CLASSICAL MECHANICS VS QUANTUM MECHANICS

By

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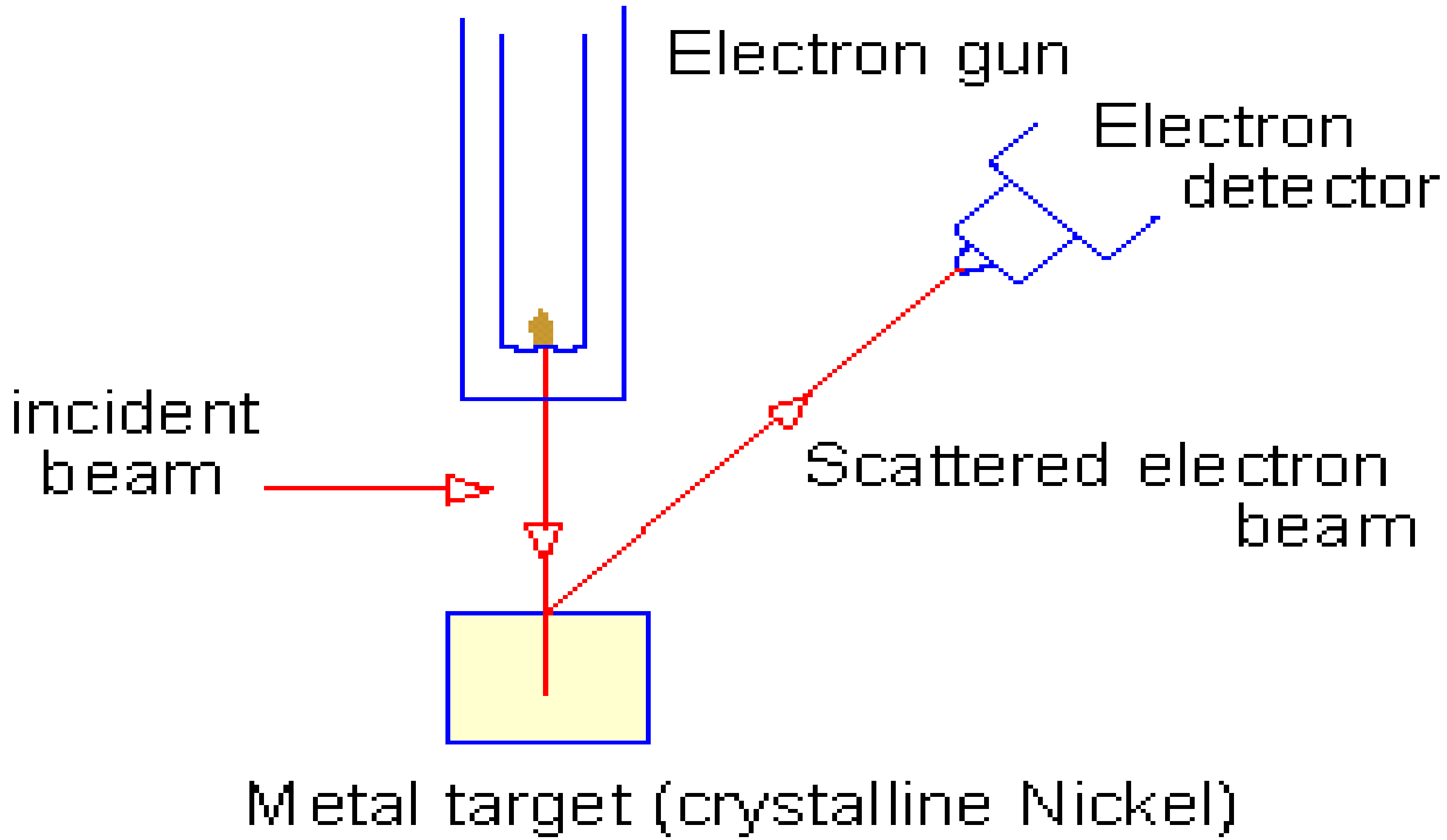
Date: 09/12/2020

