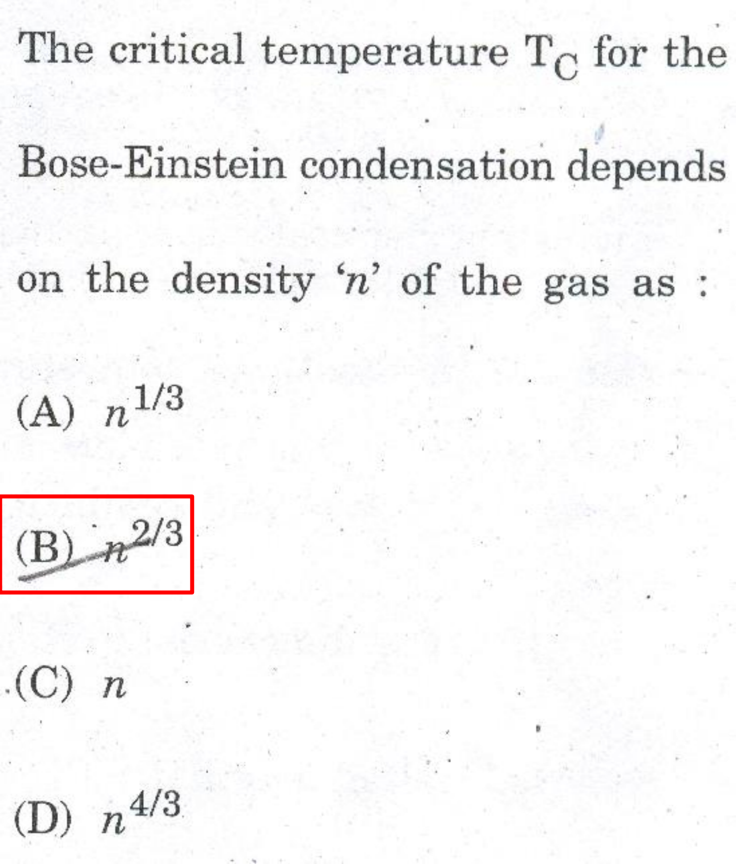
$$= \frac{g_i!}{n_i!(g_i-n_i)!}$$

$$= \frac{g_i!}{n_i!(g_i-n_i)!} = \frac{3!}{2!\times 1!}$$

$$= \frac{3\times 2\times 1}{2\times 1\times 1}$$

$$= 3$$

SET 2013



The critical temperature T_C for the Bose-Einstein condensation depends on the density 'n' of the gas as :

(A) $n^{1/3}$

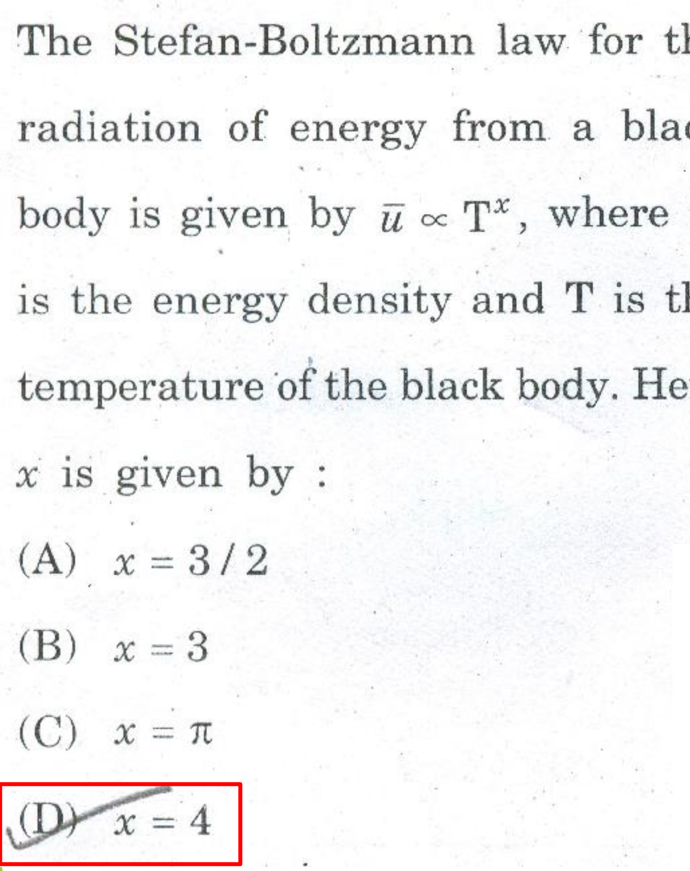
(B) $n^{2/3}$

(C) n

(D) $n^{4/3}$

$$T_c = \left[\frac{N}{\xi(3/2)V}\right]^{2/3}\left(\frac{h^2}{2\pi mk}\right)$$

SET 2013



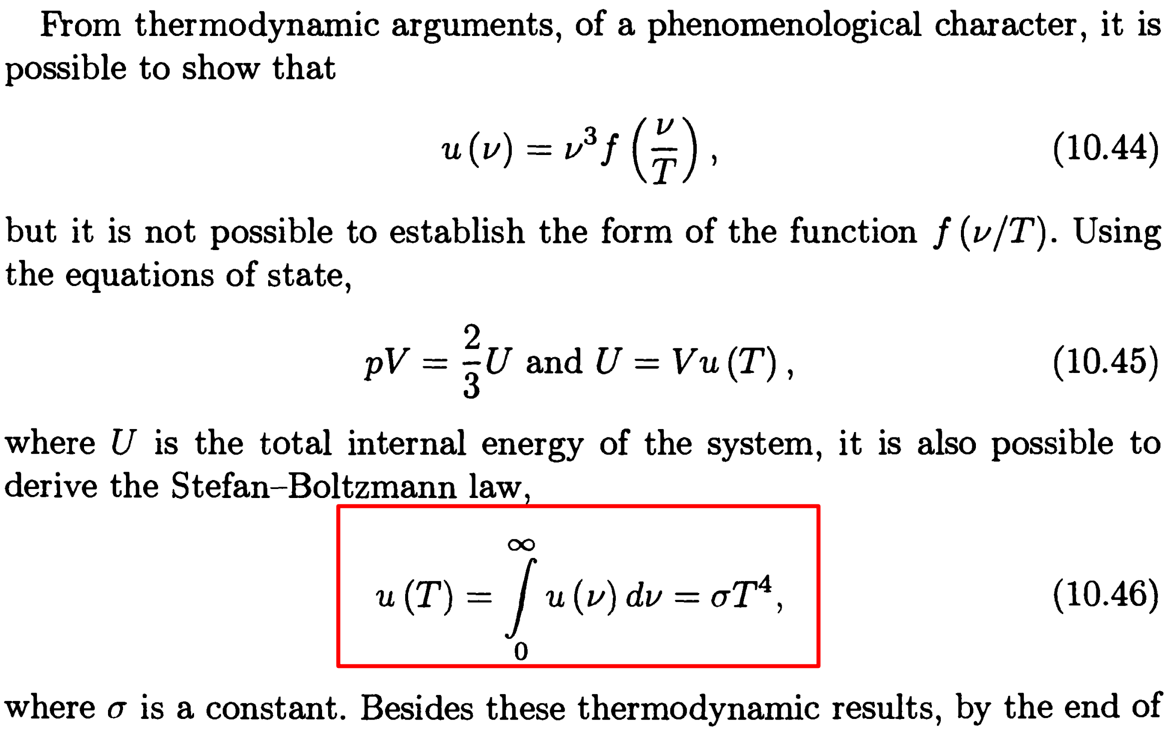
The Stefan-Boltzmann law for the radiation of energy from a black body is given by $\bar{u} \propto T^x$, where $\bar{u}$ is the energy density and T is the temperature of the black body. Here x is given by :

(A) $x = 3/2$

(B) $x = 3$

(C) $x = \pi$

(D) $x = 4$



From thermodynamic arguments, of a phenomenological character, it is possible to show that

$$u(\nu) = \nu^3 f\left(\frac{\nu}{T}\right), \tag{10.44}$$

but it is not possible to establish the form of the function $f(\nu/T)$. Using the equations of state,

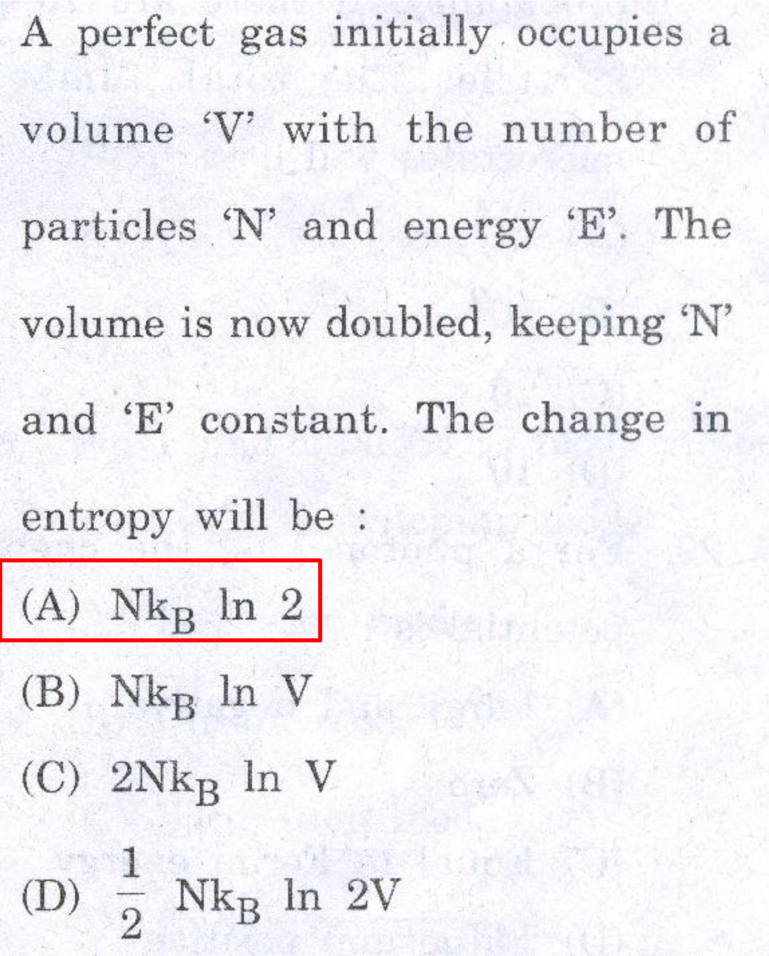
$$pV = \frac{2}{3}U \text{ and } U = Vu(T), \tag{10.45}$$

where U is the total internal energy of the system, it is also possible to derive the Stefan–Boltzmann law,

$$\boxed{u(T) = \int_0^\infty u(\nu)\, d\nu = \sigma T^4,} \tag{10.46}$$

where σ is a constant. Besides these thermodynamic results, by the end of

SET 2013



A perfect gas initially occupies a volume 'V' with the number of particles 'N' and energy 'E'. The volume is now doubled, keeping 'N' and 'E' constant. The change in entropy will be :

