Quantum Mechanics

lecture by

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Classical Mechanics

- For a given force if the initial position and the velocity of the particle is known all physical quantities such as position, momentum, angular momentum, energy etc. at all subsequent times can be calculated.

- The same law can be applied from the dust particles to massive astronomical bodies.

- The later can be decomposed in large number of particles and Newton's laws can be applied to each part separately.

Classical Mechanics

- The force could be gravitational which explains the motion of all massive bodies or it could be electromagnetic which is responsible for the movement of the charge particles as well as the current.

- If the forces are of restoring nature then they can lead to harmonic motion.

- A collection of such harmonic motions together can lead to the formation of waves such as sound waves or water waves. These waves are thus different entities as particles but their origin can be well explained with the help of particle motion in a medium which in turn obeys Newton's laws of motion.

Classical Mechanics

- Thus till nineteenth century every thing can be well explained by Newton's Laws.

- Any cavity inside a material is filled with electromagnetic radiation that is caused by the motion of an accelerated charged particle which will be in thermal equilibrium with its surroundings. This is what is known as Blackbody radiation.

- It has been found that the intensity distribution of such radiation at various temperatures cannot be explained with the help of classical theory.

Classical Mechanics

- Max Planck explained this departure by making a very drastic but unexplained assumption that electromagnetic radiation is absorbed or emitted in quanta's $\hbar\omega$ and not continuously. Such an abrupt assumption which cannot be accounted by classical Newton’s law however well explained the experimental result.

h is called Planck’s constant. $\hbar = h/2\pi$

Failure of Classical Mechanics

OR

Origin of Quantum Mechanics

Quantum mechanics

- Quantum mechanics began in 1900 when the study of light emitted by heating solids was studied, so we begin by discussing ***the nature of light***.

- In 1801Thomas Young gave convincing experimental evidence for the wave nature of light by showing that light exhibited diffraction and interference when passed through two adjacent pinholes.

- In 1860 Maxwell developed Maxwell’s equations predicted that an accelerated electric charge would radiated energy in the form of electromagnetic waves.

- light is a type of energy and has wave properties

- Electromagnetic (EM) radiation travels through space as electric energy and magnetic energy.

- Wavelength λ and frequency ν are related by:

$$\lambda \times \nu = c = 3.00 \times 10^{8} \text{ m/s}$$

- Frequency (or wavelength) determines the type of radiation All electromagnetic waves travel at speed c= 3.00×10^{10} cm/sec in vacuum.

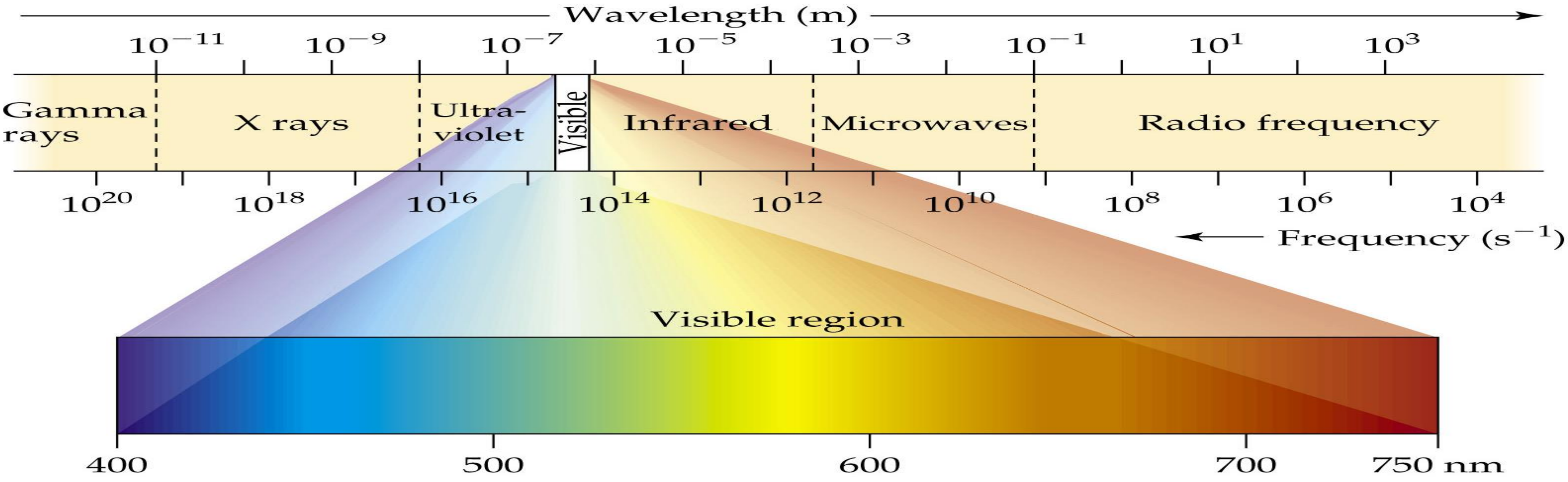
- As a wave, we can describe the energy by its wavelength, which is the distance from the crest of one wave to the crest of the next wave.

