Radioactive Decay

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Radioactive Decay

- Marie Curie and her husband Pierre discovered polonium and radium in 1898.
  - The simplest decay form is that of a gamma ray, which represents the nucleus changing from an excited state to lower energy state.
  - Other modes of decay include emission of α particles, *β* particles, protons, neutrons, and fission.

- The disintegrations or decays per unit time (**activity**).

$$\text{Activity} = -\frac{dN}{dt} = R$$

where *dN* / *dt* is negative because total number *N* decreases with time.

Radioactive Decay

- SI unit of activity is the becquerel: 1 Bq = 1 decay / s.
- Recent use is the Curie (Ci) 3.7 × 10^{10} decays / s.

- If N(t) is the number of radioactive nuclei in a sample at time *t*, and λ (**decay constant**) is the probability per unit time that any given nucleus will decay:

$$R = \lambda N(t)$$

$$dN(t) = -R\,dt = -\lambda N(t)\,dt$$

$$\int \frac{dN}{N} = -\int \lambda\,dt$$

$$\ln N = -\lambda t + \text{constant}$$

$$N(t) = e^{-\lambda t + \text{constant}}$$

- If we let *N*(*t* =

$$N(t) = N_0 e^{-\lambda t}$$ ----- **radioactive decay law**

Radioactive Decay

- The activity *R* is

$$R = \lambda N(t) = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

where R_0 is the initial activity at $t = 0$.

- It is common to refer to the half-life $t_{1/2}$ or the mean lifetime τ rather than its decay constant.

$$N(t_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

$$\ln\left(\tfrac{1}{2}\right) = \ln(e^{-\lambda t_{1/2}}) = -\lambda t_{1/2}$$

- The half-life is

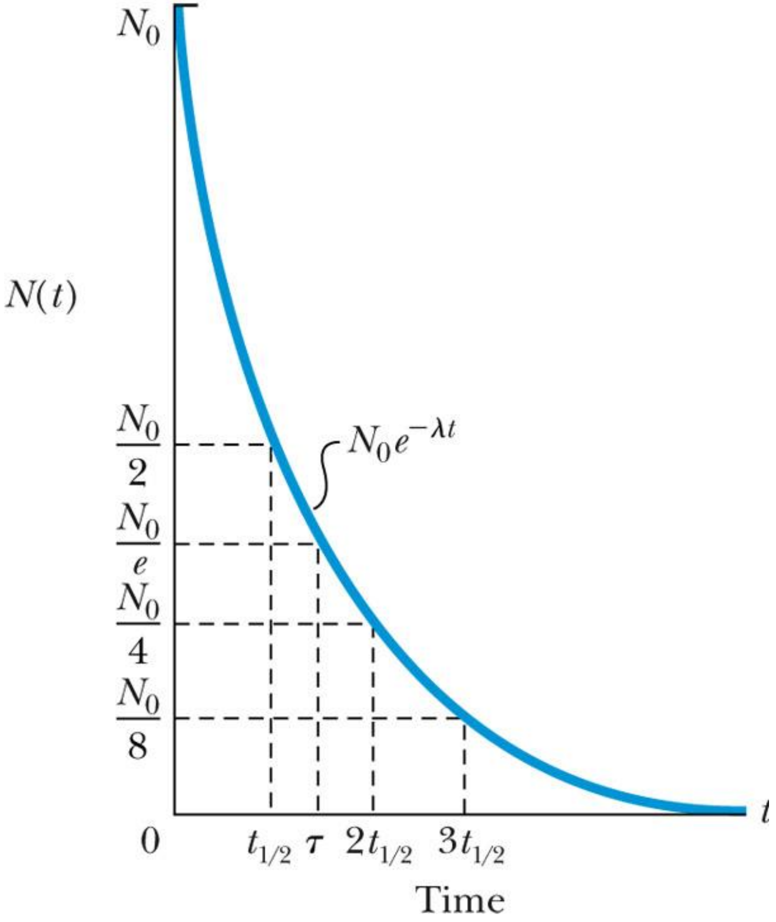
$$t_{1/2} = \frac{-\ln(1/2)}{\lambda} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}$$

- The mean lifetime is

$$\tau = \frac{1}{\lambda} = \frac{t_{1/2}}{\ln(2)}$$

Radioactive Decay

- The number of radioactive nuclei as a function of time



12.7: Alpha, Beta, and Gamma Decay

When a nucleus decays, all the conservation laws must be observed:

- Mass-energy
- Linear momentum
- Angular momentum
- Electric charge
- **Conservation of nucleons**
  - The total number of nucleons (*A*, the mass number) must be conserved in a low-energy nuclear reaction or decay.