(A) $Nk_B \ln 2$

(B) $Nk_B \ln V$

(C) $2Nk_B \ln V$

(D) $\frac{1}{2} Nk_B \ln 2V$

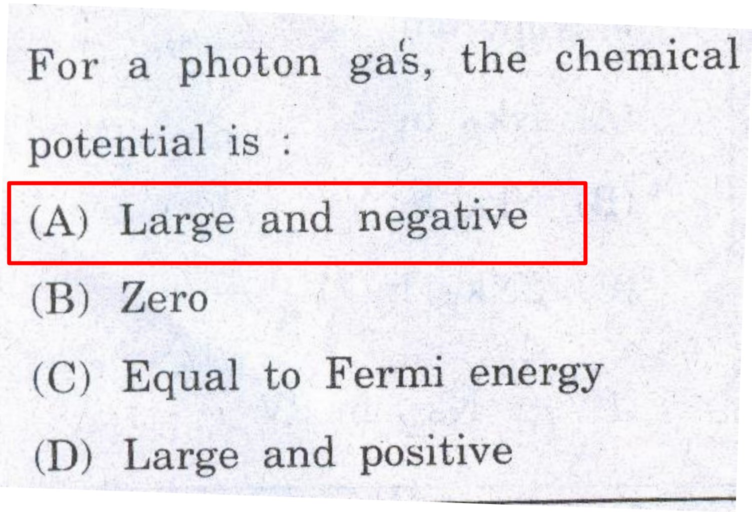
$$S = NkLn\left[V\left(\frac{E}{N}\right)^{3/2}\left(\frac{4\pi m}{3h^2}\right)^{3/2}\right] + \frac{3}{2}Nk$$

$$S' = NkLn\left[2V\left(\frac{E}{N}\right)^{3/2}\left(\frac{4\pi m}{3h^2}\right)^{3/2}\right] + \frac{3}{2}Nk$$

$$S' = NkLn2 + NkLn\left[V\left(\frac{E}{N}\right)^{3/2}\left(\frac{4\pi m}{3h^2}\right)^{3/2}\right] + \frac{3}{2}Nk$$

$$S' = NkLn2 + S$$

SET 2013



For a photon gas, the chemical potential is :

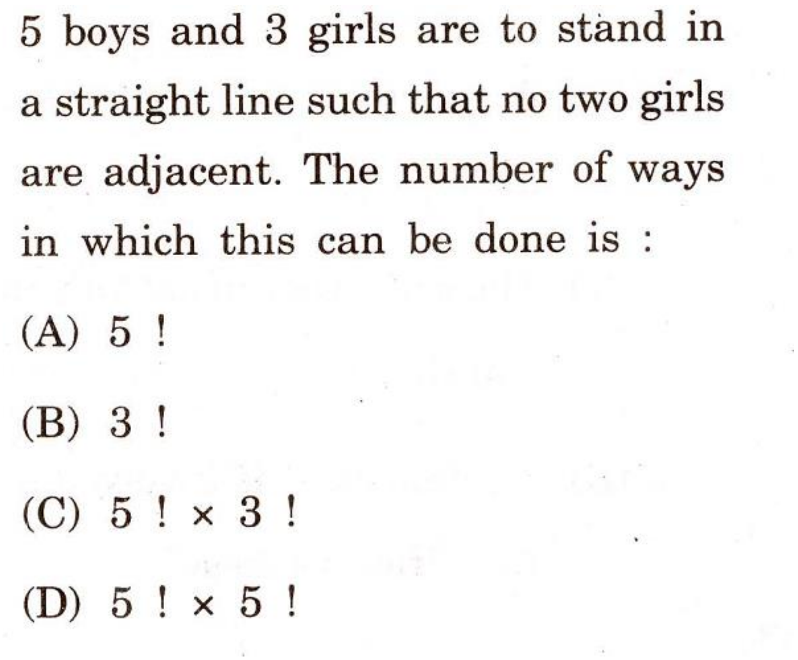
[✓ (A) Large and negative]

(B) Zero

(C) Equal to Fermi energy

(D) Large and positive

Aug 2011



5 boys and 3 girls are to stand in a straight line such that no two girls are adjacent. The number of ways in which this can be done is :

(A) 5 !

(B) 3 !

(C) 5 ! × 3 !

(D) 5 ! × 5 !

Nov 2011

Consider a system of 4 spins with spin $S = \frac{1}{2}$ and magnetic moment μ each. It is placed in an external magnetic field H. The magnetic moments can either be parallel or antiparallel to the magnetic field. Consider a macrostate of the system with energy $-2\mu H$. Using the postulate of equal a priori probability, the probability of finding

Nov 2011

the system with the magnetic

moment -2μ is given by :

(A) 1/16

(B) 1/4

(C) 1/8

(D) 1/2

Nov 2011

Consider N particles with spin angular momentum S each. Each spin has $2S + 1$ projections along the axis of quantization. The total number of microstates of the system will be :

(A) $N(2S + 1)$

(B) $(2S + 1)^N$

(C) $N^{2S + 1}$

(D) $N(2S + 1)\,!$

GATE 2005

The total number of accessible states of N noninteracting particles of spin 1/2 is

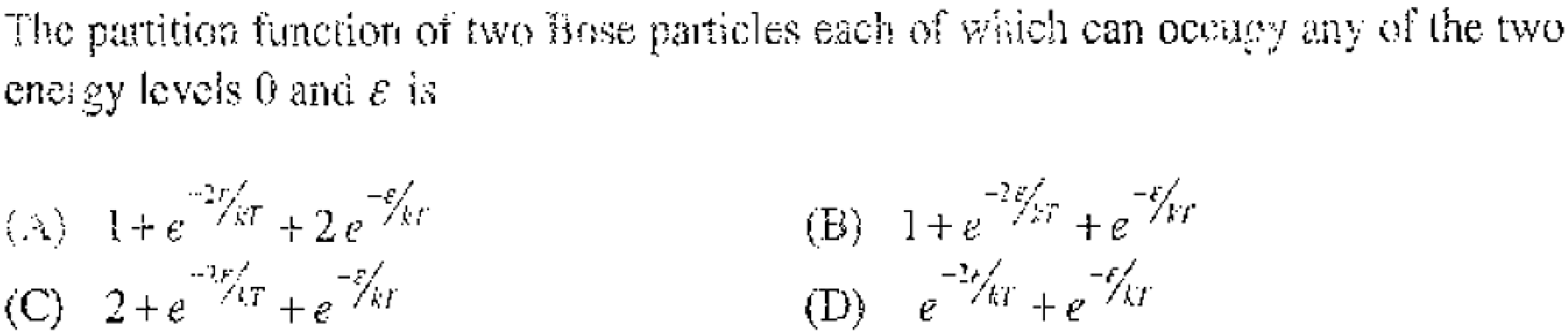
(A) 2^N

(B) N^2

(C) $2^{N/2}$

(D) N

GATE 2005



The partition function of two Bose particles each of which can occupy any of the two energy levels 0 and ε is

(A) $1+e^{-2\varepsilon/kT}+2e^{-\varepsilon/kT}$

(B) $1+e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$

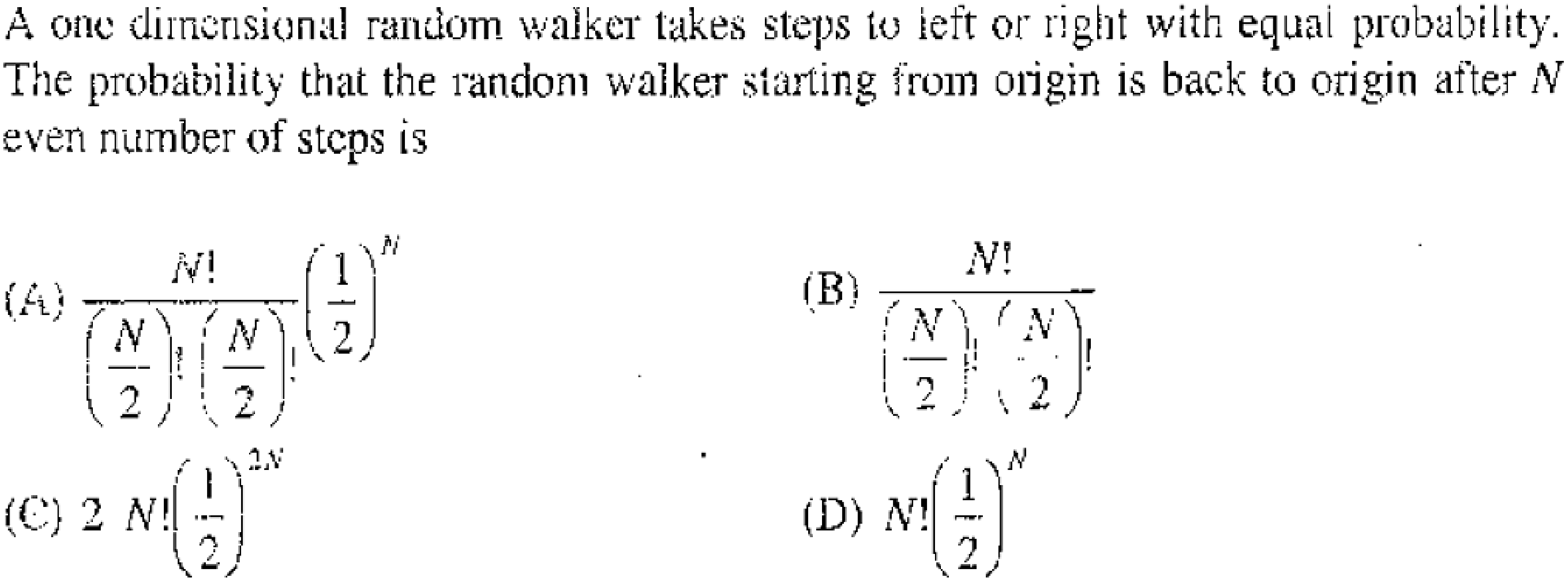
(C) $2+e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$

(D) $e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$

$$\sum_n e^{-E/kT}=e^{0/kT}+e^{-\varepsilon/kT}$$

$$n=\int_0^\infty g(\varepsilon)f(\varepsilon)d\varepsilon=\int_0^\infty \frac{g(\varepsilon)}{\exp\left(\frac{\varepsilon-\mu}{k_BT}\right)-1}d\varepsilon$$

GATE 2005



A one dimensional random walker takes steps to left or right with equal probability. The probability that the random walker starting from origin is back to origin after N even number of steps is

(A) $\dfrac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}\left(\dfrac{1}{2}\right)^{N}$

(B) $\dfrac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}$

(C) $2N!\left(\dfrac{1}{2}\right)^{2N}$

(D) $N!\left(\dfrac{1}{2}\right)^{N}$

GATE 2005

The number of states for a system of N identical free particles in a three dimensional space having total energy between E and $E+\delta E$ ($\delta E << E$), is proportional to

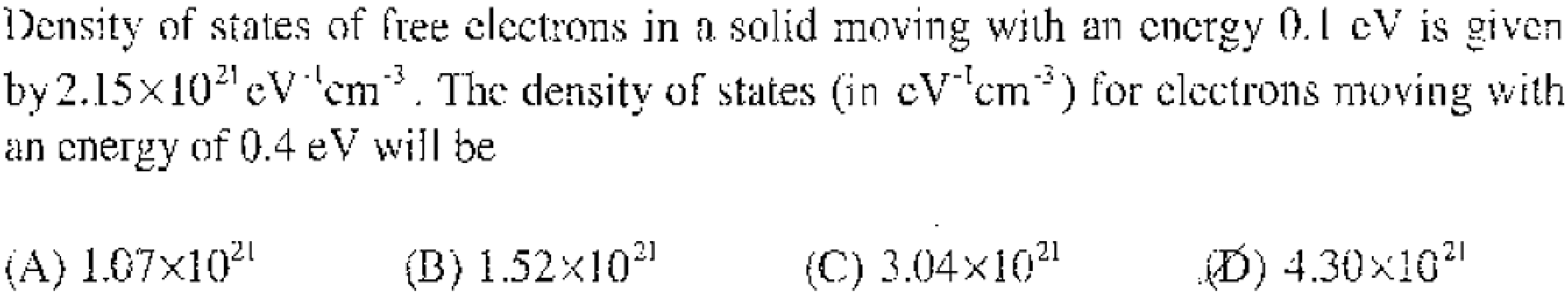
(A) $\left(E^{\frac{3N}{2}-1}\right)\delta E$