DAVISSION GERMER EXPERIMENT

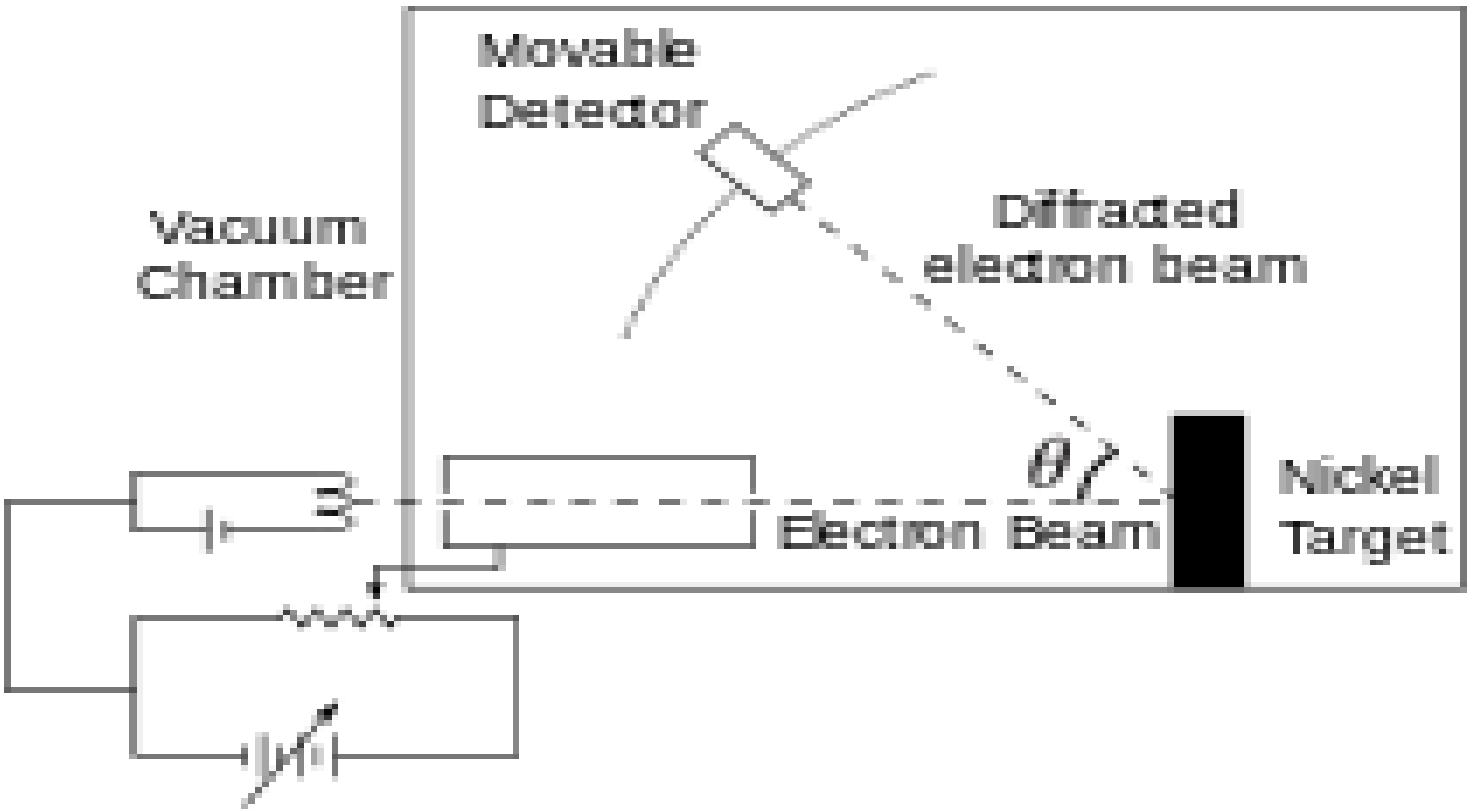
AIM OF EXPERIMENT

- To demonstrate diffraction phenomenon of electron to support wave character of material.
- This experiment gave establishment of quantum mechanics and schrodinger wave equation.


Electron gun
Electron detector
incident beam
Scattered electron beam
Metal target (crystalline Nickel)

- The experiment consisted of firing an electron beam from an electron gun directed to a piece of nickel crystal at normal incidence (i.e. perpendicular to the surface of the crystal). The experiment included an electron gun consisting of a heated filament that released thermally excited electrons, which were then accelerated through a potential difference giving them a certain amount of kinetic energy towards the nickel crystal.

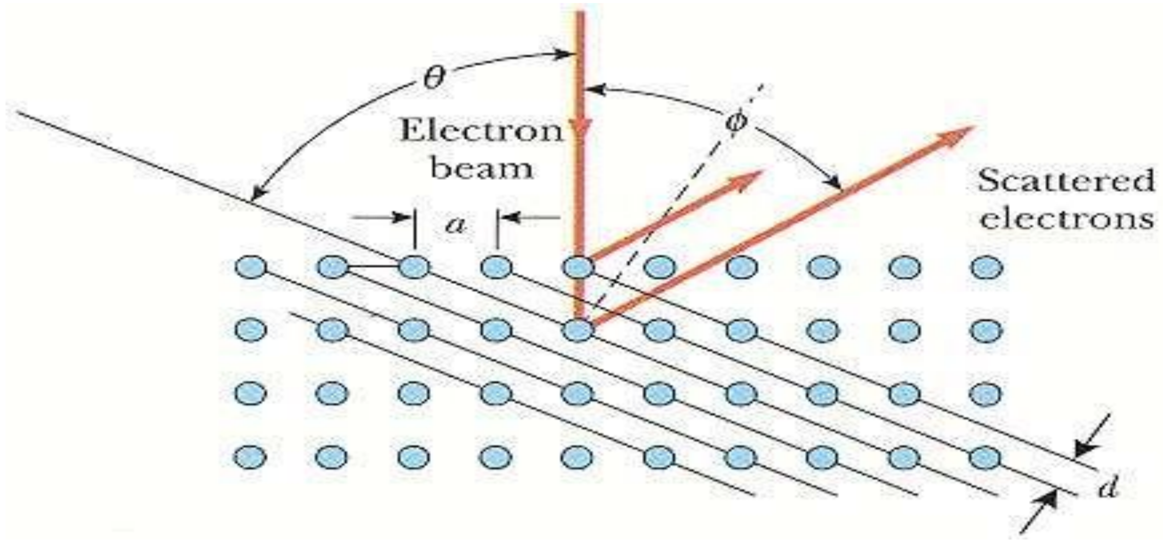
- To avoid collisions of the electrons with other molecules on their way towards the surface, the experiment was conducted in a vacuum chamber. To measure the number of electrons that were scattered at different angles, an electron detector that could be moved on an arc path about the crystal was used. The detector was designed to accept only elastically scattered electrons.