- The wavelength of electromagnetic radiation can range from miles (radio waves) to inches (microwaves in a microwave oven) to millionths of an inch (the light we see) to billionths of an inch (x-rays).

- The wavelength of light is more commonly stated in nanometers (nm). One nanometer is one billionth of a meter.

- Visible light has wavelengths of roughly 400 nm to roughly 700 nm. This range of wavelengths is called the visible spectrum.

The Wave Nature of Light



$$c = \lambda \cdot \nu$$

$$E = h \cdot \nu$$

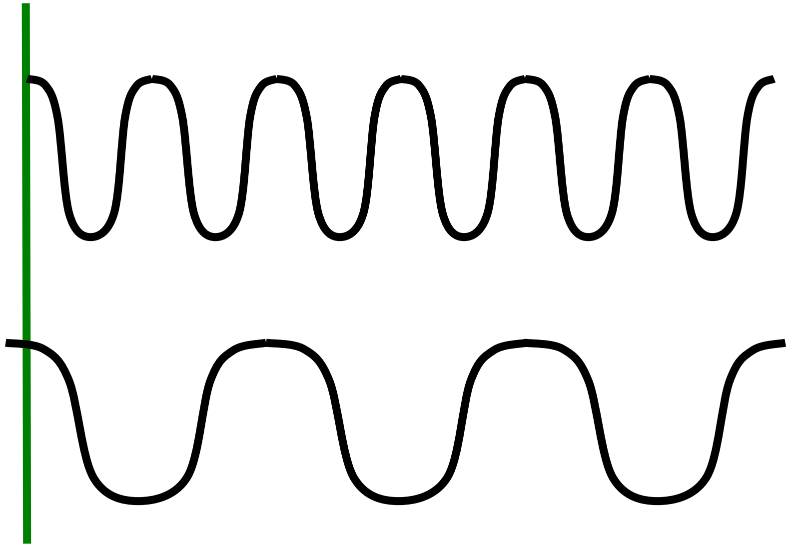
[The speed of light is constant]

The speed of EM waves

- Observe what happens when a radio wave and a visible wave move through space (at same speed of c)

Visual 6 Hz

Radio 3 Hz



- The longer the wavelength, the smaller the frequency has to be to keep c constant

Energy and Matter

Size of Matter	Particle Property	Wave Property
Large – macroscopic	Mainly	Unobservable
Intermediate – electron	Some	Some
Small – photon	Few	Mainly