Alpha, Beta, and Gamma Decay

- Let the radioactive nucleus $^{A}_{Z}X$ alled the parent and have the mass $M\left({}^{A}_{Z}X\right)$
- Two or more products can be produced in the decay.
- Let the lighter one be M_y and the mass of the heavier one (*daughter*) be M_D.
- The conservation of energy is

$$M\left({}^{A}_{Z}X\right) = M_D + M_y + Q/c^2$$

where Q is the energy released (**disintegration energy**) and equal to the total kinetic energy of the reaction products.

$$Q = \left[M\left({}^{A}_{Z}X\right) - M_D - M_y\right]c^2$$

- If $B > 0$, a nuclide is bound and stable;
- If $Q > 0$, a nuclide is unbound, unstable, and may decay.
- If $Q < 0$, decay emitting nucleons do not occur.

Alpha Decay

- The nucleus ^{4}He has a binding energy of 28.3 MeV.
- If the last two protons and two neutrons in a nucleus are bound by less than 28.3 MeV, then the emission of an alpha particle (alpha decay) is possible.

$$ {}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}D + \alpha $$

$$ Q = \left[M\left({}_{Z}^{A}X\right) - M\left({}_{Z-2}^{A-4}D\right) - M({}^{4}\text{He}) \right] c^{2} $$

- If $Q > 0$, alpha decay is possible.

  EX. ${}_{92}^{230}\text{U} \rightarrow \alpha + {}_{90}^{226}\text{Th}$

  The appropriate masses are

$$ M\left({}_{92}^{230}\text{U}\right) = 230.033927\ \text{u};\ M({}^{4}\text{He}) = 4.002603\ \text{u};\ M({}_{90}^{226}\text{Th}) = 226.024891\ \text{u} $$

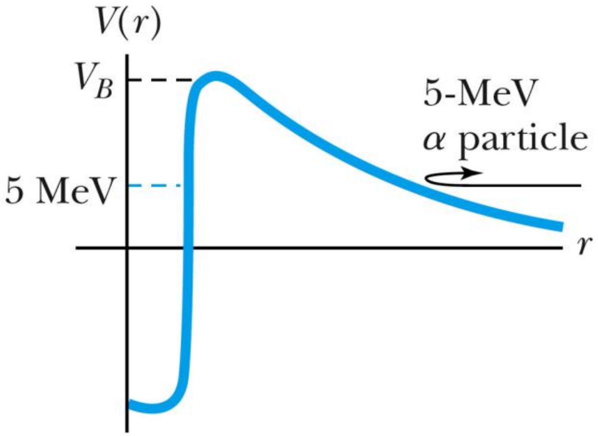
Alpha Decay

- Insert into Eq.(12.31)

$$Q = \left[M(^{230}\text{U}) - M(^{226}\text{Th}) - M(^{4}\text{He}) \right] c^2$$

$$= [230.033927\ \text{u} - 226.024891\ \text{u} - 4.002603\ \text{u}]c^2 \left(\frac{931.5\ \text{MeV}}{c^2 \cdot \text{u}} \right) = 6.0\ \text{MeV}$$

- In order for alpha decay to occur, two neutrons and two protons group together within the nucleus prior to decay and the alpha particle has difficulty in overcoming the nuclear attraction from the remaining nucleons to escape.

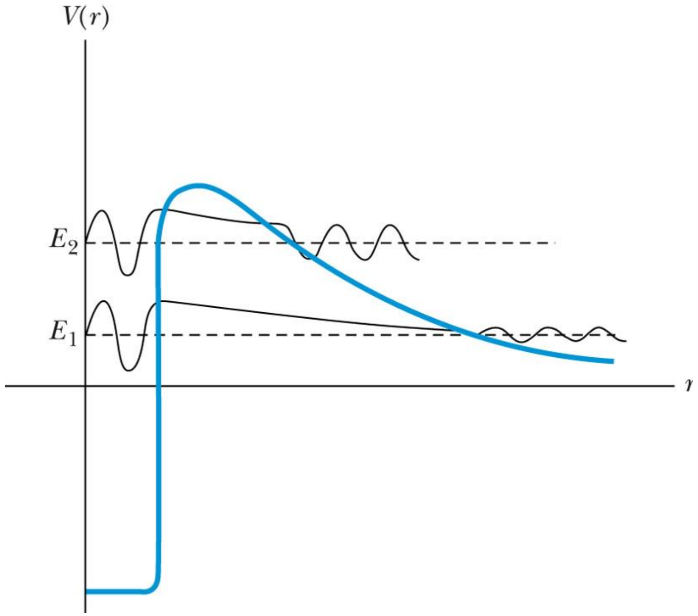


The potential energy diagram of alpha particle

Alpha Decay

- The barrier height V_B is greater than 20 MeV.
- The kinetic energies of alpha particles emitted from nuclei range from 4-10 MeV.