Movable Detector
Vacuum Chamber
Diffracted electron beam
θ
Nickel Target
Electron Beam

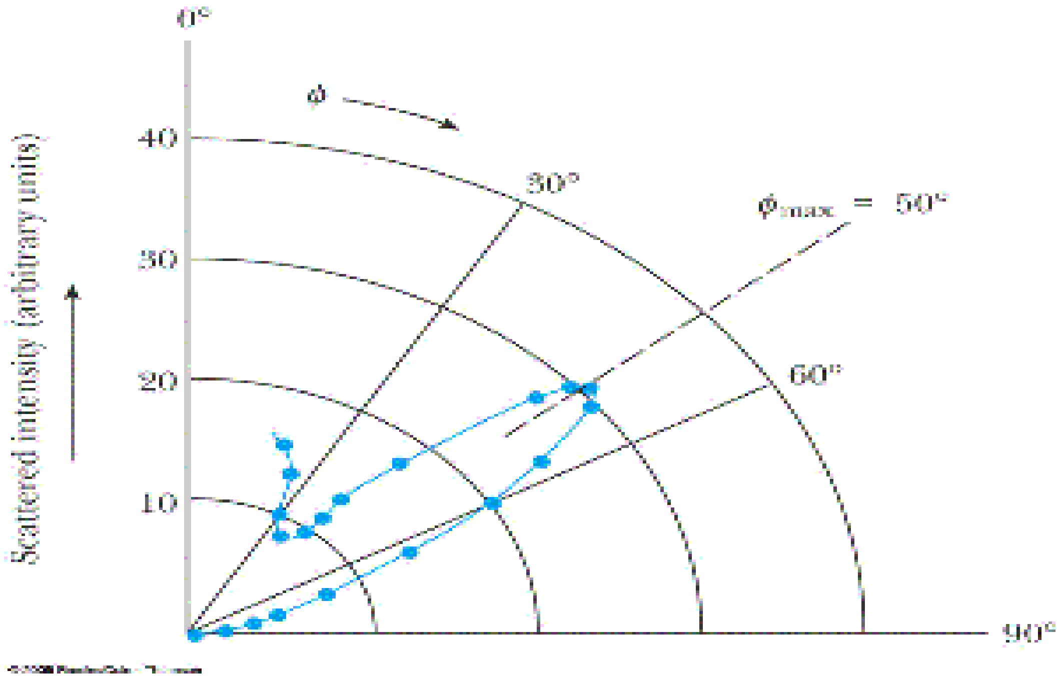
- As Max von Laue proved in 1912 the crystal structure serves as a type of three dimensional diffraction grating. The angles of maximum reflection are given by Bragg's condition for constructive interference from an array, Bragg's law
- for $n = 1, \theta = 50°$, and for the spacing of the crystalline planes of nickel ($d = 0.091$ nm) obtained from previous X-ray scattering experiments on crystalline nickel.

$$n\lambda = 2d\sin\left(90^\circ - \frac{\theta}{2}\right),$$

This is 3-d grating of nickel target where scattering of electron takes place.

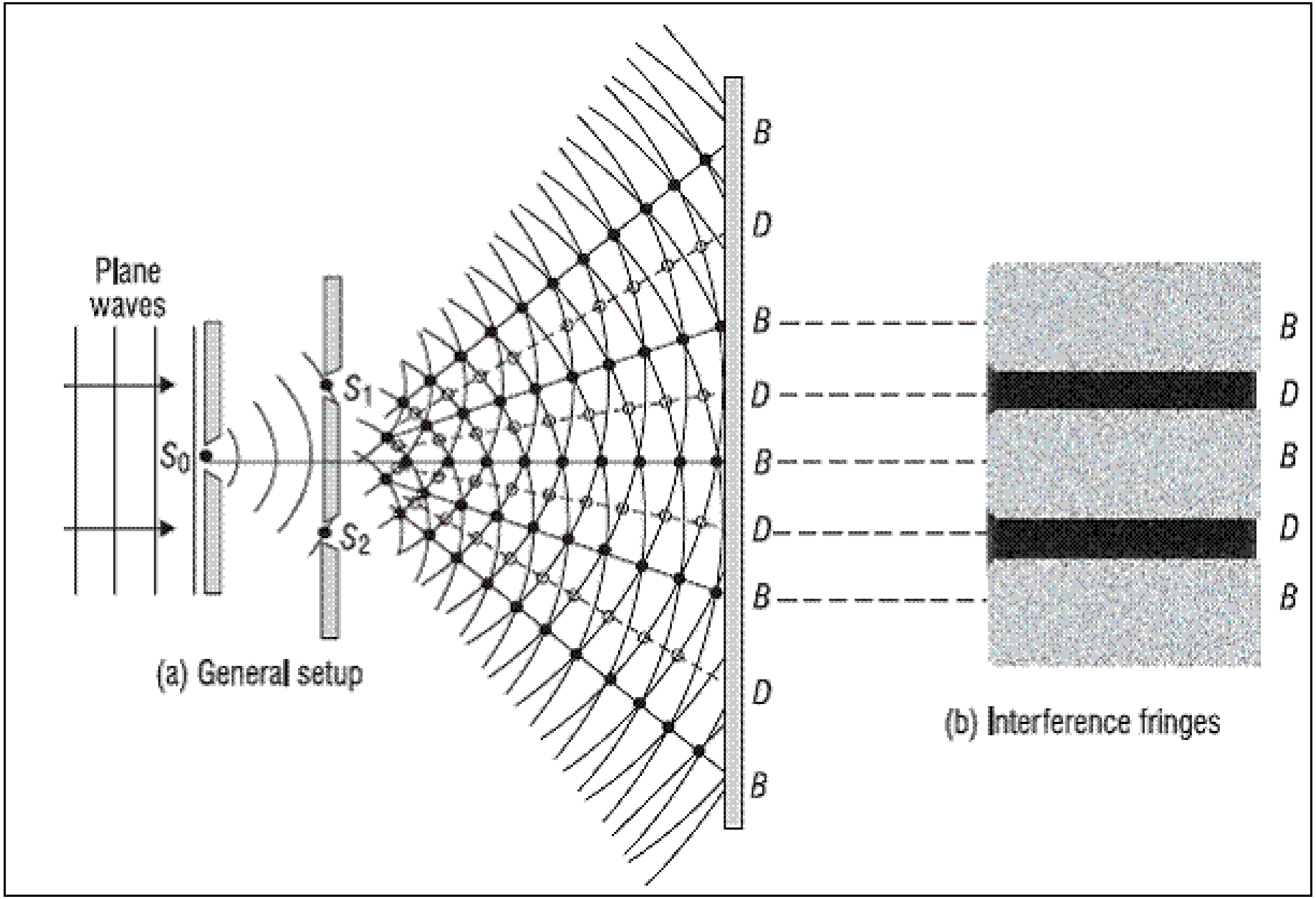


- By varying the applied voltage to the electron gun, the maximum intensity of electrons diffracted by the atomic surface was found at different angles. The highest intensity was observed at an angle $\theta = 50°$ with a voltage of 54 V, giving the electrons a kinetic energy of 54 eV.