For matter $E = m c^2$

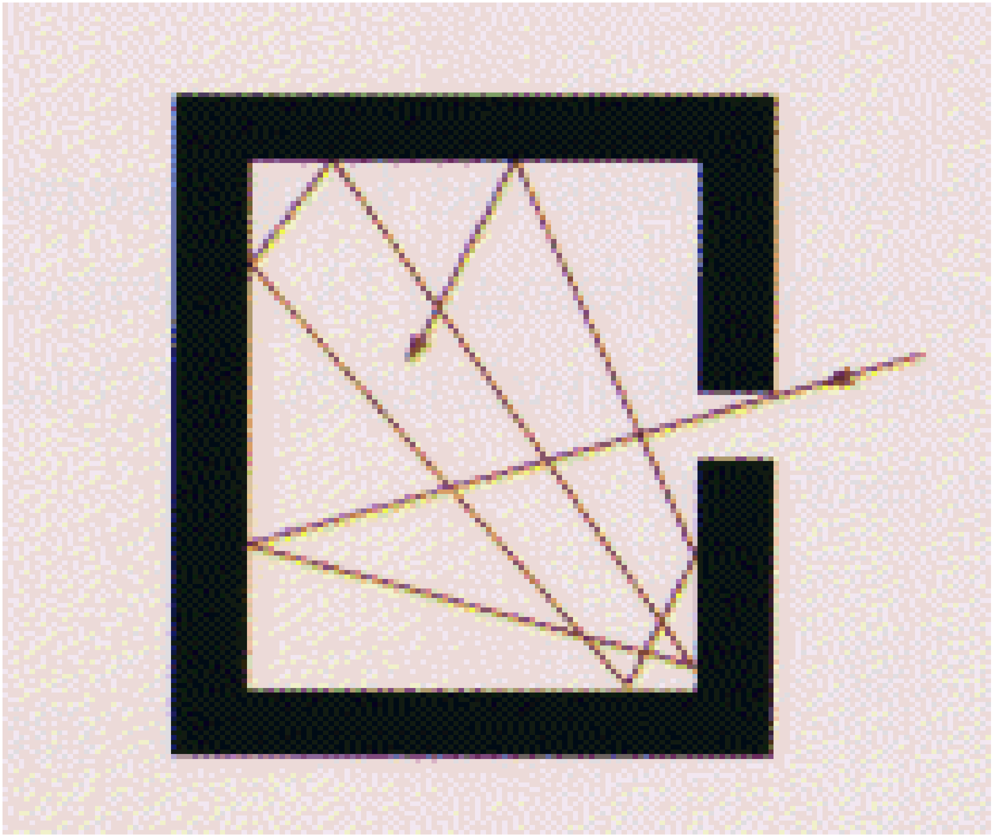
For waves $E = h \nu$

Black body radiation

- A black body is a theoretical object that absorbs 100% of the radiation that hits it. Therefore it reflects no radiation and appears perfectly black.
- Roughly we can say that the stars radiate like blackbody radiators. This is important because it means that we can use the theory for blackbody radiators to infer things about stars.
- At a particular temperature the black body would emit the maximum amount of energy possible for that temperature.
- Blackbody radiation does not depend on the type of object emitting it. Entire spectrum of blackbody radiation depends on only one parameter, the temperature, T.