(B) $E^{\frac{N}{2}}\,\delta E$

(C) $NE^{\frac{1}{2}}\delta E$

(D) $E^{N}\,\delta E$

GATE 2005



Density of states of free electrons in a solid moving with an energy 0.1 eV is given by $2.15\times10^{21}\ \text{eV}^{-1}\text{cm}^{-3}$. The density of states (in $\text{eV}^{-1}\text{cm}^{-3}$) for electrons moving with an energy of 0.4 eV will be

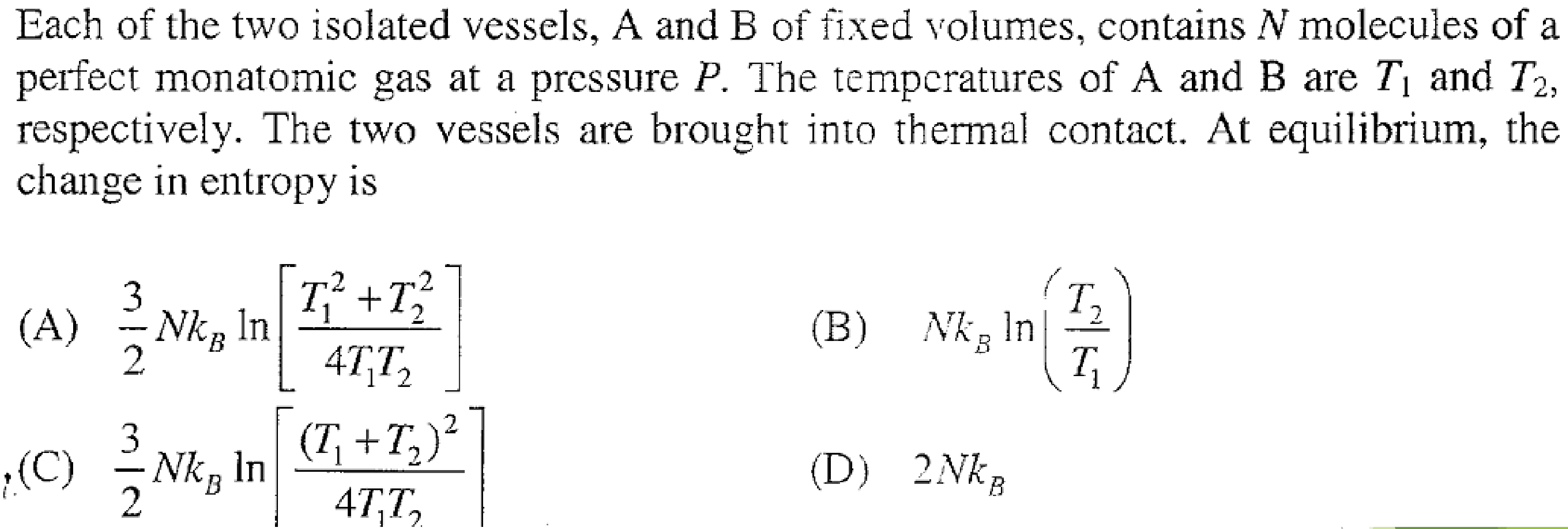
(A) 1.07×10^{21} (B) 1.52×10^{21} (C) 3.04×10^{21} (D) 4.30×10^{21}

GATE 2005

The effective density of states at the conduction band edge of Ge is 1.04×10^{19} cm^{-3} at room temperature (300K). Ge has an optical bandgap of 0.66eV. The intrinsic carrier concentration (in cm^{-3}) in Ge at room temperature (300 K) is approximately

(A) 3×10^{13} ✓(B) 3×10^{15} (C) 3×10^{16} (D) 6×10^{16}

GATE 2006



Each of the two isolated vessels, A and B of fixed volumes, contains N molecules of a perfect monatomic gas at a pressure P. The temperatures of A and B are T_1 and T_2, respectively. The two vessels are brought into thermal contact. At equilibrium, the change in entropy is

(A) $\frac{3}{2} N k_B \ln\left[\frac{T_1^2 + T_2^2}{4T_1T_2}\right]$

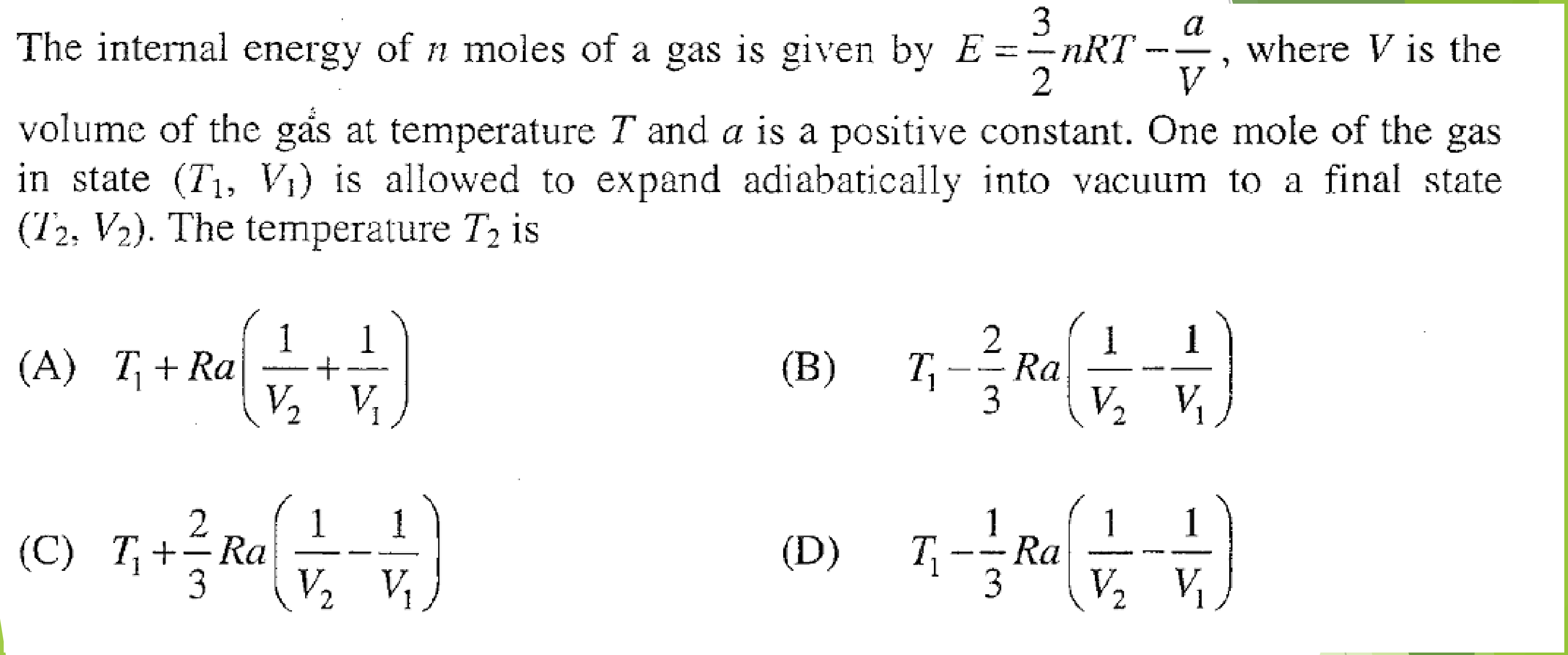
(B) $N k_B \ln\left(\frac{T_2}{T_1}\right)$

(C) $\frac{3}{2} N k_B \ln\left[\frac{(T_1 + T_2)^2}{4T_1T_2}\right]$

(D) $2Nk_B$

$$S = Nk\left\{Ln\left(\frac{V}{N}\right) + \frac{3}{2} LnT + \frac{3}{2} Ln\left(\frac{2\pi mk}{h^2}\right) + \frac{5}{2}\right\}$$

GATE 2006



The internal energy of n moles of a gas is given by $E = \frac{3}{2}nRT - \frac{a}{V}$, where V is the volume of the gas at temperature T and a is a positive constant. One mole of the gas in state (T_1, V_1) is allowed to expand adiabatically into vacuum to a final state (T_2, V_2). The temperature T_2 is

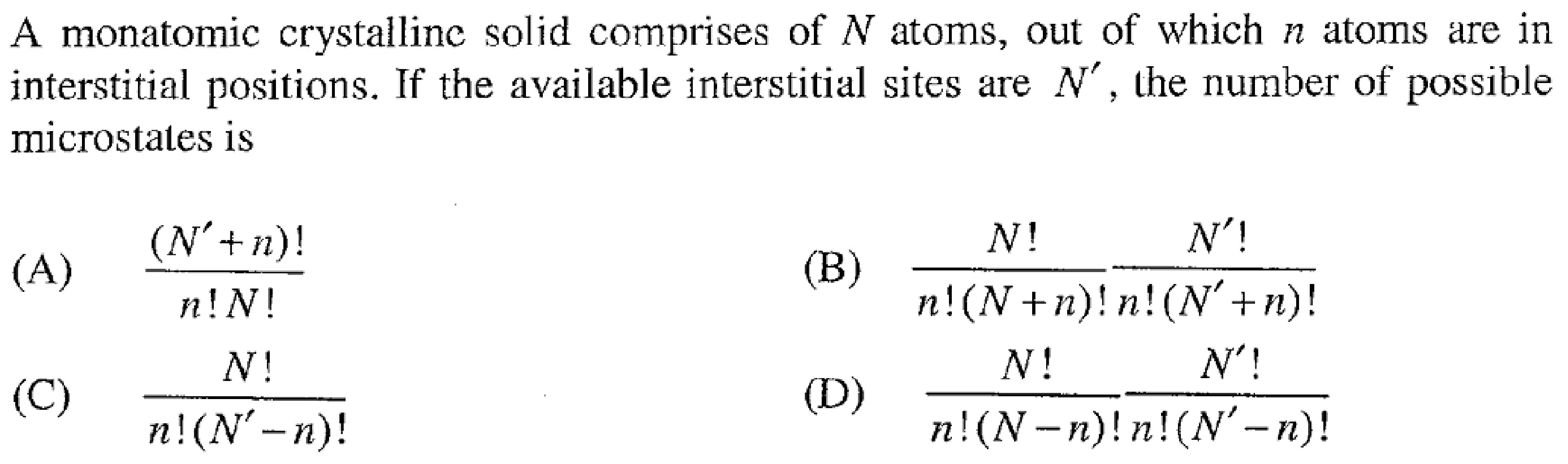
(A) $T_1 + Ra\left(\frac{1}{V_2} + \frac{1}{V_1}\right)$

(B) $T_1 - \frac{2}{3}Ra\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$

(C) $T_1 + \frac{2}{3}Ra\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$

(D) $T_1 - \frac{1}{3}Ra\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$

GATE 2006



A monatomic crystalline solid comprises of N atoms, out of which n atoms are in interstitial positions. If the available interstitial sites are N', the number of possible microstates is