0°
φ
40
30
20
10
30°
ϕ_{max} = 50°
60°
90°
Scattered intensity (arbitrary units)

Double Slit Experiment

- Let us try to understand what is meant by when we associate wave like properties with electrons.
- Consider an electron gun emitting electrons with a given wavelength which are allowed to pass through a double slit like arrangements.
- As the Davidson Germer experiment suggest the resulting intensity pattern will follow a two slit interference pattern on the slit.
- Now suppose we make the intensity of the electron beam so low that we can observe the electrons falling on the detector one by one. What are we going to see?
- First of all we cannot say through which slit the electron will pass even though are they are all identical and injected from the same source with same energy (which means classically they have same initial position and momentum and they are under the same force.


Plane waves
S_0
S_1
S_2
(a) General setup
B
D
B
D
B
D
B
D
B
B
D
B
D
B
(b) Interference fringes

Electron Double Slit Experiment

- If electrons behave like a particle then of course we could have determined through which slit it is coming out by following it's trajectory.

- On the other hand if a single electron itself behave like a wave in that case after coming out of the slit it's would have spread over the entire screen. But when a single electron comes through either of the slit it just forms a spot. Thus in that way it also retains its particle like identity.

- The wave like interference pattern was generated because two such identical electron when comes out of either of the slit under exactly identical condition they can form two spots at two completely arbitrary location. When many such electrons come through two such slits the resulting pattern is the famous Young's double slit pattern. Actually the above experiment was done by Tonomura much later at Hitachi Lab in Japan. But after Davidson Germer experiment this was what exactly expected.

Born Interpretation

- This puzzling behavior of electron where its collective behavior looks to have a wave like properties where a single electron till behaves as it is was finally explained by Max Born.

- The idea is unlike a classical particle the trajectory of an electron is not determined. Thus if we make a large number of measurement on identical electrons the result of the experiment can only be explained in terms of probability even though all these electrons start from the same location and are under the same potential.

- It is this probability amplitude which behaves like a wave and as a result resulting intensity pattern is same like a double slit interference pattern.

- All the physical quantities associated with electrons are thus to be calculated using this probability amplitude.

Classical Mechanics	Quantum Mechanics
(i) It deals with macroscopic particles.	(i) It deals with microscopic particles.
(ii)It is based upon Newton's laws of motion	(ii) It takes into account Heisenberg's uncertainty principle and de Broglie concept of dual nature of matter (particle nature and wave nature)
(iii) It is based on Maxwell's electromagnetic wave theory according to which any amount of energy may be emitted or absorbed continuously.	(iii) It is based on Planck's quantum theory according to which only discrete values of energy are emitted or absorbed.
(iv) The state of a system is defined by specifying all the forces acting on the particles as well as their positions and velocities (moment). The future state then can be predicted with certainty.	(iv) It gives probabilities of finding the particles at various locations in space.

LINE SPECTRUM

Continuous spectrum is the superimposed image of both absorption and emission spectra	Line spectrum is either an absorption spectrum or an emission spectrum
Contain no observable gaps	There are huge gaps between lines
Contain all the wavelengths of a given range	Contain only a few wavelengths
Rainbow and black body radiation are examples	Emission spectra of hydrogen and absorption spectra of hydrogen are examples