$\longrightarrow$ It is impossible classically for the alpha particle to reach the nucleus, but the alpha particles are able to tunnel through the barrier.

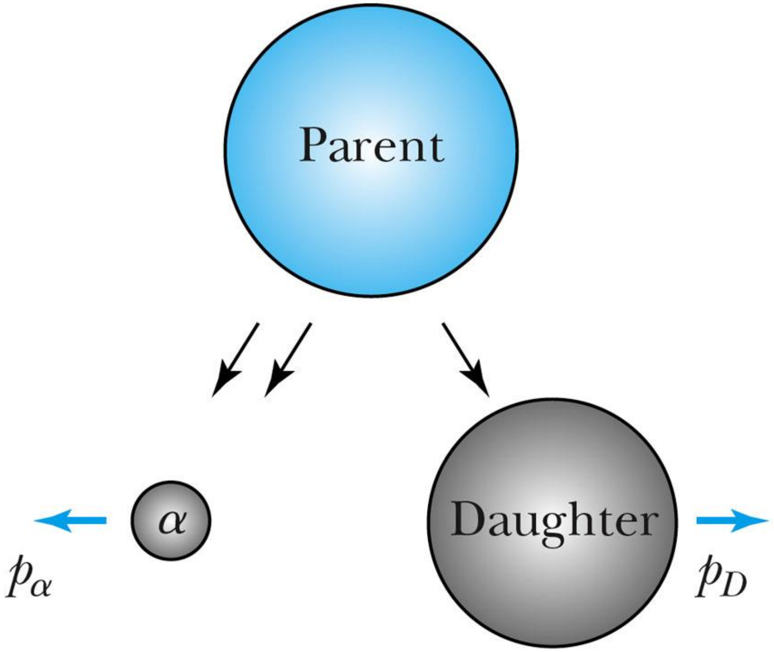


A higher energy E_2 has much higher probability than does a lower energy E_1.

There is a correlation between lower energies and greater difficulty of escaping (longer lifetimes).

Alpha Decay

- Assume the parent nucleus is initially at rest so that the total momentum is zero.
- The final momenta of the daughter p_D and alpha particle p_α have the same magnitude and opposite directions.



Alpha Decay

- From the conservation of energy and conservation of linear momentum, determine a unique energy for the alpha particle.

$$Q = K_\alpha + K_D$$

$$p_\alpha = p_D$$

$$K_\alpha = Q - K_D = Q - \frac{{p_D}^2}{2M_D} = Q - \frac{{p_\alpha}^2}{2M_D}$$

$$K_\alpha = Q - \frac{2M_a K_\alpha}{2M_D} = Q - \frac{M_\alpha}{M_D} K_\alpha$$

$$K_\alpha \left(1 + \frac{M_\alpha}{M_D} \right) = Q$$

$$K_\alpha = \frac{M_D}{M_D + M_\alpha} Q \approx \left(\frac{A-4}{A} \right) Q$$