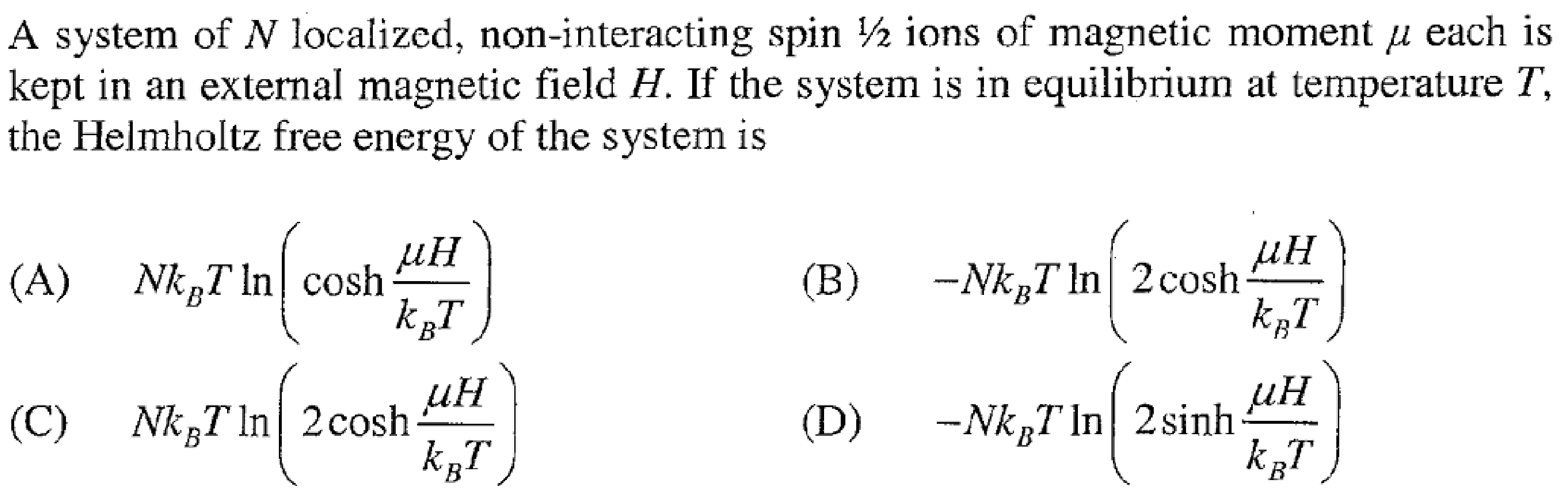
(A) $\dfrac{(N'+n)!}{n!\,N!}$

(B) $\dfrac{N!}{n!\,(N+n)!}\,\dfrac{N'!}{n!\,(N'+n)!}$

(C) $\dfrac{N!}{n!\,(N'-n)!}$

(D) $\dfrac{N!}{n!\,(N-n)!}\,\dfrac{N'!}{n!\,(N'-n)!}$

GATE 2006



A system of N localized, non-interacting spin ½ ions of magnetic moment μ each is kept in an external magnetic field H. If the system is in equilibrium at temperature T, the Helmholtz free energy of the system is

(A) $Nk_BT\ln\left(\cosh\dfrac{\mu H}{k_BT}\right)$

(B) $-Nk_BT\ln\left(2\cosh\dfrac{\mu H}{k_BT}\right)$

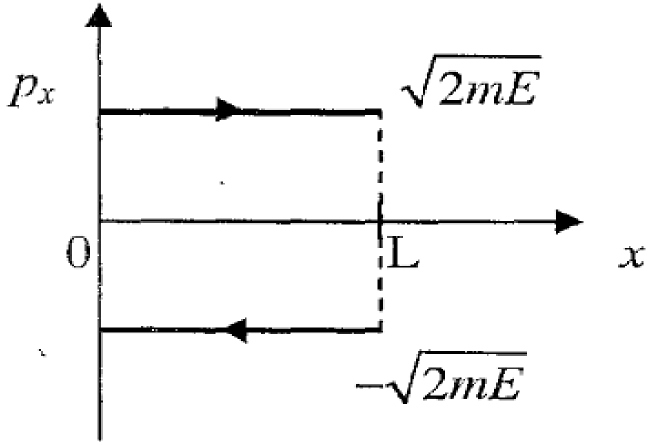
(C) $Nk_BT\ln\left(2\cosh\dfrac{\mu H}{k_BT}\right)$

(D) $-Nk_BT\ln\left(2\sinh\dfrac{\mu H}{k_BT}\right)$

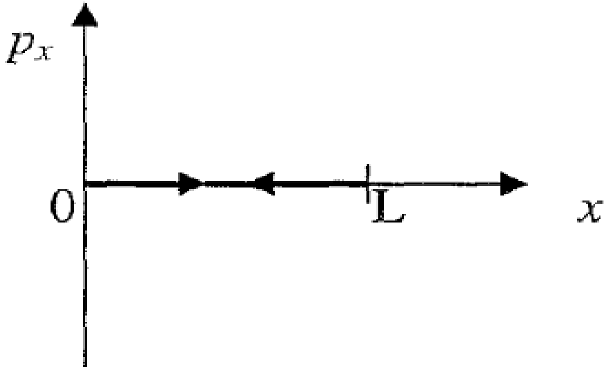
GATE 2006

The phase diagram of a free particle of mass m and kinetic energy E, moving in a one-dimensional box with perfectly elastic walls at $x = 0$ and $x = L$, is given by

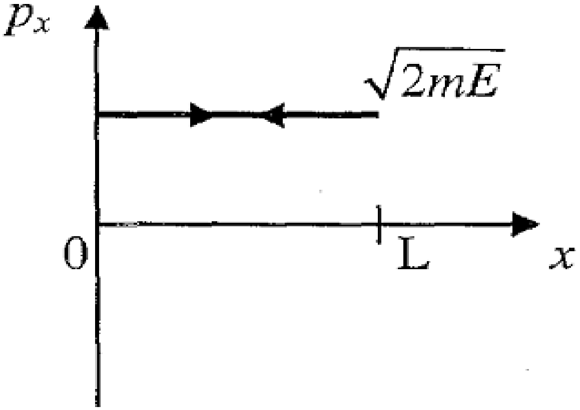
(A)



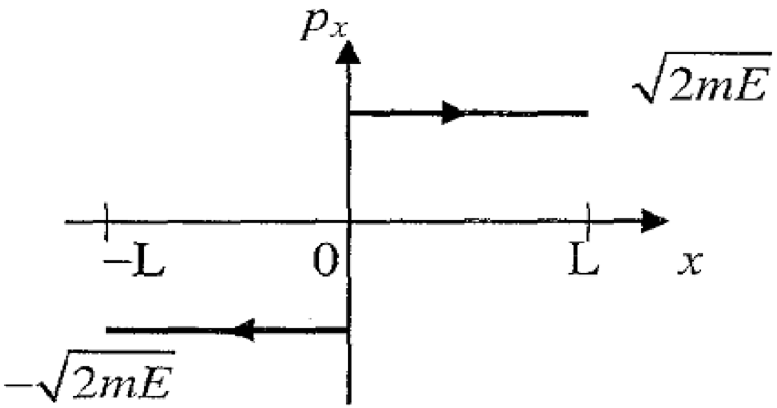
(B)



(C)



(D)



GATE 2006

Statement for Linked Answer Questions

Consider a radiation cavity of volume V at temperature T.

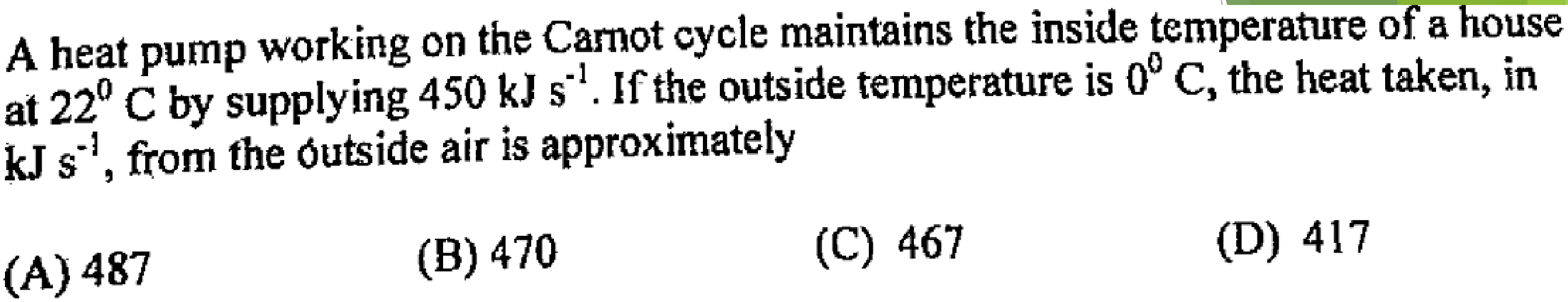
The density of states at energy E of the quantized radiation (photons) is

(A) $\frac{8\pi V}{h^3c^3}E^2$ (B) $\frac{8\pi V}{h^3c^3}E^{\frac{3}{2}}$ (C) $\frac{8\pi V}{h^3c^2}E$ (D) $\frac{8\pi V}{h^3c^2}E^{\frac{1}{2}}$

The average number of photons in equilibrium inside the cavity is proportional to

(A) T (B) T^2 (C) T^3 (D) T^4

GATE 2007



A heat pump working on the Carnot cycle maintains the inside temperature of a house at 22^0 C by supplying 450 kJ s^{-1}. If the outside temperature is 0^0 C, the heat taken, in kJ s^{-1}, from the outside air is approximately

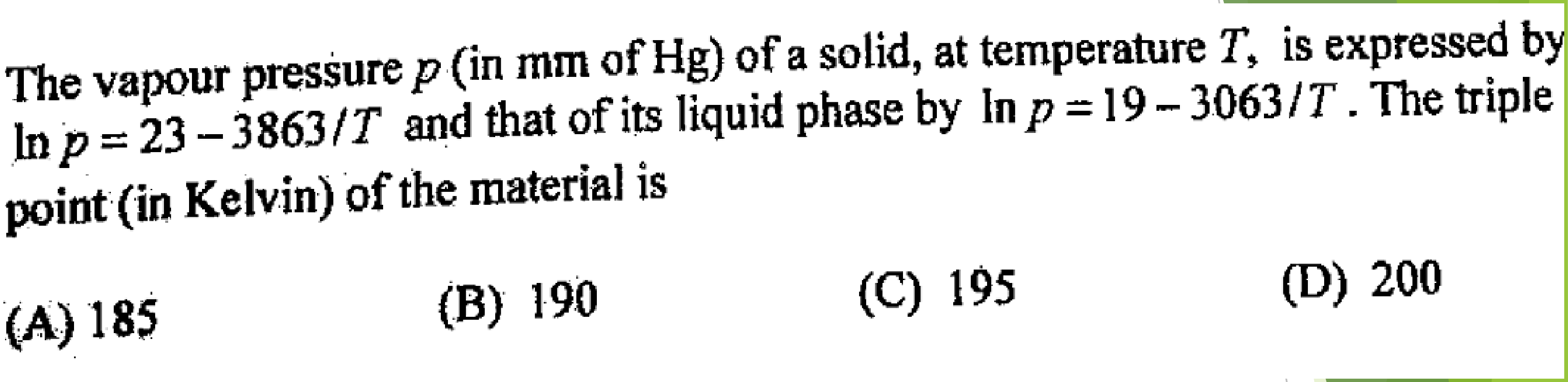
(A) 487

(B) 470

(C) 467

(D) 417

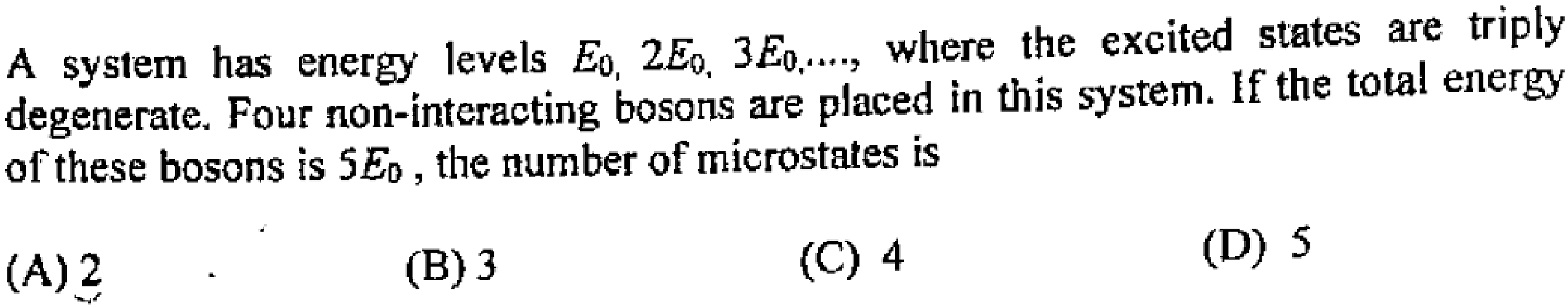
GATE 2007



The vapour pressure p (in mm of Hg) of a solid, at temperature T, is expressed by $\ln p = 23 - 3863/T$ and that of its liquid phase by $\ln p = 19 - 3063/T$. The triple point (in Kelvin) of the material is