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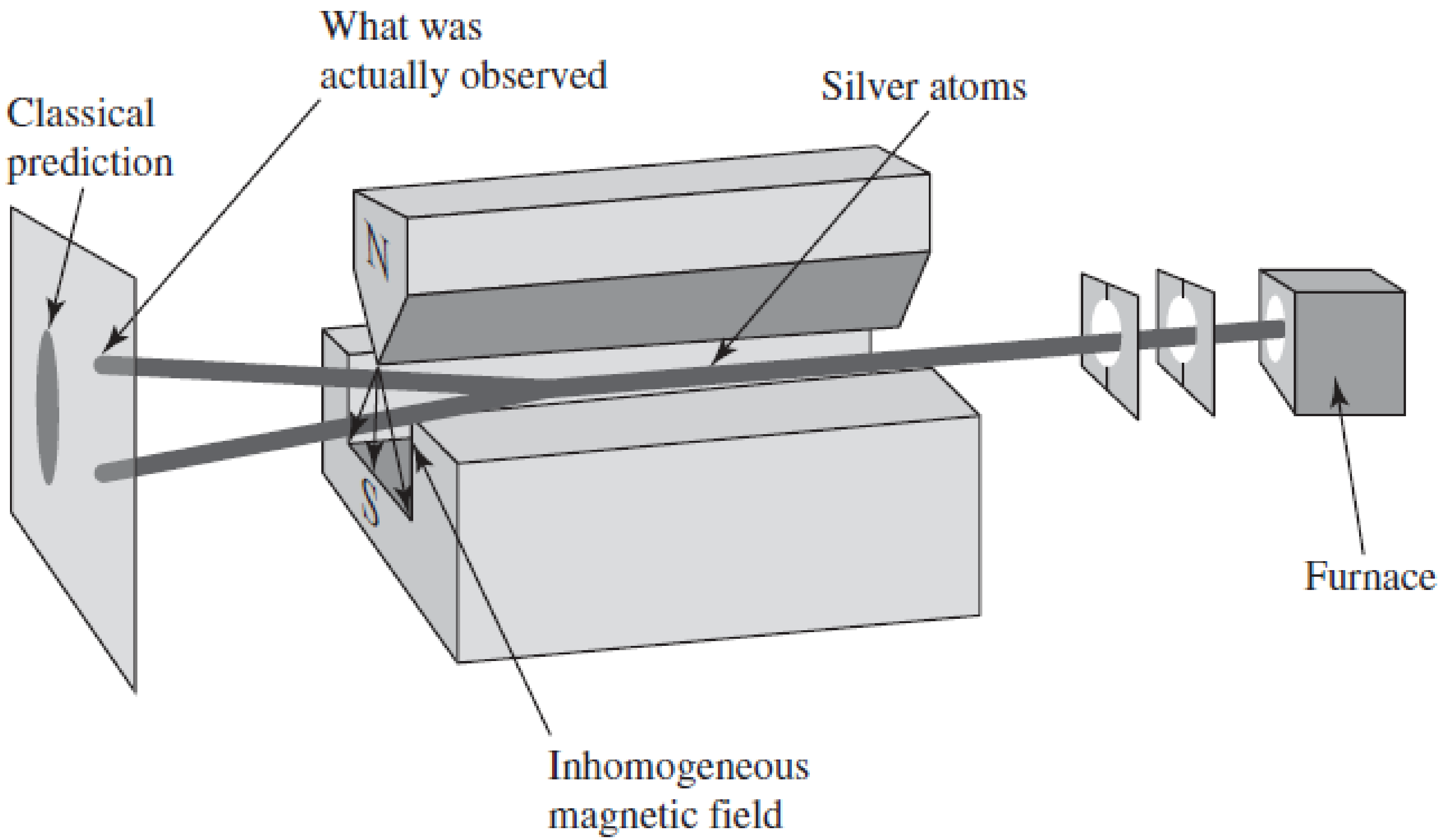
THE STERN-GERLACH EXPERIMENT



The Stern-Gerlach experiment.

- The silver atom is made up of a nucleus and 47 electrons, where 46 out of the 47 electrons can be visualized as forming a spherically symmetrical electron cloud with no net angular momentum. If we ignore the nuclear spin, which is irrelevant to our discussion, we see that the atom as a whole does have an angular momentum, which is due solely to the spin—intrinsic as opposed to orbital—angular momentum of the single 47th (5*s*) electron. The 47 electrons are attached to the nucleus, which is ∼2×10^5 times heavier than the electron; as a result, the heavy atom as a whole possesses a magnetic moment equal to the spin magnetic moment of the 47th electron. In other words, the magnetic moment μ of the atom is proportional to the electron spin **S**,

$$\mu \propto \mathbf{S},$$

where the precise proportionality factor turns out to be e/mc to an accuracy of about 0.2%.

Because the interaction energy of the magnetic moment with the magnetic field is just $-\mu\cdot\mathbf{B}$, the *z*-component of the force experienced by the atom is given by

$$F_z = \frac{\partial}{\partial z}(\boldsymbol{\mu} \cdot \mathbf{B}) \simeq \mu_z \frac{\partial B_z}{\partial z},$$

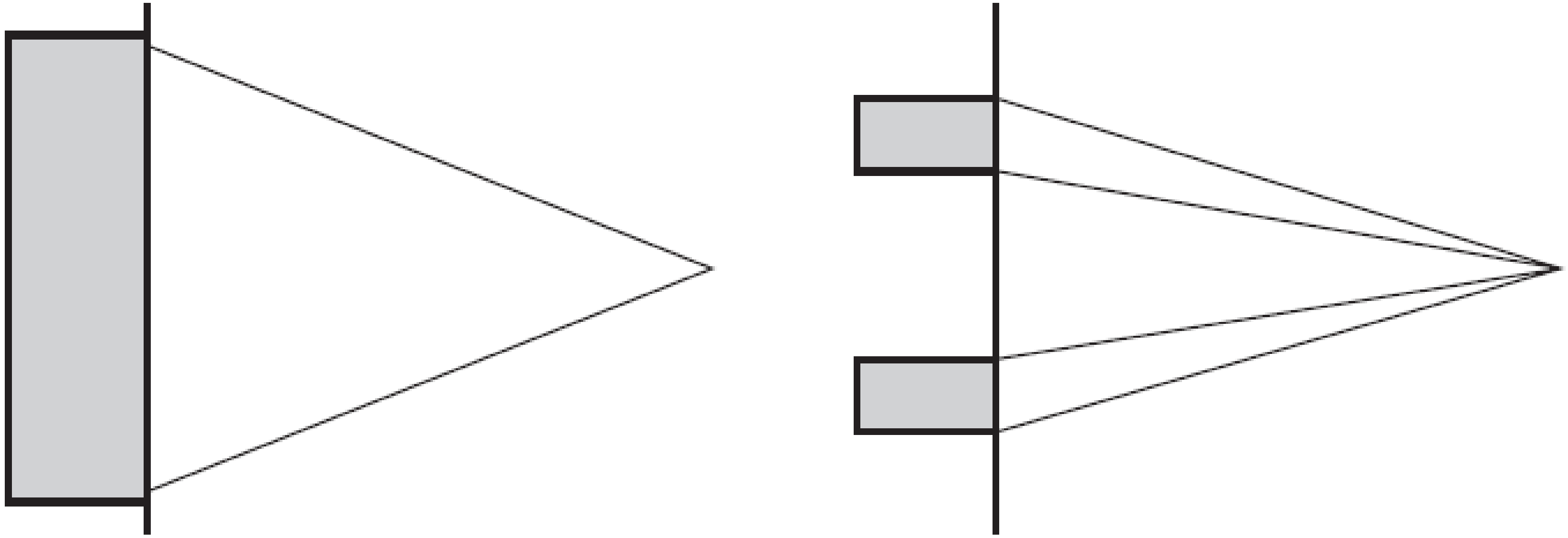
where we have ignored the components of **B** in directions other than the *z* direction. Because the atom as a whole is very heavy, we expect that the classical concept of trajectory can be legitimately applied.