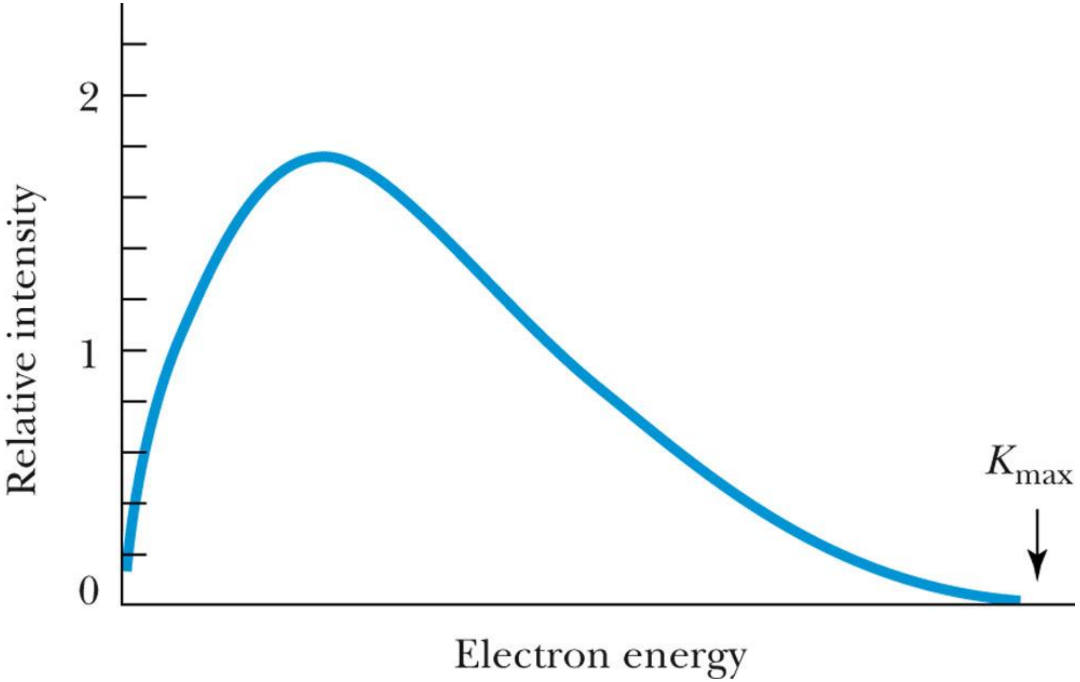
Beta Decay

- Unstable nuclei may move closer to the line of stability by undergoing beta decay.
- The decay of a free neutron is

$$n \rightarrow p + \beta^-$$

- The beta decay of ^{14}C (unstable) to form ^{14}N, a stable nucleus, can be written as

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \beta^-$$



The electron energy spectrum from the beta decay

Beta Decay

- There was a problem in neutron decay, the spin ½ neutron cannot decay to two spin ½ particles, a proton and an electron. ^{14}C has spin 0, ^{14}N has spin 1, and the electron has spin ½.

→ we cannot combine spin ½ & 1 to obtain a spin 0.

- Wolfgang Pauli suggested a **neutrino** ν that must be produced in beta decay. It has spin quantum number ½, charge 0, and carries away the additional energy missing in Fig. (12.14).

Beta Decay

- An occasional electron is detected with the kinetic energy K_{max} required to conserve energy, but in most cases the electron's kinetic energy is less than K_{max}.

→ the neutrino has little or no mass, and its energy may be all kinetic.

- Neutrinos have no charge and do not interact *electromagnetically*.
- They are not affected by the *strong* force of the nucleus.
- They are the *weak* interaction.
- The electromagnetic and weak forces are the *electroweak* force.

β^- Decay

- There are *anti*neutrinos $\bar{\nu}$.
- The beta decay of a free neutron of ^{14}C is written as

$$n \rightarrow p + \beta^- + \bar{\nu} \qquad \beta^- \text{ decay}$$

$$^{14}\text{C} \rightarrow {}^{14}\text{N} + \beta^- + \bar{\nu} \qquad \beta^- \text{ decay}$$

- In the general beta decay of the parent nuclide $^A_Z X$ to the daughter $^A_{Z+1}D$, the reactio $^A_Z X$

$$^A_Z X \rightarrow {}_{Z+1}^{\;\;A}D + \beta^- + \bar{\nu} \qquad \beta^- \text{ decay}$$

- The disintegration energy *Q* is

$$Q = \left[M\left(^A_Z X \right) - M\left({}_{Z+1}^{\;\;A}D \right) \right] c^2 \qquad \beta^- \text{ decay}$$

- In order for *β*⁻ to occur, we must have *Q* > 0.
- The nucleus *A* is constant, but *Z* charges to *Z* + 1.

β^+ Decay

- What happens for unstable nuclides with too many protons?
- Positive electron (positron) is produced.
- Positron is the antiparticle of the electron.
- A free proton does not decay when $t_{1/2} > 10^{32}$ y.
- The nucleus ^{14}O is unstable and decays by emitting a positron to become stable ^{14}N.

$$^{14}\mathrm{O} \rightarrow {}^{14}\mathrm{N} + \beta^+ + \nu \qquad \beta^+ \text{ decay}$$

- The general β^+ decay is

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}D + \beta^+ + \nu \qquad \beta^+ \text{ decay}$$

- The disintegration energy Q is

$$Q = \left[M\left({}^{A}_{Z}X \right) - M\left({}^{A}_{Z-1}D \right) - 2m_e \right] c^2 \qquad \beta^+ \text{ decay}$$