(A) 185 (B) 190 (C) 195 (D) 200

GATE 2007



A system has energy levels E_0, $2E_0$, $3E_0$,...., where the excited states are triply degenerate. Four non-interacting bosons are placed in this system. If the total energy of these bosons is $5E_0$, the number of microstates is

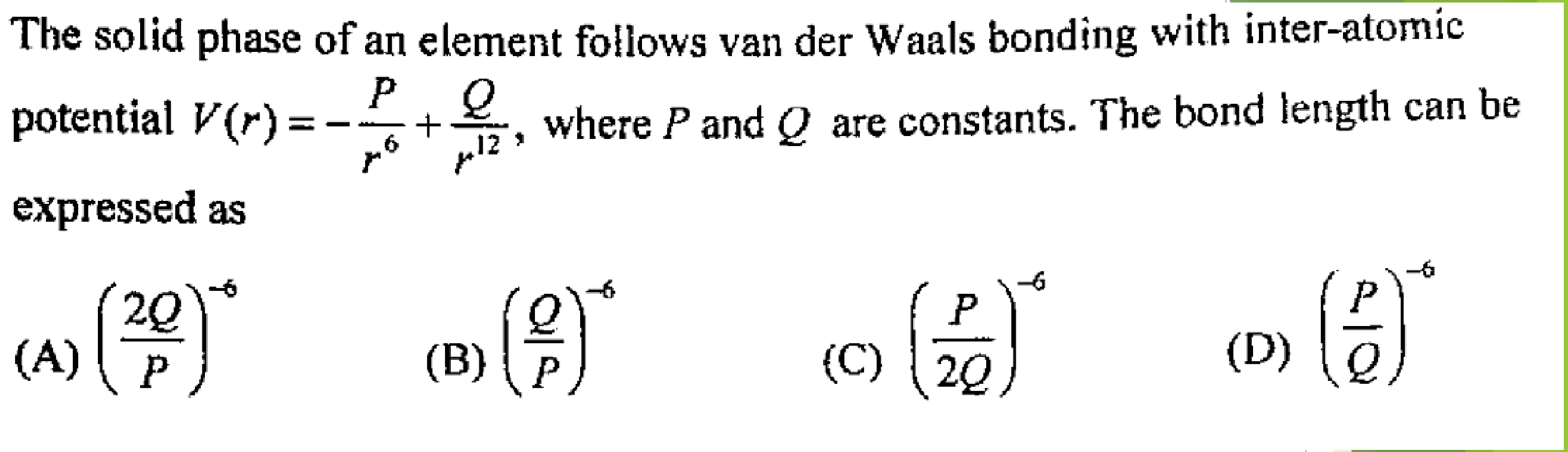
(A) 2

(B) 3

(C) 4

(D) 5

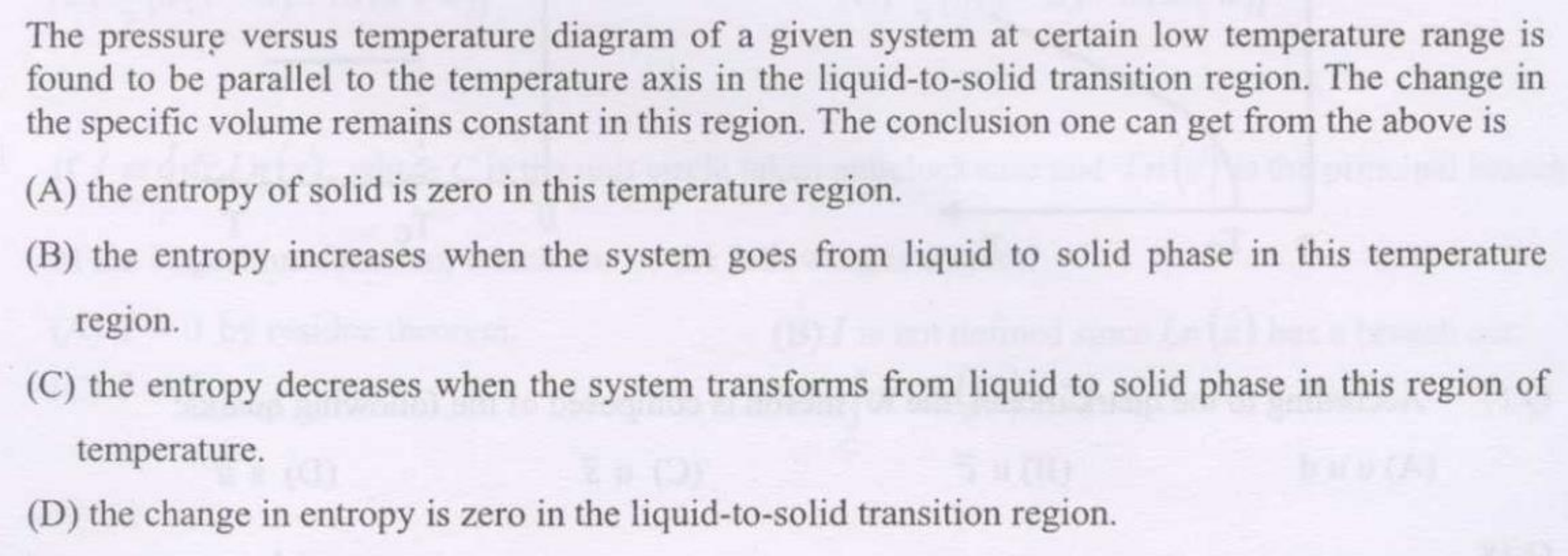
GATE 2007



The solid phase of an element follows van der Waals bonding with inter-atomic potential $V(r) = -\frac{P}{r^6} + \frac{Q}{r^{12}}$, where P and Q are constants. The bond length can be expressed as

(A) $\left(\frac{2Q}{P}\right)^{-6}$ (B) $\left(\frac{Q}{P}\right)^{-6}$ (C) $\left(\frac{P}{2Q}\right)^{-6}$ (D) $\left(\frac{P}{Q}\right)^{-6}$

GATE 2008



The pressure versus temperature diagram of a given system at certain low temperature range is found to be parallel to the temperature axis in the liquid-to-solid transition region. The change in the specific volume remains constant in this region. The conclusion one can get from the above is

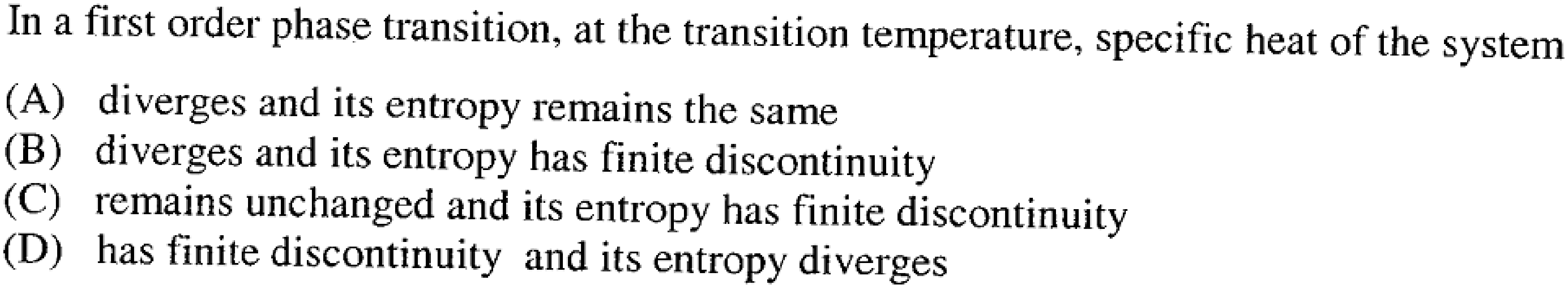
(A) the entropy of solid is zero in this temperature region.

(B) the entropy increases when the system goes from liquid to solid phase in this temperature region.

(C) the entropy decreases when the system transforms from liquid to solid phase in this region of temperature.

(D) the change in entropy is zero in the liquid-to-solid transition region.

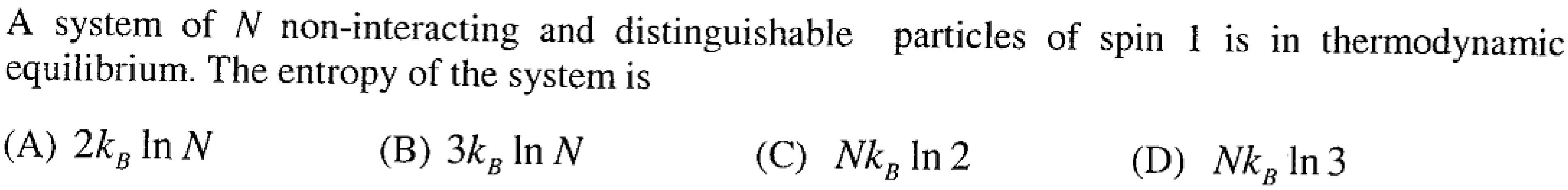
GATE 2011



In a first order phase transition, at the transition temperature, specific heat of the system

(A) diverges and its entropy remains the same
(B) diverges and its entropy has finite discontinuity
(C) remains unchanged and its entropy has finite discontinuity
(D) has finite discontinuity and its entropy diverges

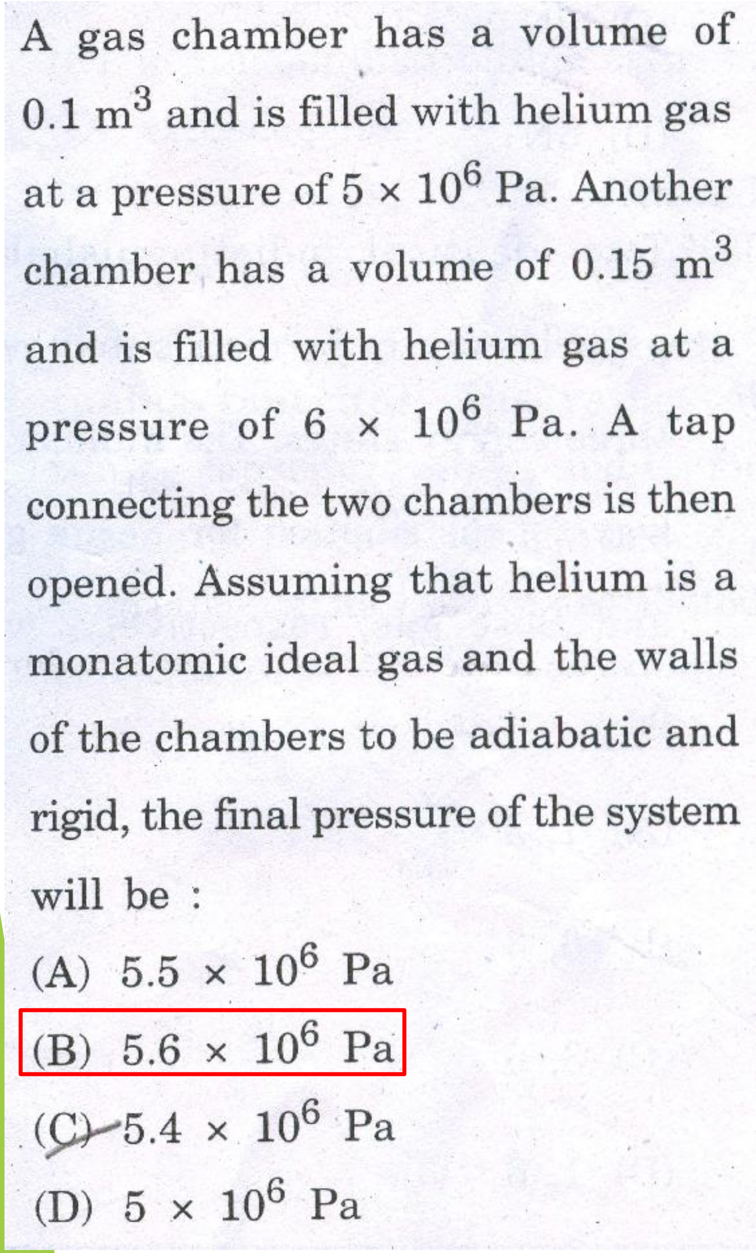
GATE 2011



A system of N non-interacting and distinguishable particles of spin 1 is in thermodynamic equilibrium. The entropy of the system is