With the arrangement of Figure 1.1, the $\mu_z > 0$ ($S_z < 0$) atom experiences a downward force, while the $\mu_z < 0$ ($S_z > 0$) atom experiences an upward force. The beam is then expected to get split according to the values of μ_z . In other words, the SG (Stern-Gerlach) apparatus "measures" the *z*-component of μ or, equivalently, the *z*-component of **S** up to a proportionality factor.

The atoms in the oven are randomly oriented; there is no preferred direction for the orientation of μ. If the electron were like a classical spinning object, we would expect all values of μ_z to be realized between |μ| and −|μ|. This would lead us to expect a continuous bundle of beams coming out of the SG apparatus, as indicated in Figure 1.1, spread more or less evenly over the expected range. Instead, what we experimentally observe is more like the situation also shown in Figure.1, where two "spots" are observed, corresponding to one "up" and one "down" orientation.

In other words, the SG apparatus splits the original silver beam from the oven into *two distinct* components, a phenomenon referred to in the early days of quantum theory as “space quantization.” To the extent that μ can be identified within a proportionality factor with the electron spin **S**, only two possible values of the *z*-component of **S** are observed to be possible: *Sz* up and *Sz* down, which we call *Sz*+ and *Sz*−. The two possible values of *Sz* are multiples of some fundamental unit of angular momentum; numerically it turns out that *Sz* = .*h*/2 and −.*h*/2, where

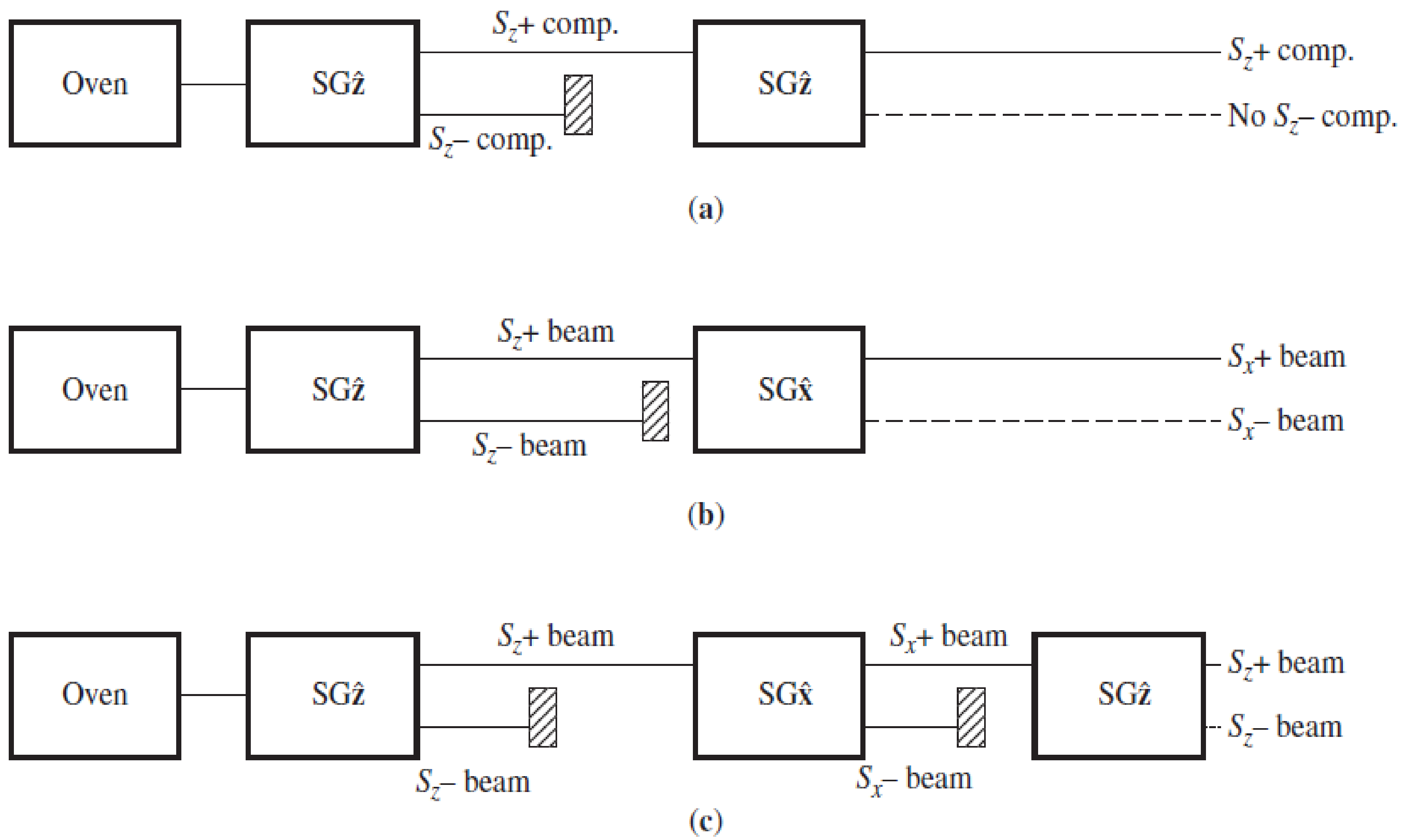
$$\begin{aligned}\hbar &= 1.0546 \times 10^{-27}\,\text{erg-s}\\ &= 6.5822 \times 10^{-16}\,\text{eV-s.}\end{aligned}$$



(a) Classical physics prediction for results from the Stern-Gerlach experiment. The beam should have been spread out vertically, over a distance corresponding to the range of values of the magnetic moment times the cosine of the orientation angle. Stern and Gerlach, however, observed the result in (b), namely that only two orientations of the magnetic moment manifested themselves. These two orientations did not span the entire expected range.