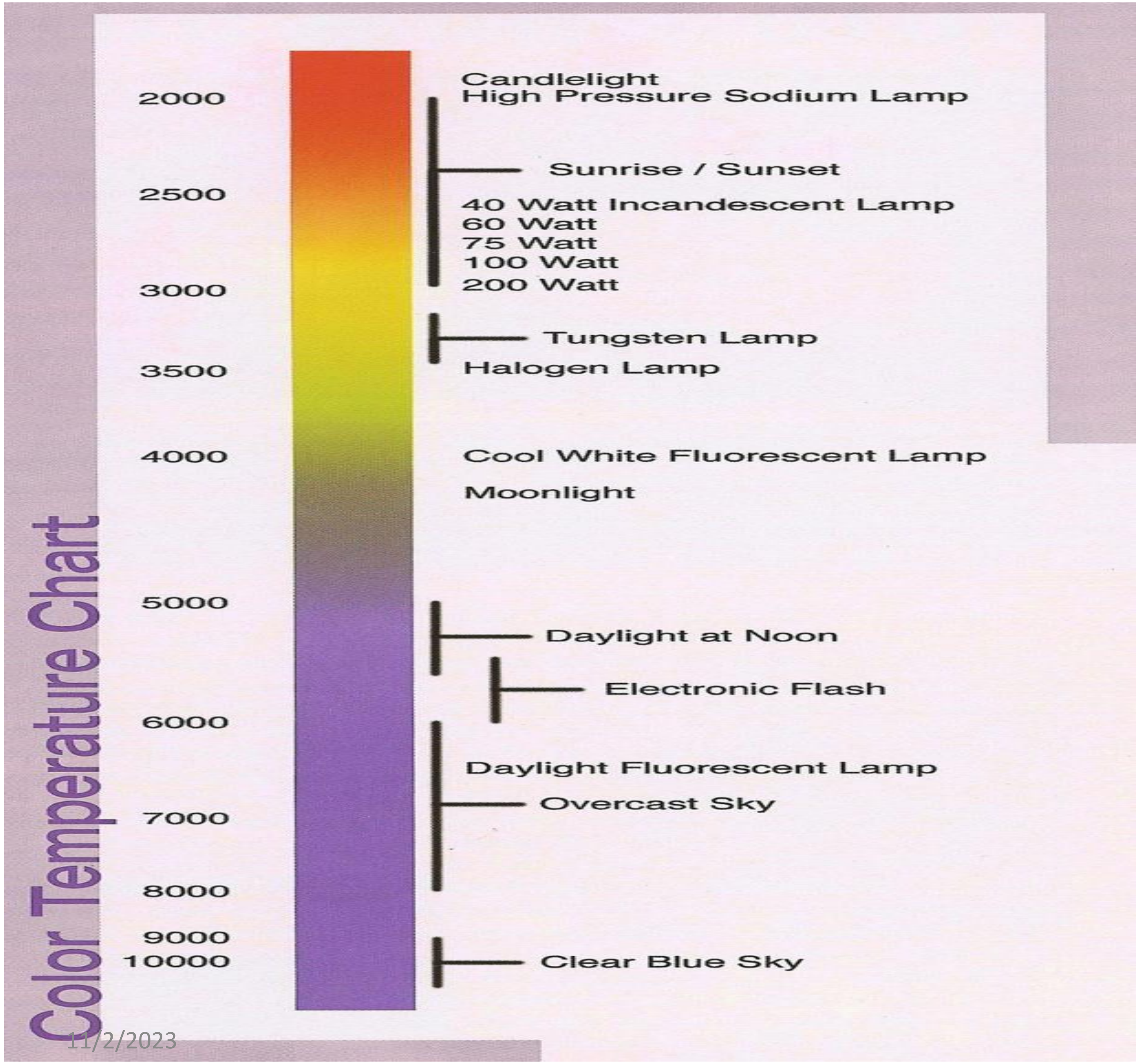
Definition of a black body

A black body is an ideal body which allows the whole of the incident radiation to pass into itself (without reflecting the energy) and absorbs within itself this whole incident radiation (without passing on the energy). This propety is valid for radiation corresponding to all wavelengths and to all angels of incidence. Therefore, the black body is an ideal absorber of incident radaition.



The relationships between temperature, wavelength, and energy emitted by an ideal thermal radiator (blackbody).

1. Based on everyday observations, the bodies at different temperatures emit radiation (heat energy) of different wavelengths or colors. For example, the wires in a heater begin to glow red when heated then the color of the wire will changed by increasing the temperature.
2. Blackbody radiation is the theoretical maximum radiation expected for temperature-related thermal self-radiation.
3. This radiation can have a peak energy distribution in the infrared, visible, or ultraviolet region of the electromagnetic spectrum.
4. The hotter the emitter, the more energy emitted and the shorter the wavelength. An object at room temperature has its peak radiation in the infrared while the sun has its peak in the visible region.
5. The equations for calculating radiation based on temperature use the Kelvin temperature scale. (Be sure to use the Kelvin scale for all calculations).



Color temperature

A light spectrum of Blackbodies is often characterized in terms of its temperature even if it's not exactly a blackbody.