Electron Capture

- Classically, inner K-shell and L-shell electrons are tightly bound and their orbits are highly elliptical, these electrons spend a time passing through the nucleus, thereby the possibility of atomic electron capture.
- The reaction for a proton is $p + e^{-} \rightarrow n + v$
- The general reaction is

$$ {}^{A}_{Z}X + e^{-} \rightarrow {}^{A}_{Z-1}D + v \qquad \text{Electron capture} $$

- The disintegration energy Q is

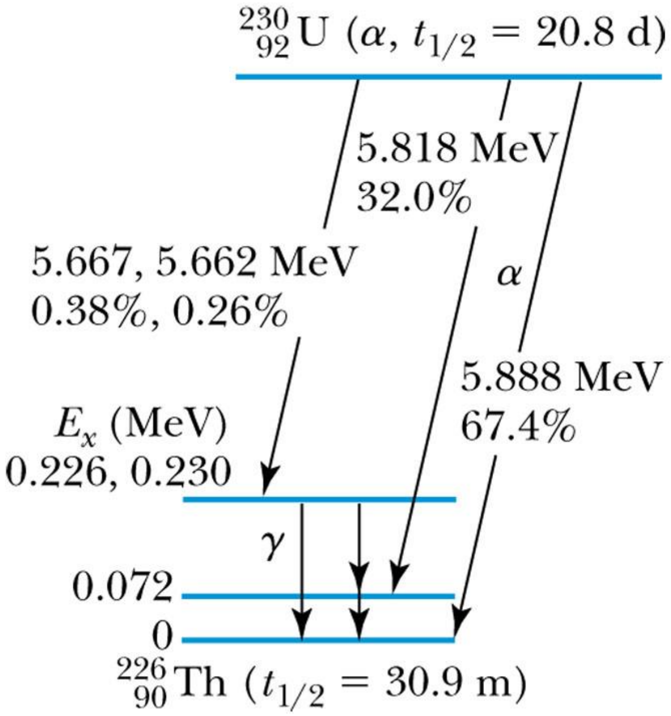
$$ Q = \left[M\left({}^{A}_{Z}X \right) - M\left({}^{A}_{Z-1}D \right) \right] c^{2} \qquad \text{Electron capture} $$

Gamma Decay

- If the decay proceeds to an excited state of energy E_x rather than to the ground state, then Q for the transition to the excited state can be determined with respect to the transition to the ground state. The disintegration energy Q to the ground state Q_0.

- Q for a transition to the excited state E_x is

$$Q = Q_0 - E_x$$



Gamma Decay

- The excitation energies tend to be much larger, many keV or even MeV.
- The possibilities for the nucleus to rid itself of this extra energy is to emit a photon (gamma ray).
- The gamma-ray energy *hf* is given by the difference of the higher energy state $E_>$ and lower one $E_<$.

$$hf = E_> - E_<$$

- The decay of an excited stat e * is an excited state) to its ground state is

$$^{A}X^* \rightarrow {}^{A}X + \gamma$$

- A transition between two nuclear excited states $E_>$ and $E_<$ is

$$^{A}X^*(E_>) \rightarrow {}^{A}X^*(E_<) + \gamma$$

Gamma Decay

- The gamma rays are normally emitted soon after the nucleus is created in an excited state.

- Sometimes selection rules prohibit a certain transition, and the excited state may live for a long time.
- These states are called **isomers** or **isomeric states** and are denoted by a small m for *metastable*.

- Ex: the spin 9 state of a ${}^{210m}_{83}\mathrm{Bi}$. MeV excitation energy does not gamma decay because of a large spin difference transition.
- Even though is another example of prohibited (the probability of occurring is sma ${}^{93m}_{41}\mathrm{Nb}$ ay to the ground state, it does gamma decay.

12.8: Radioactive Nuclides

- The unstable nuclei found in nature exhibit natural radioactivity.