Figure a shows the result one would have expected from the experiment. According to classical physics, the beam should have spread itself over a vertical distance corresponding to the (continuous) range of orientation of the magnetic moment. Instead, one observes Figure b, which is completely at odds with classical physics. The beam mysteriously splits itself into two parts, one corresponding to spin “up” and the other to spin “down.” Of course, there is nothing sacred about the up-down direction or the *z*-axis. We could just as well have applied an inhomogeneous field in a horizontal direction, say in the *x*-direction, with the beam proceeding in the *y*-direction. In this manner we could have separated the beam from the oven into $Sx+$ component and $Sx-$ component.

Sequential Stern-Gerlach Experiments



Sequential Stern-Gerlach experiments.

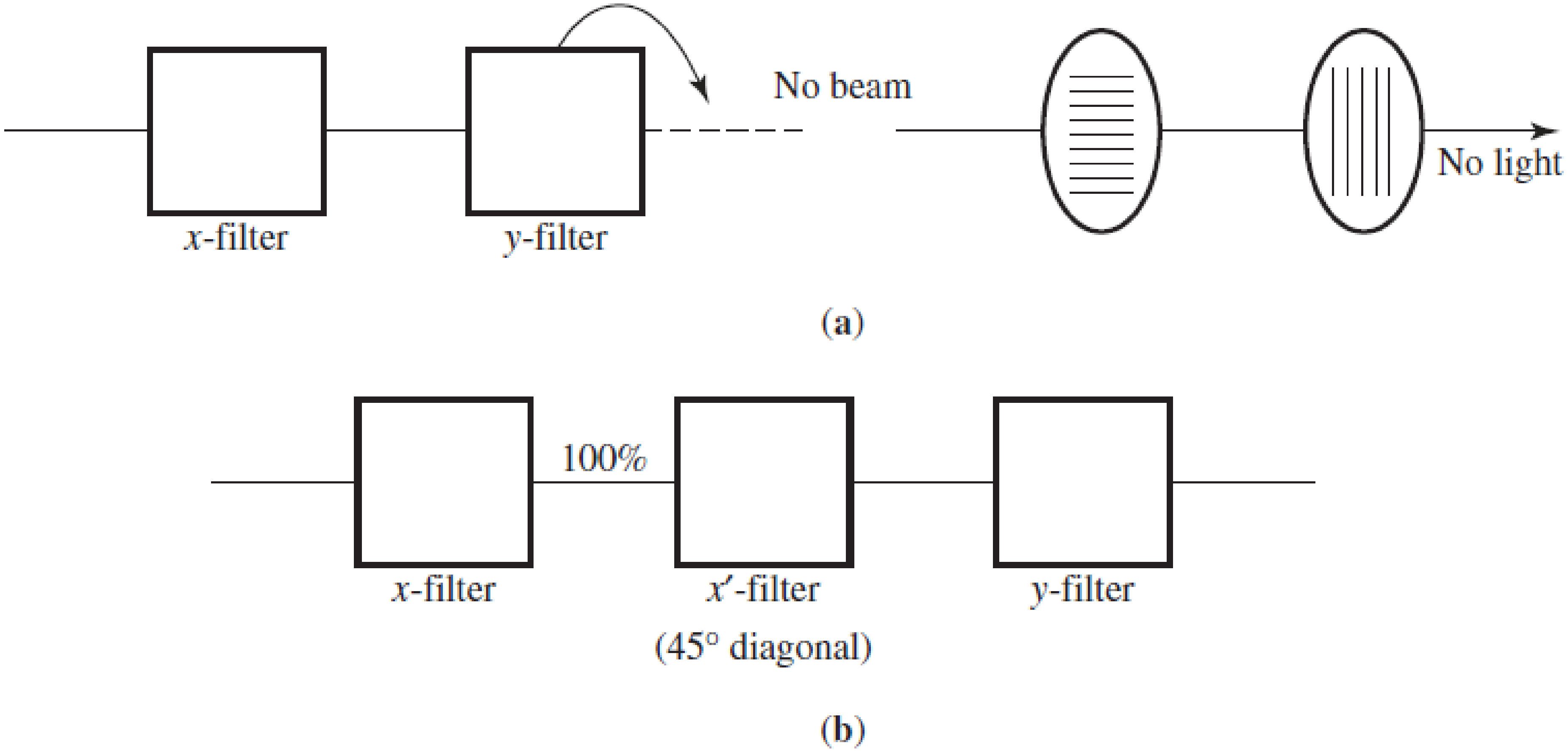
Analogy with Polarization of Light

Consider a monochromatic light wave propagating in the z-direction. A linearly polarized (or plane polarized) light with a polarization vector in the x-direction, which we call for short an *x-polarized light*, has a space-time– dependent electric field oscillating in the x-direction