Thermal radiation spectral radiance

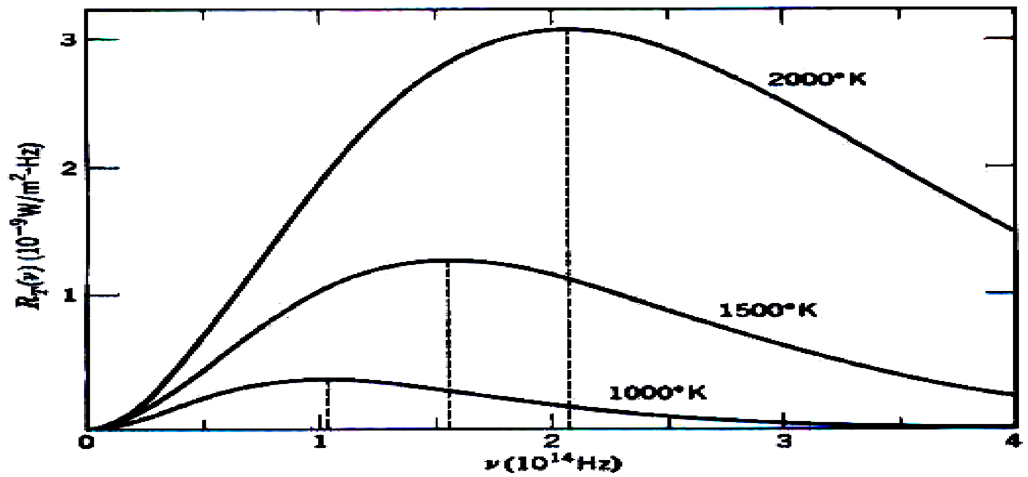
thermal radiation

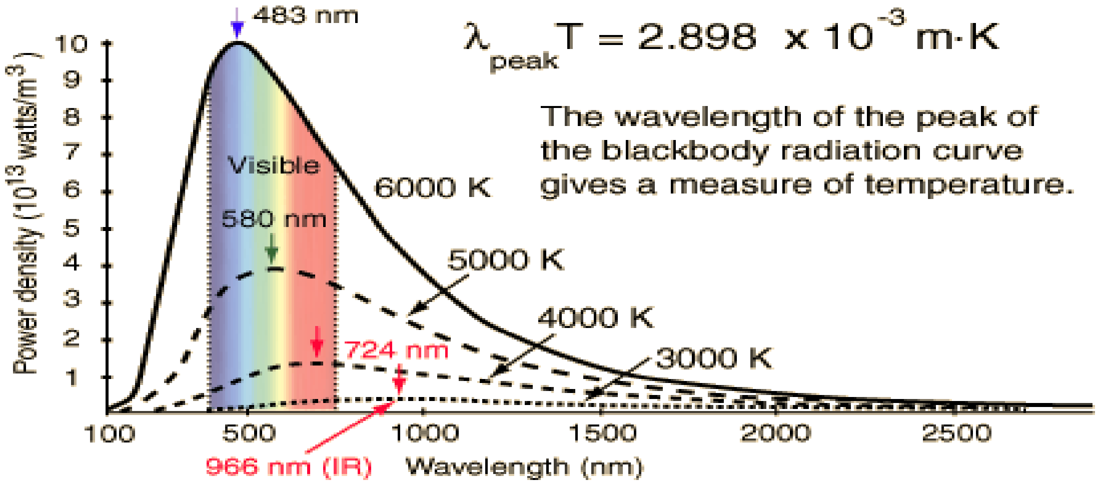
- **Thermal radiation: The radiation emitted by a body as a result of temperature.**

- **Blackbody : A body that its surface absorbs and emit all the thermal radiation incident on them.**

- **Spectral radiance: The spectral distribution of blackbody radiation.**

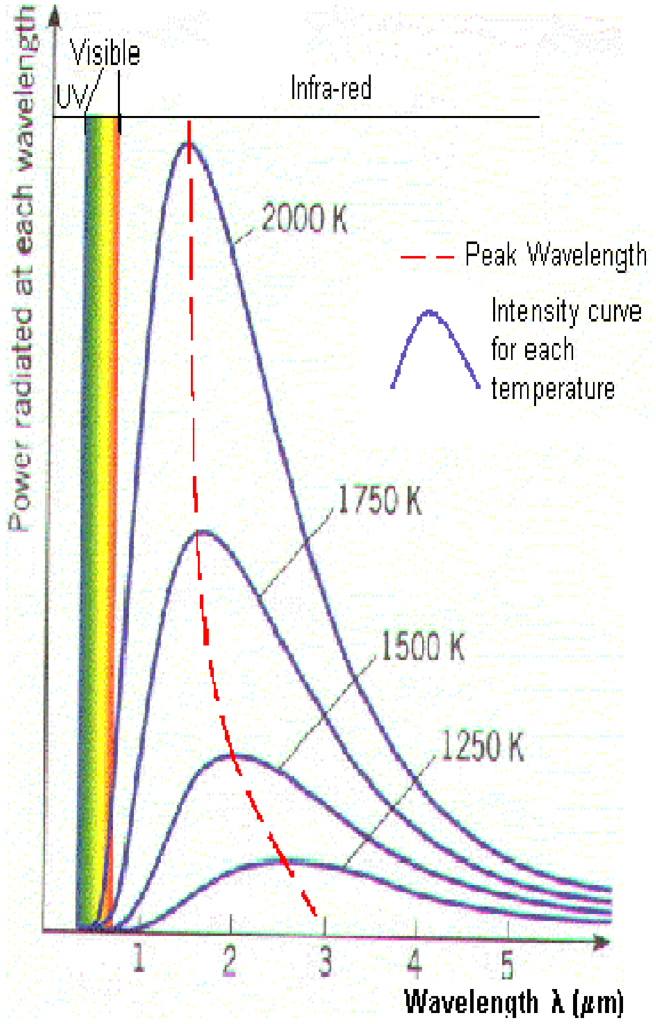
$R_T(\nu)d\nu$: $R_T(\nu)$ **represents the emitted energy from a unit area per unit time between** ν **and** $\nu + d\nu$ **at absolute temperature T.**