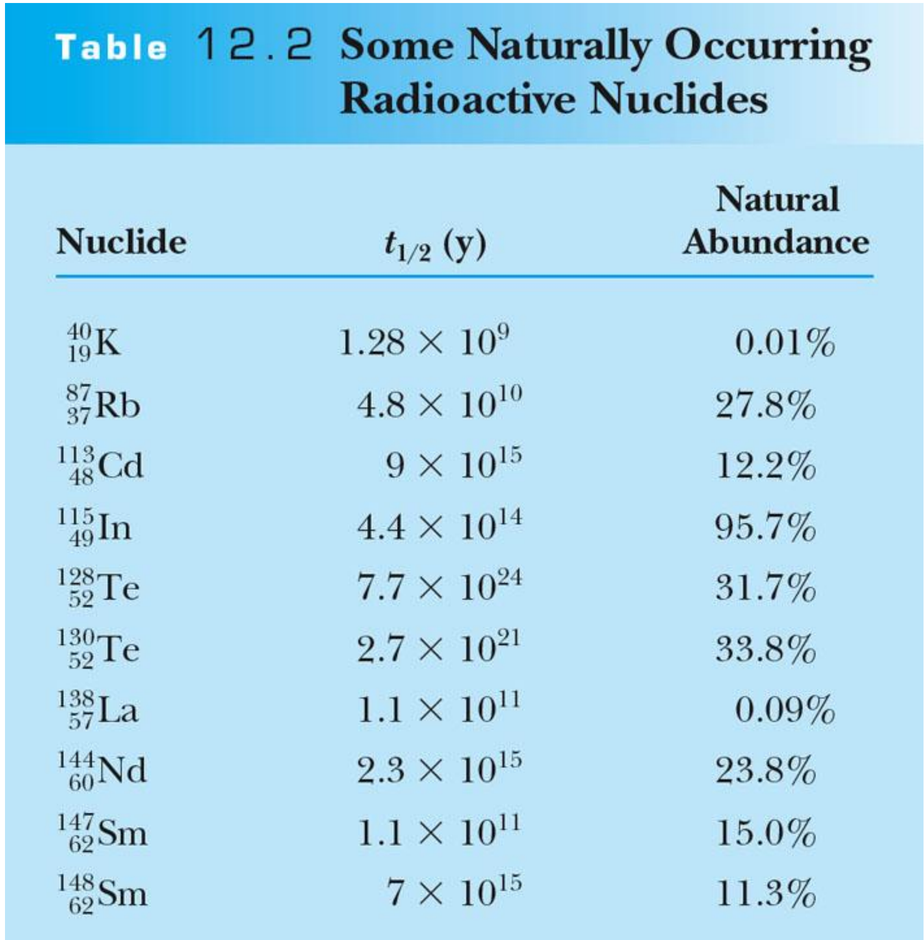


Table 12.2 Some Naturally Occurring Radioactive Nuclides

Nuclide	$t_{1/2}$ (y)	Natural Abundance
$^{40}_{19}$K	1.28×10^{9}	0.01%
$^{87}_{37}$Rb	4.8×10^{10}	27.8%
$^{113}_{48}$Cd	9×10^{15}	12.2%
$^{115}_{49}$In	4.4×10^{14}	95.7%
$^{128}_{52}$Te	7.7×10^{24}	31.7%
$^{130}_{52}$Te	2.7×10^{21}	33.8%
$^{138}_{57}$La	1.1×10^{11}	0.09%
$^{144}_{60}$Nd	2.3×10^{15}	23.8%
$^{147}_{62}$Sm	1.1×10^{11}	15.0%
$^{148}_{62}$Sm	7×10^{15}	11.3%

Radioactive Nuclides

- The radioactive nuclides made in the laboratory exhibit artificial radioactivity.
- Heavy radioactive nuclides can change their mass number only by alpha decay ($^{A}X \rightarrow {}^{A-4}D$) but can change their charge number Z by either alpha or beta decay.
- There are only four paths that the heavy naturally occurring radioactive nuclides may take as they decay.
- Mass numbers expressed by either:
  - $4n$
  - $4n + 1$
  - $4n + 2$
  - $4n + 3$