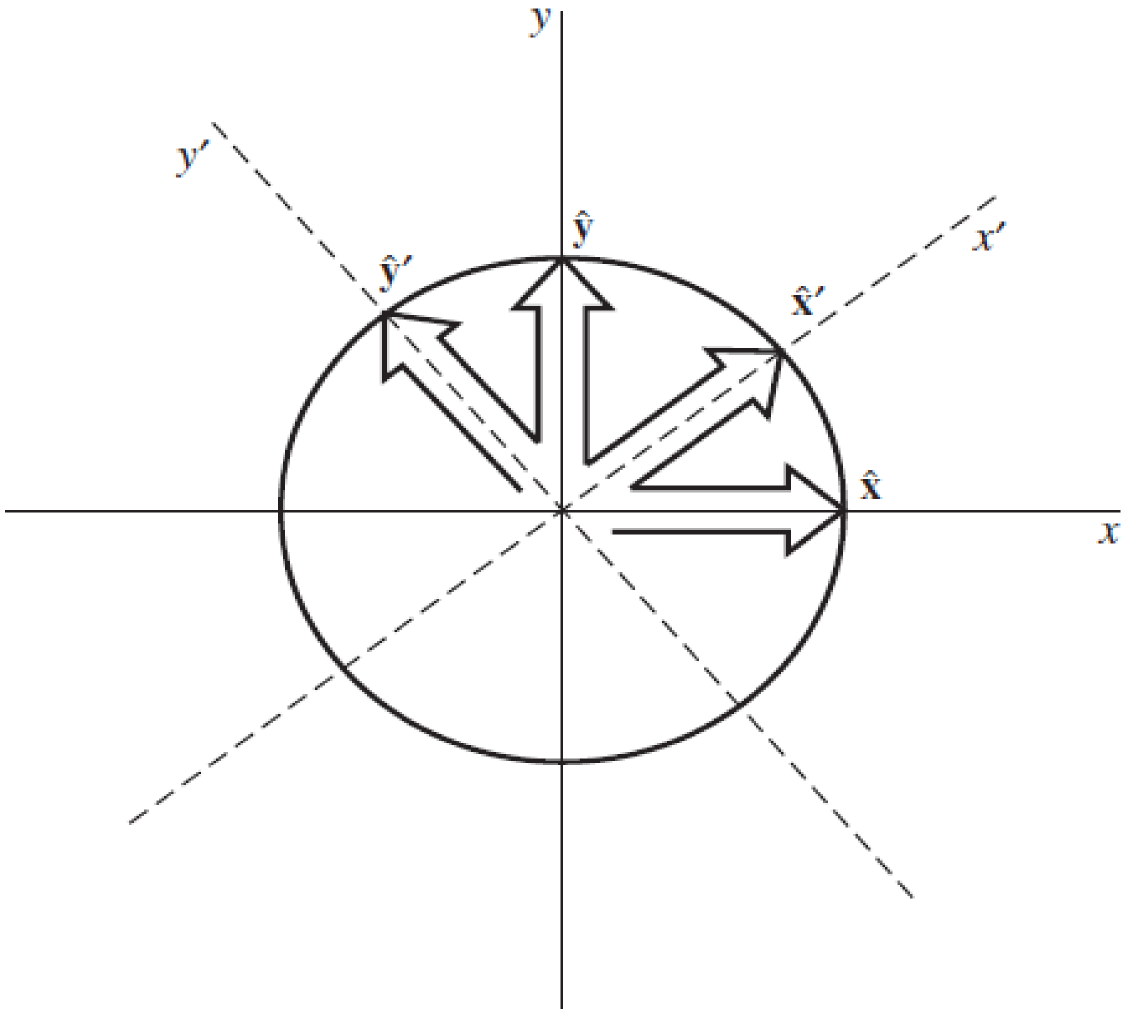
$$\mathbf{E} = E_0 \hat{\mathbf{x}} \cos(kz - \omega t).$$

Likewise, we may consider a y-polarized light, also propagating in the z-direction,

$$\mathbf{E} = E_0 \hat{\mathbf{y}} \cos(kz - \omega t).$$



Light beams subjected to Polaroid filters.



$S_z \pm \text{atoms} \leftrightarrow x\text{-},\ y\text{-polarized light}$

$S_x \pm \text{atoms} \leftrightarrow x'\text{-},\ y'\text{-polarized light},$

Orientations of the x'- and y'-axes.

Let us examine how we can quantitatively describe the behavior of 45◦- polarized beams (x'- and y'-polarized beams) within the framework of classical electrodynamics.

$$E_0\hat{\mathbf{x}}'\cos(kz-\omega t) = E_0\left[\frac{1}{\sqrt{2}}\hat{\mathbf{x}}\cos(kz-\omega t) + \frac{1}{\sqrt{2}}\hat{\mathbf{y}}\cos(kz-\omega t)\right],$$

$$E_0\hat{\mathbf{y}}'\cos(kz-\omega t) = E_0\left[-\frac{1}{\sqrt{2}}\hat{\mathbf{x}}\cos(kz-\omega t) + \frac{1}{\sqrt{2}}\hat{\mathbf{y}}\cos(kz-\omega t)\right].$$

$$\mathbf{E} = E_0\left[\frac{1}{\sqrt{2}}\hat{\mathbf{x}}\cos(kz-\omega t) + \frac{1}{\sqrt{2}}\hat{\mathbf{y}}\cos\left(kz-\omega t+\frac{\pi}{2}\right)\right].$$

$$\mathrm{Re}(\boldsymbol{\epsilon}) = \mathbf{E}/E_0.$$

For a right circularly polarized light, we can then write

$$\boldsymbol{\epsilon} = \left[\frac{1}{\sqrt{2}} \hat{\mathbf{x}} e^{i(kz-\omega t)} + \frac{i}{\sqrt{2}} \hat{\mathbf{y}} e^{i(kz-\omega t)} \right],$$

where we have used $i = e^{i\pi/2}$.

We can make the following analogy with the spin states of silver atoms:

$$S_y + \text{atom} \leftrightarrow \text{right circularly polarized beam},$$

$$S_y - \text{atom} \leftrightarrow \text{left circularly polarized beam}.$$