The spectral radiance of blackbody radiation shows that:

1. The higher the temperature, the more the emission and the shorter the average wavelength.
2. little power radiation at very low wavelength.
3. The power radiation increases rapidly as λ increases from very small value.
4. The power radiation is most intense at certain wavelength λ_{max} or υ_{max} for particular temperature.
5. $\lambda < \lambda_{max}$ and R_T drops slowly, but continuously as λ increases, and $R_{T(\lambda \to \infty)} \to 0.$
6. λ_{max} increases linearly with increasing temperature.
7. The total radiation for all ν (radiance $R_T = \int R_T(\upsilon)\, d\upsilon$, increases less rapidly than linearly with increasing temperature.



Now it is very important to find mathematical equations shows the following relationships:

1. Inverse proportionality between λ_{max} and T.

2. Direct proportionality between (R_T dυ) or (E_λ dλ) with T.

3. The relation between (R_T dυ) or (E_λ dλ) and λ.

Classical quantum mechanics of the black body radiation:

The power radiated per unit surface area of the radiator is given by the Stefan-Boltzmann law.

The Stefan-Boltzmann Law

* The amount of energy radiated is proportional to the temperature of the object raised to the fourth power.

➡ The Stefan Boltzmann equation

$$\mathbf{F = \sigma T^4}$$

F = flux of energy (W/m^2)

T = temperature (K)

$\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$ (Stefan-Boltzmann constant)

This law gives the total energy being emitted at all wavelengths by the blackbody (which is the area under the Planck Law curve).

* Explains the growth in the height of the curve as the temperature increases. Notice that this growth is very abrupt.

Classical quantum mechanics of the black body radiation:

Basic Laws of Radiation

1) All objects emit radiant energy.

2) Hotter objects emit more energy than colder objects (per unit area). The amount of energy radiated is proportional to the temperature of the object.

3) The hotter the object, the shorter the wavelength (λ) of emitted energy.

➡This is Wien's Law

$$\lambda_{max} \cong \frac{3000\ \mu m}{T(K)}$$

Classical quantum mechanics of the black body radiation:

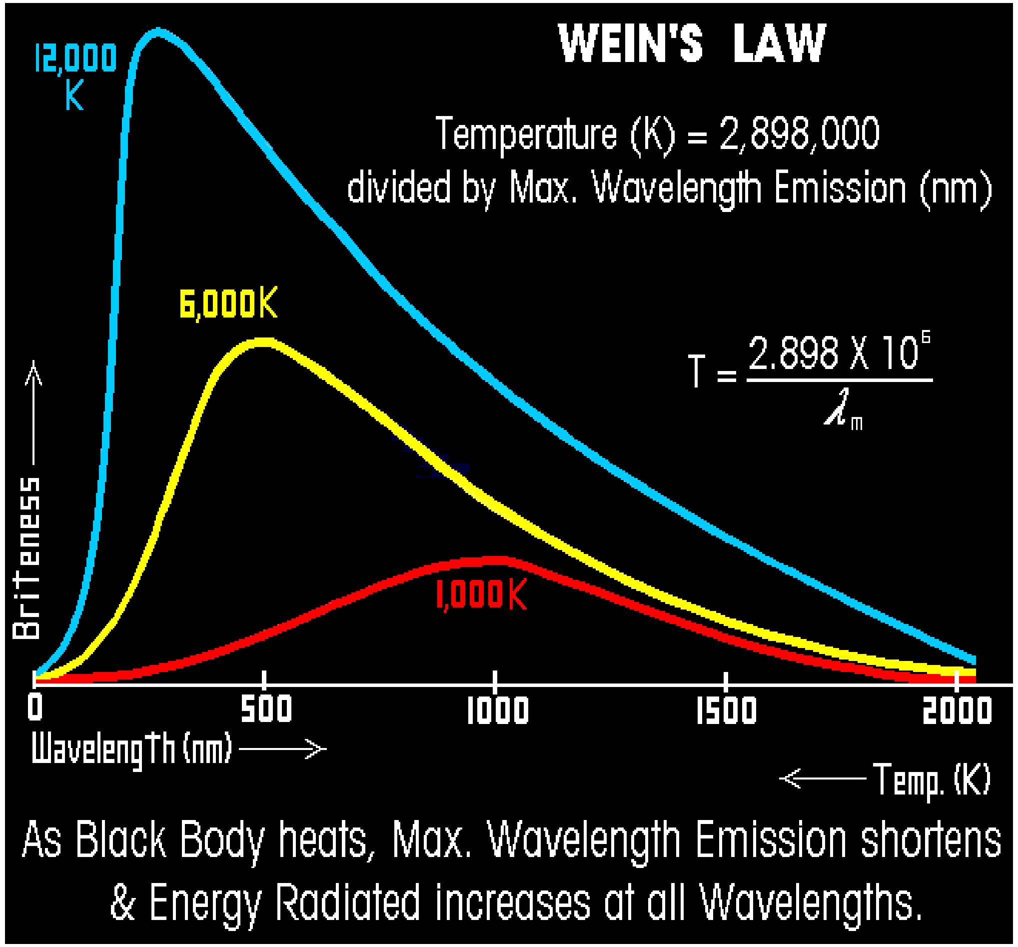
$$\lambda_{max} = \frac{b}{T}$$

Wein Displacement Law

- It tells us as we heat an object up, its color changes from red to orange to white hot.
- You can use this to calculate the temperature of stars.

The surface temperature of the Sun is 5778 K, this temperature corresponds to a peak emission = 502 nm = about 5000 Å.

- b is a constant of proportionality, called Wien's displacement constant and equals 2.897 768 × 10^{-3} m K = 2.897768 × 10^{6} nm K.



Classical quantum mechanics of the black body radiation:

- ➡ Stefan Boltzmann Law.

$$F = \sigma T^4$$

F = flux of energy (W/m^2)
T = temperature (K)
$\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$ (a constant)

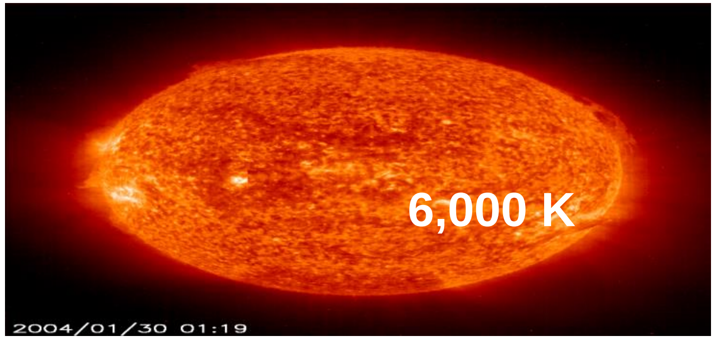
- ➡ Wien's Law

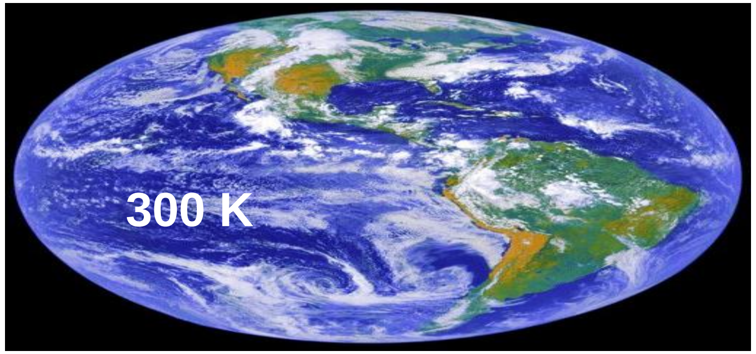
$$\lambda_{max} \cong \frac{3000\ \mu m}{T(K)}$$

λ_{peak} vs Temperature

T	$\lambda_{peak} = \dfrac{2.9 \times 10^{-3}\ m}{T(Kelvin)}$	
310^0K (body temp)	$\dfrac{2.9 \times 10^{-3}\ m}{310^0}$	$=9x10^{-6}m$ infrared light
5800^0K (Sun's surface)	$\dfrac{2.9 \times 10^{-3}\ m}{5800^0}$	visible light $=0.5x10^{-6}m$

We can use these equations to calculate properties of energy radiating from the Sun and the Earth.