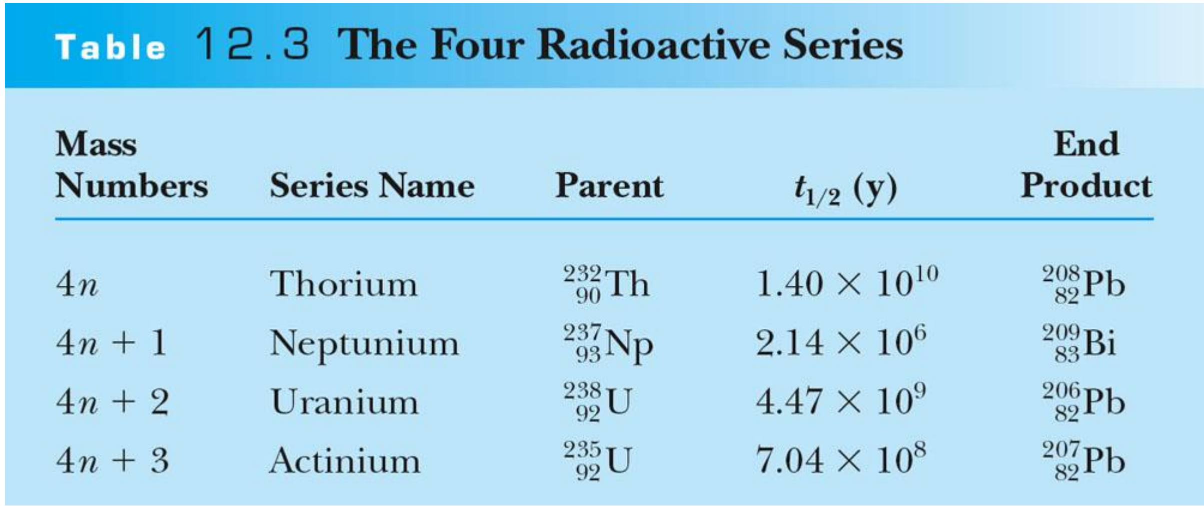
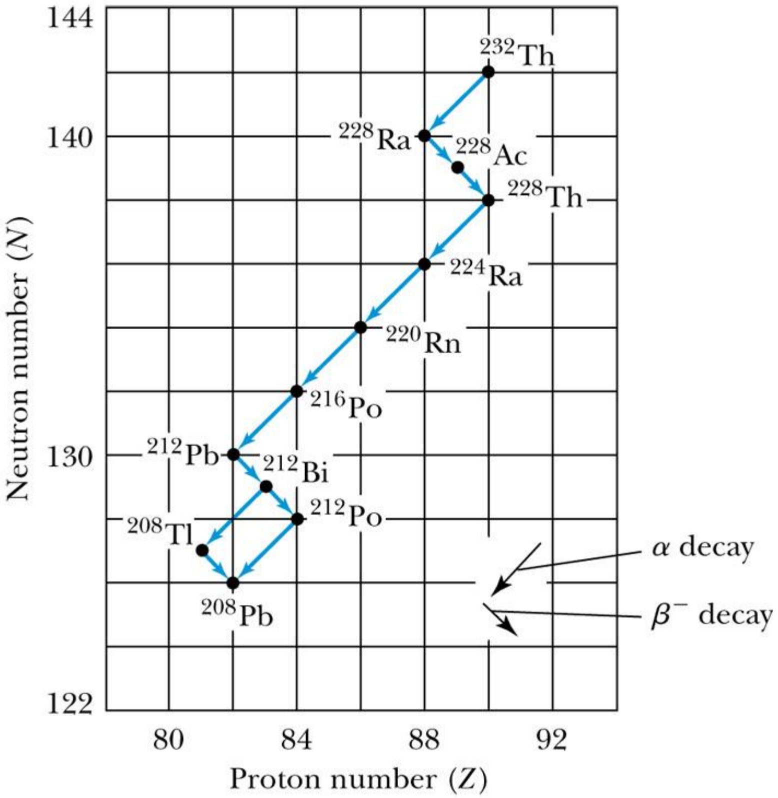


Table 12.3 The Four Radioactive Series

Mass Numbers	Series Name	Parent	$t_{1/2}$ (y)	End Product
$4n$	Thorium	$^{232}_{90}\text{Th}$	1.40×10^{10}	$^{208}_{82}\text{Pb}$
$4n + 1$	Neptunium	$^{237}_{93}\text{Np}$	2.14×10^{6}	$^{209}_{83}\text{Bi}$
$4n + 2$	Uranium	$^{238}_{92}\text{U}$	4.47×10^{9}	$^{206}_{82}\text{Pb}$
$4n + 3$	Actinium	$^{235}_{92}\text{U}$	7.04×10^{8}	$^{207}_{82}\text{Pb}$