(A) $2k_B \ln N$ (B) $3k_B \ln N$ (C) $Nk_B \ln 2$ (D) $Nk_B \ln 3$

SET 2013



A gas chamber has a volume of 0.1 m^3 and is filled with helium gas at a pressure of 5×10^6 Pa. Another chamber has a volume of 0.15 m^3 and is filled with helium gas at a pressure of 6×10^6 Pa. A tap connecting the two chambers is then opened. Assuming that helium is a monatomic ideal gas and the walls of the chambers to be adiabatic and rigid, the final pressure of the system will be :

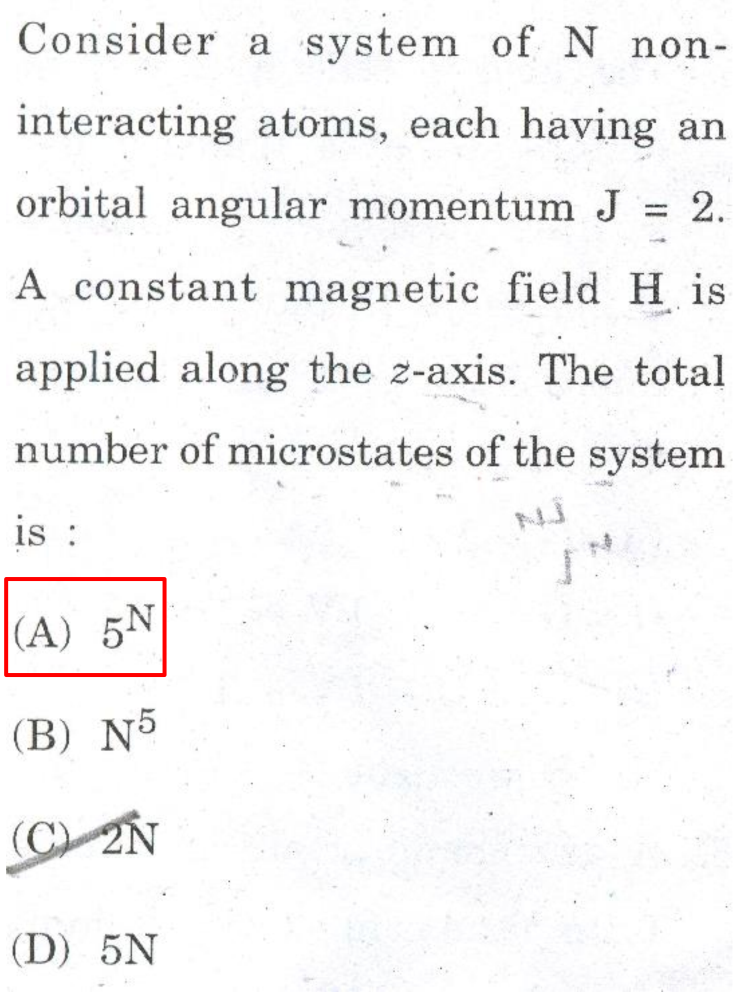
(A) 5.5×10^6 Pa

[✓ (B) 5.6×10^6 Pa]

(C) 5.4×10^6 Pa

(D) 5×10^6 Pa

SET 2013



Consider a system of N non-interacting atoms, each having an orbital angular momentum J = 2. A constant magnetic field H is applied along the z-axis. The total number of microstates of the system is :

NJ

J^N

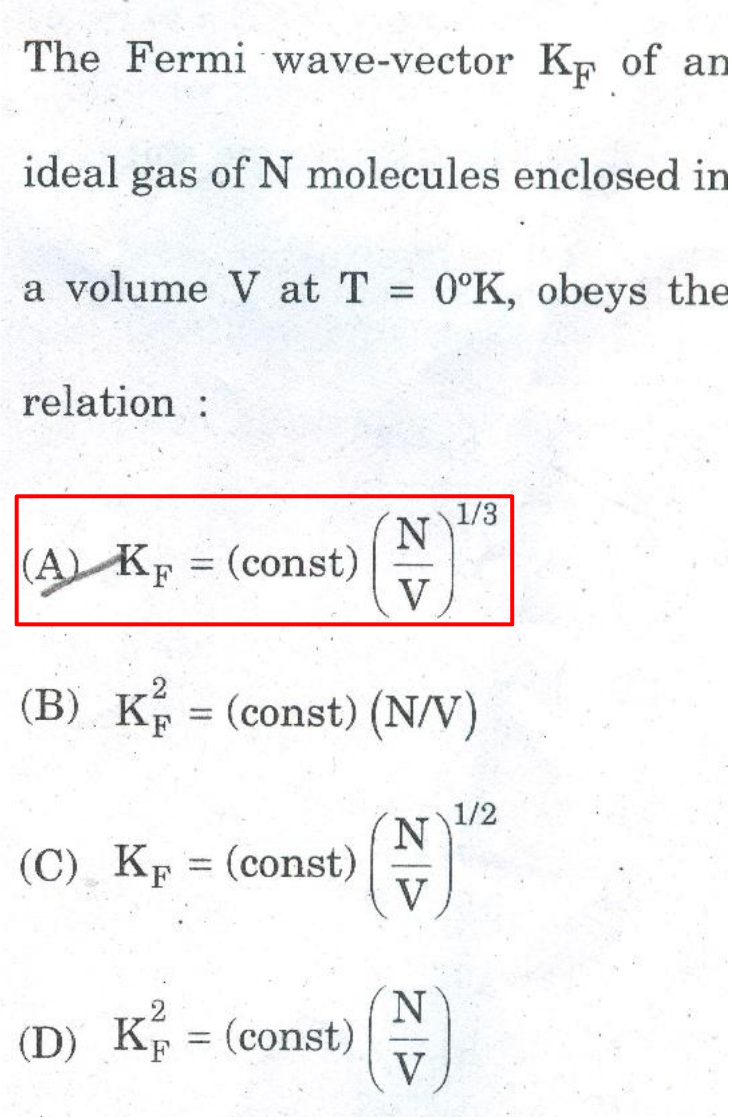
(A) 5^N

(B) N^5

~~(C) 2N~~

(D) 5N

SET 2013



The Fermi wave-vector K_F of an ideal gas of N molecules enclosed in a volume V at T = 0°K, obeys the relation :

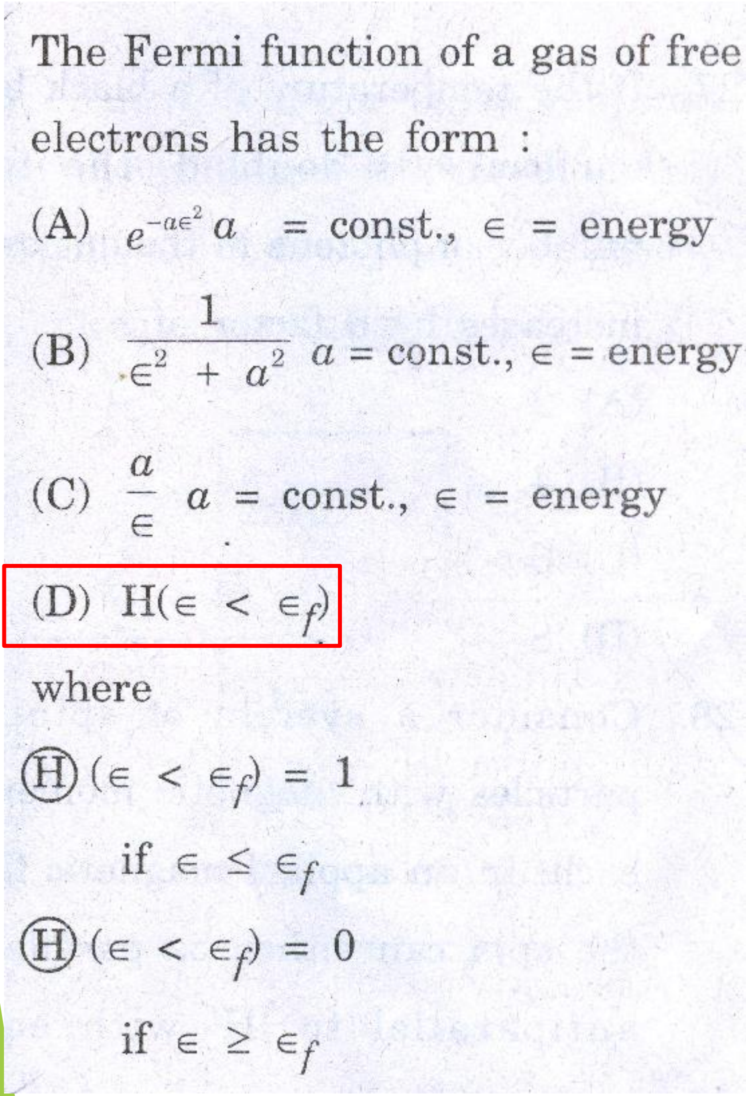
[✓ (A)] $K_F = (\text{const}) \left(\frac{N}{V}\right)^{1/3}$

(B) $K_F^2 = (\text{const})\,(N/V)$

(C) $K_F = (\text{const}) \left(\frac{N}{V}\right)^{1/2}$

(D) $K_F^2 = (\text{const}) \left(\frac{N}{V}\right)$

SET 2013



The Fermi function of a gas of free electrons has the form :

(A) $e^{-a\in^2}$ a = const., $\in$ = energy

(B) $\frac{1}{\in^2 + a^2}$ a = const., $\in$ = energy

(C) $\frac{a}{\in}$ a = const., $\in$ = energy

[✓ (D) $H(\in < \in_f)$]

where

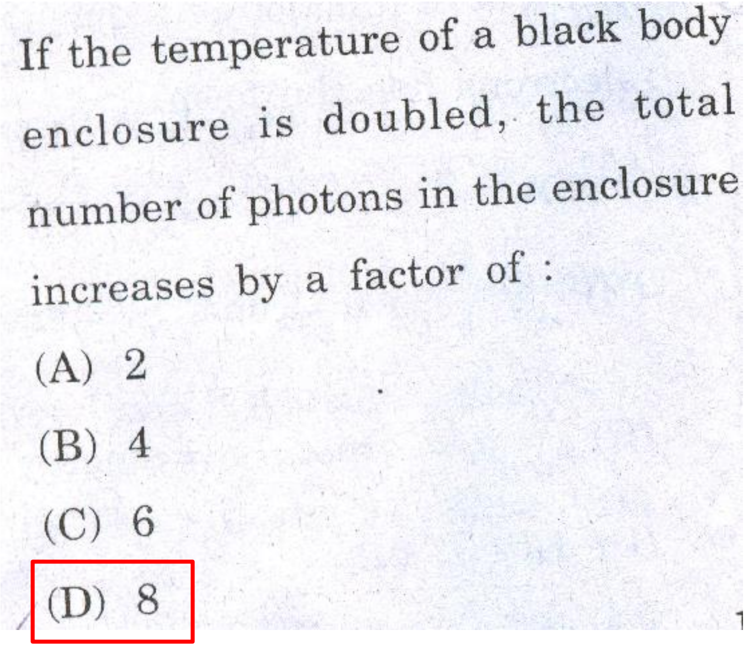
Ⓗ $(\in < \in_f) = 1$

if $\in \leq \in_f$

Ⓗ $(\in < \in_f) = 0$

if $\in \geq \in_f$

SET 2013



If the temperature of a black body enclosure is doubled, the total number of photons in the enclosure increases by a factor of :

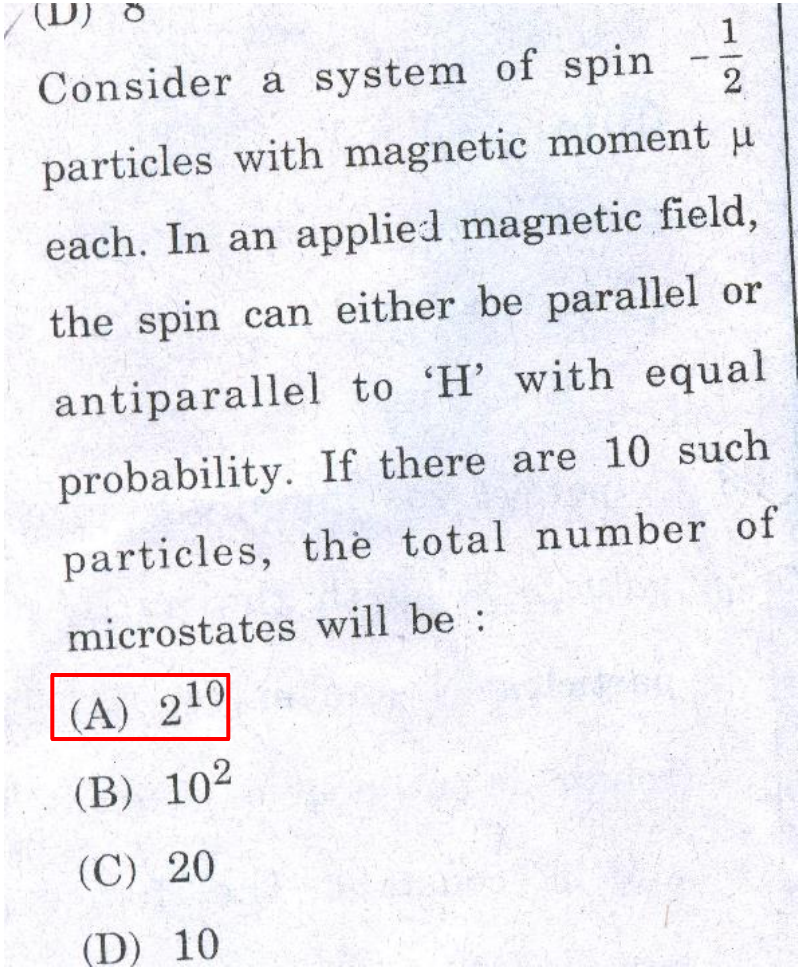
(A) 2

(B) 4

(C) 6

[✓ (D) 8]

SET 2013



(D) 8

Consider a system of spin $-\frac{1}{2}$ particles with magnetic moment μ each. In an applied magnetic field, the spin can either be parallel or antiparallel to 'H' with equal probability. If there are 10 such particles, the total number of microstates will be :

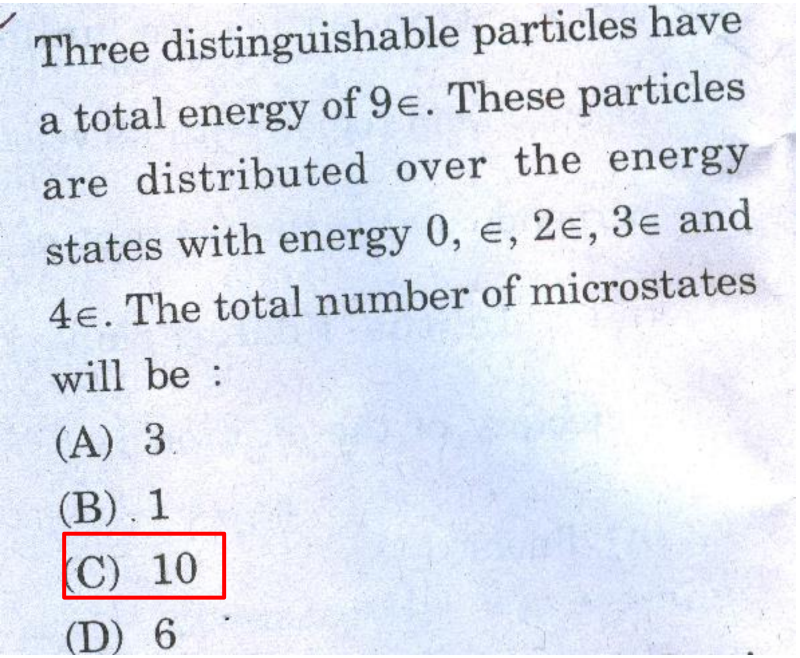
[✓ (A) 2^{10}]

(B) 10^2

(C) 20

(D) 10

SET 2013



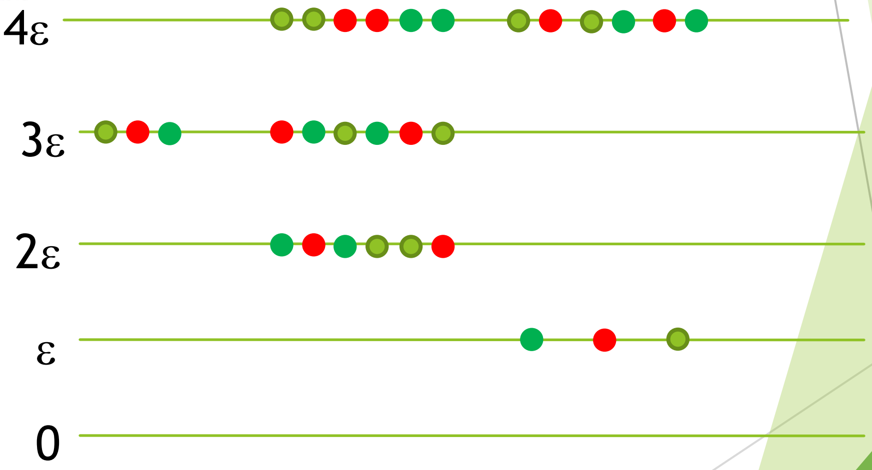
Three distinguishable particles have a total energy of 9∈. These particles are distributed over the energy states with energy 0, ∈, 2∈, 3∈ and 4∈. The total number of microstates will be :

(A) 3

(B) 1

[✓ (C) 10]

(D) 6



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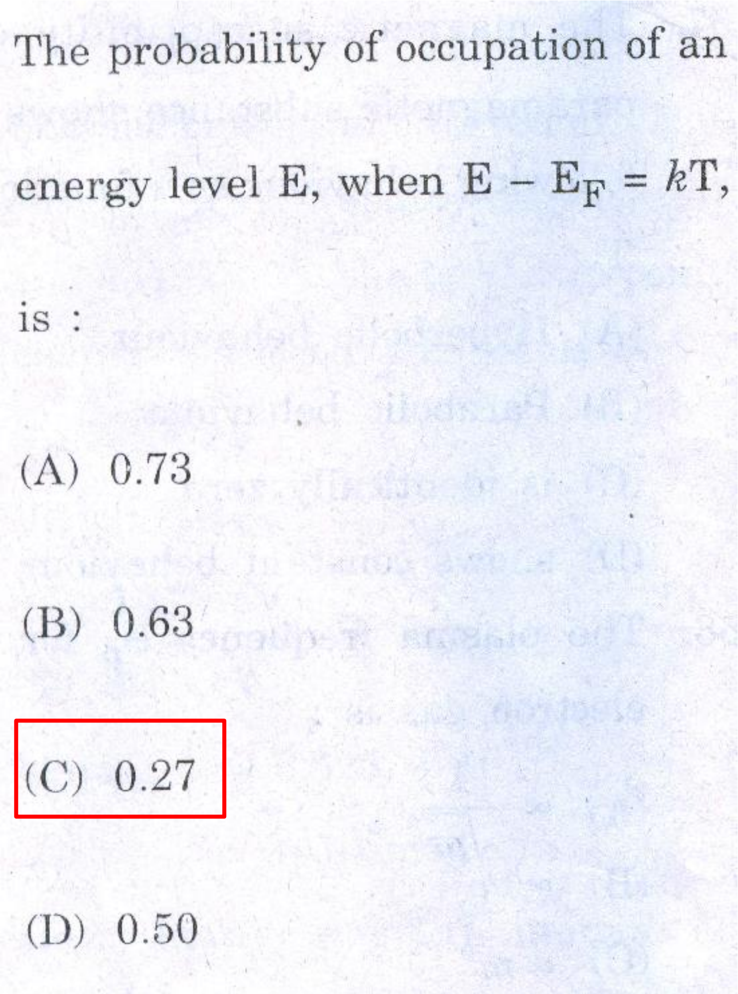


The probability of occupation of an energy level E, when $E - E_F = kT$, is :

(A) 0.73

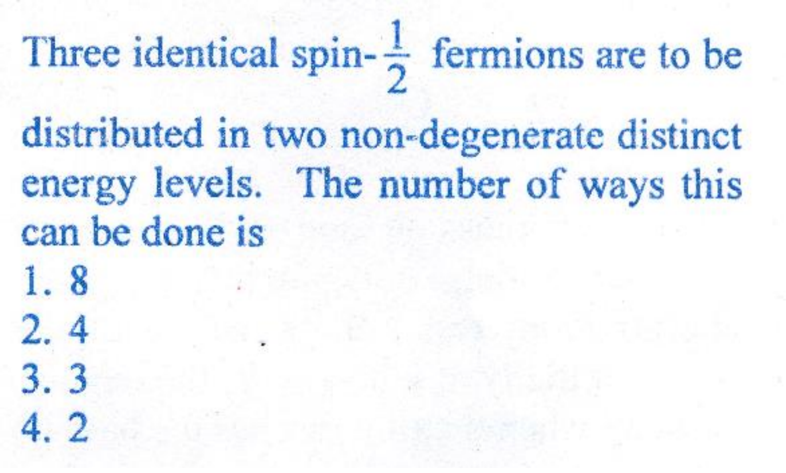
(B) 0.63

(C) 0.27

(D) 0.50

$$P = \frac{1}{e^{(E-E_F)/kT}+1} = \frac{1}{e^{1}+1} = \frac{1}{2.718282+1} = 0.268941 \sim 0.27$$