Radioactive Nuclides

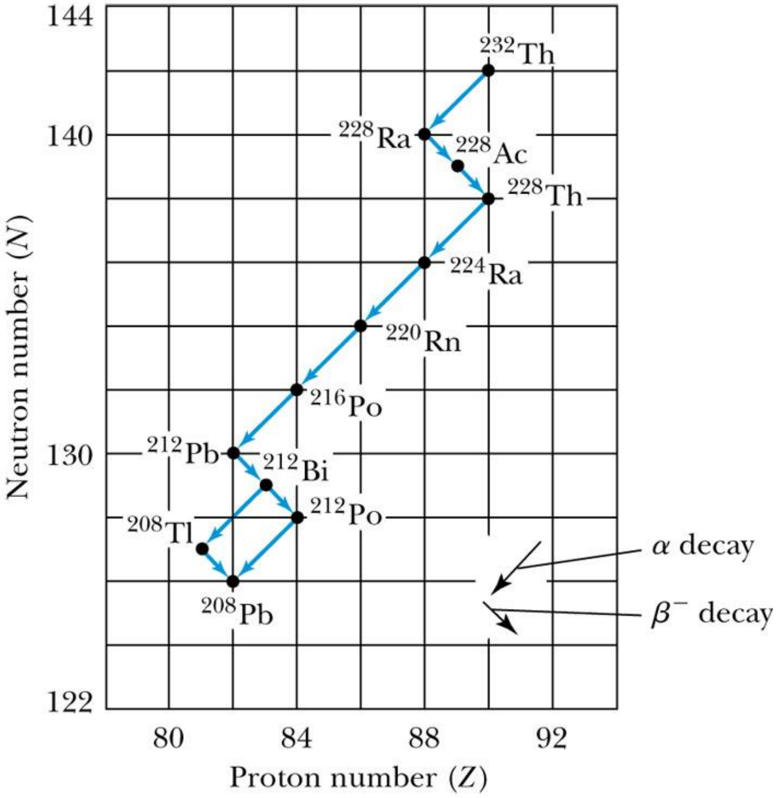
- The sequence of one of the radioactive series ^{232}Th



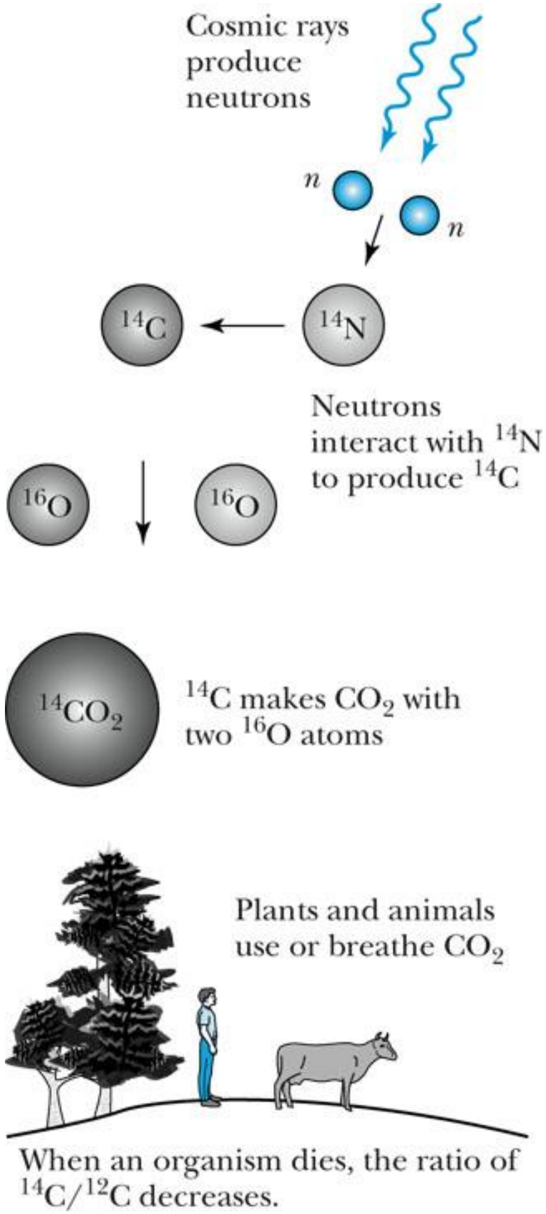
- ^{212}Bi can decay by either alpha or beta decay (*branching*).

Time Dating Using Lead Isotopes

- A plot of the abundance ratio of ^{206}Pb / ^{204}Pb versus ^{207}Pb / ^{204}Pb can be a sensitive indicator of the age of lead ores. Such techniques have been used to show that meteorites, believed to be left over from the formation of the solar system, are 4.55 billion years old.



Radioactive Carbon Dating



- Radioactive ^{14}C is produced in our atmosphere by the bombardment of ^{14}N by neutrons produced by cosmic rays.

$$n + {}^{14}\mathrm{N} \rightarrow {}^{14}\mathrm{C} + p$$

- When living organisms die, their intake of ^{14}C ceases, and the ratio of ^{14}C / ^{12}C (= R) decreases as ^{14}C decays. The period just before 9000 years ago had a higher ^{14}C / ^{12}C ratio by factor of about 1.5 than it does today.
- Because the half-life of ^{14}C is 5730 years, it is convenient to use the ^{14}C / ^{12}C ratio to determine the age of objects over a range up to 45,000 years ago.