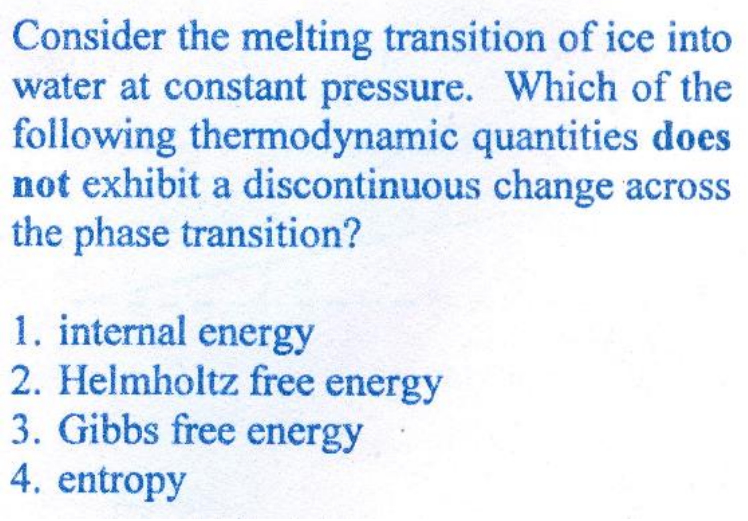
NET 2013



Three identical spin-$\frac{1}{2}$ fermions are to be distributed in two non-degenerate distinct energy levels. The number of ways this can be done is

1. 8
2. 4
3. 3
4. 2

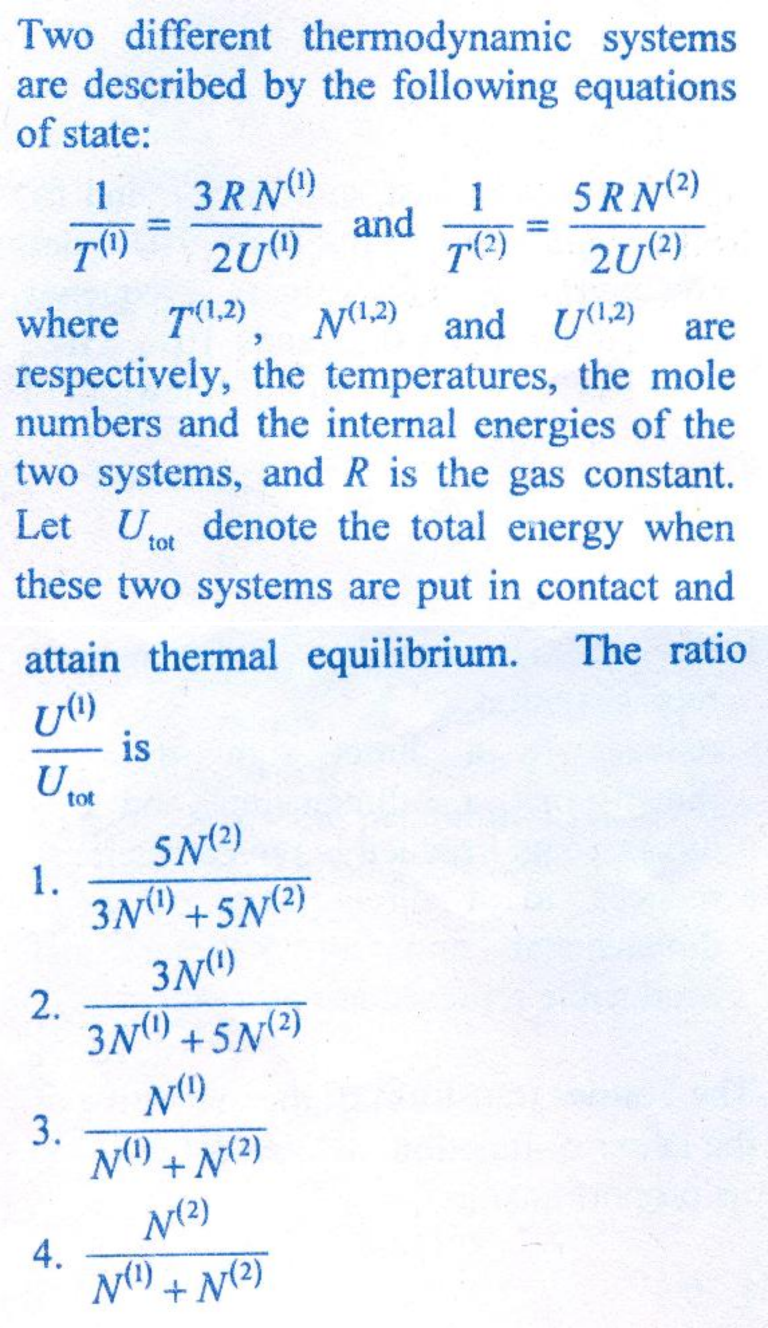
NET 2013



Consider the melting transition of ice into water at constant pressure. Which of the following thermodynamic quantities **does not** exhibit a discontinuous change across the phase transition?

1. internal energy
2. Helmholtz free energy
3. Gibbs free energy
4. entropy

NET 2013



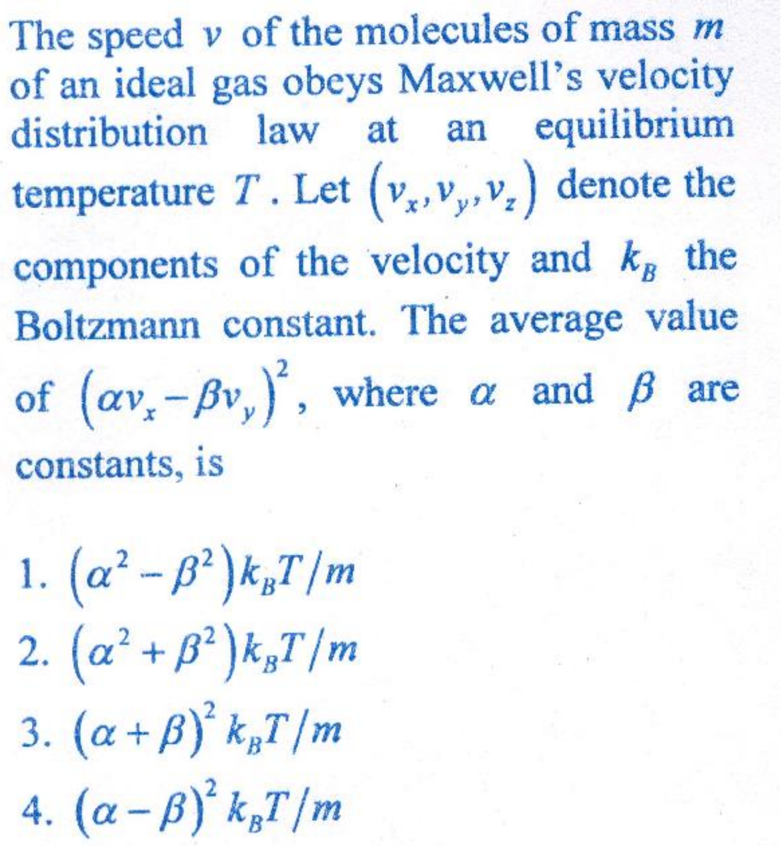
Two different thermodynamic systems are described by the following equations of state:

$$\frac{1}{T^{(1)}} = \frac{3RN^{(1)}}{2U^{(1)}} \quad \text{and} \quad \frac{1}{T^{(2)}} = \frac{5RN^{(2)}}{2U^{(2)}}$$

where $T^{(1,2)}$, $N^{(1,2)}$ and $U^{(1,2)}$ are respectively, the temperatures, the mole numbers and the internal energies of the two systems, and R is the gas constant. Let U_{tot} denote the total energy when these two systems are put in contact and attain thermal equilibrium. The ratio $\frac{U^{(1)}}{U_{\text{tot}}}$ is

1. $\frac{5N^{(2)}}{3N^{(1)}+5N^{(2)}}$
2. $\frac{3N^{(1)}}{3N^{(1)}+5N^{(2)}}$
3. $\frac{N^{(1)}}{N^{(1)}+N^{(2)}}$
4. $\frac{N^{(2)}}{N^{(1)}+N^{(2)}}$

NET 2013

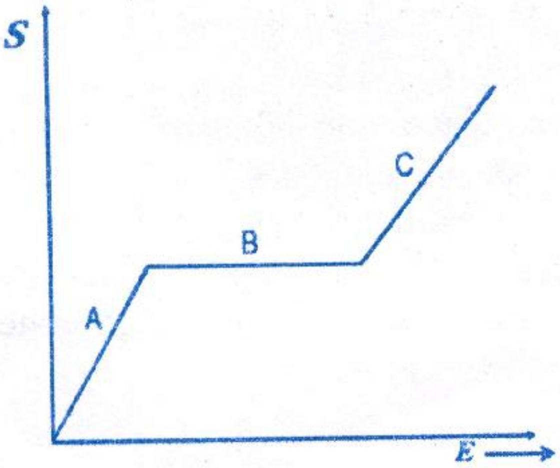


The speed v of the molecules of mass m of an ideal gas obeys Maxwell's velocity distribution law at an equilibrium temperature T. Let (v_x, v_y, v_z) denote the components of the velocity and k_B the Boltzmann constant. The average value of $(\alpha v_x - \beta v_y)^2$, where α and β are constants, is

1. $(\alpha^2 - \beta^2) k_B T/m$
2. $(\alpha^2 + \beta^2) k_B T/m$
3. $(\alpha + \beta)^2 k_B T/m$
4. $(\alpha - \beta)^2 k_B T/m$

NET 2013

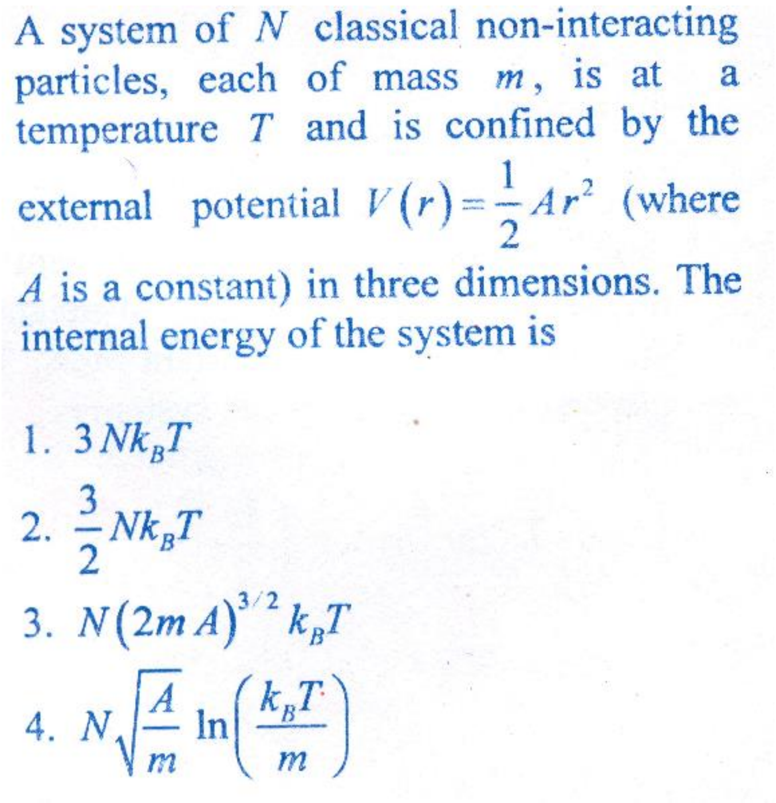
The entropy S of a thermodynamic system as a function of energy E is given by the following graph



The temperatures of the phases A, B and C, denoted by T_A, T_B and T_C, respectively, satisfy the following inequalities:

1. $T_C > T_B > T_A$
2. $T_A > T_C > T_B$
3. $T_B > T_C > T_A$
4. $T_B > T_A > T_C$

NET 2013



A system of N classical non-interacting particles, each of mass m, is at a temperature T and is confined by the external potential $V(r)=\frac{1}{2}Ar^2$ (where A is a constant) in three dimensions. The internal energy of the system is

1. $3Nk_BT$
2. $\frac{3}{2}Nk_BT$
3. $N(2mA)^{3/2}k_BT$
4. $N\sqrt{\frac{A}{m}}\ln\left(\frac{k_BT}{m}\right)$