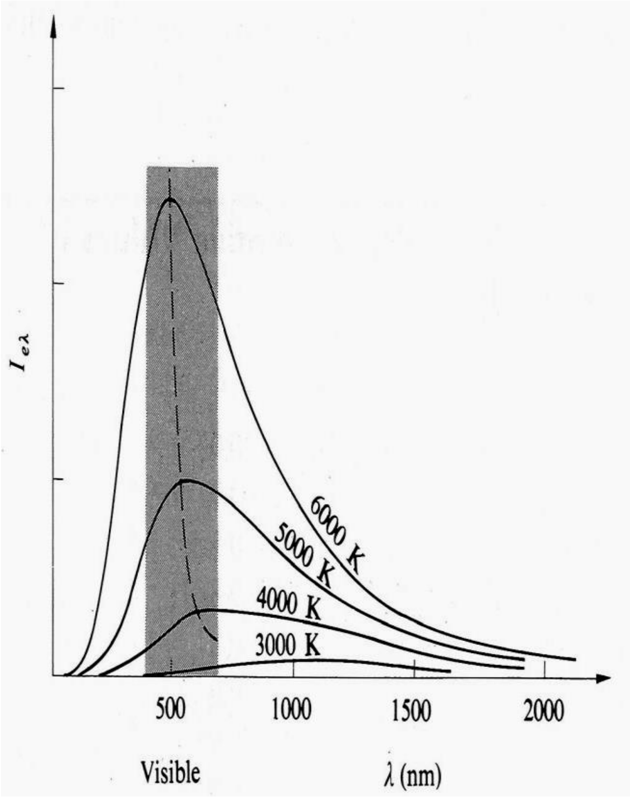


	T (K)	λ_{max} (μm)	region in spectrum	F (W/m^2)
Sun	6000	0.5	Visible (green)	7×10^7
Earth	300	10	infrared	460

Classical quantum mechanics of the black body radiation:

Wien's Law

•To find a relation describe the relation between the intensity of black body radiation versus wavelengths for several temperatures as given in the following figure. Many theoretical physicists tried to derive expression but they were all unsuccessful. One expression that is derived according to the laws of nineteenth century is ***Wien Law***



$$E_\lambda \, d\lambda = \frac{C_1}{\lambda^5} \; e^{-C_2/\lambda T} \; d\lambda$$

$C_2 = h\,C/\,k$, $C_1 = 8\pi\, h\, C$

And it was found that for small wavelengths (high frequencies), the previous equation fitted the experimental data but it deviate at large wavelengths infrared waves.

Classical quantum mechanics of the black body radiation:

The Rayleigh-Jeans Law

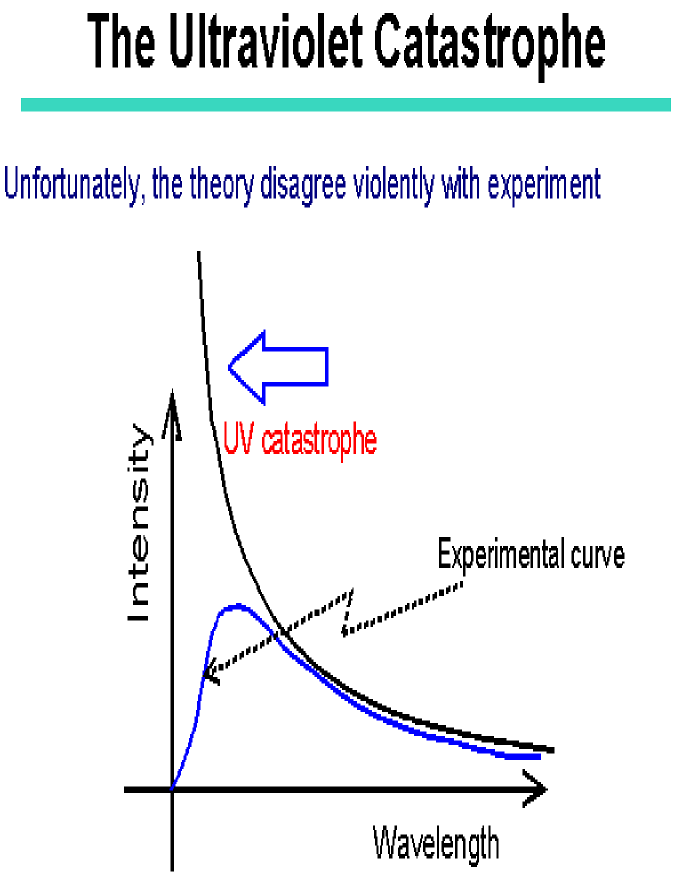
- The second atempt was by Rayleigh and Jeans how use some assumptions to obtane the following equation:

$$E_\lambda \, d\lambda = \frac{C_1}{C_2} \frac{T}{\lambda^4} \, d\lambda$$

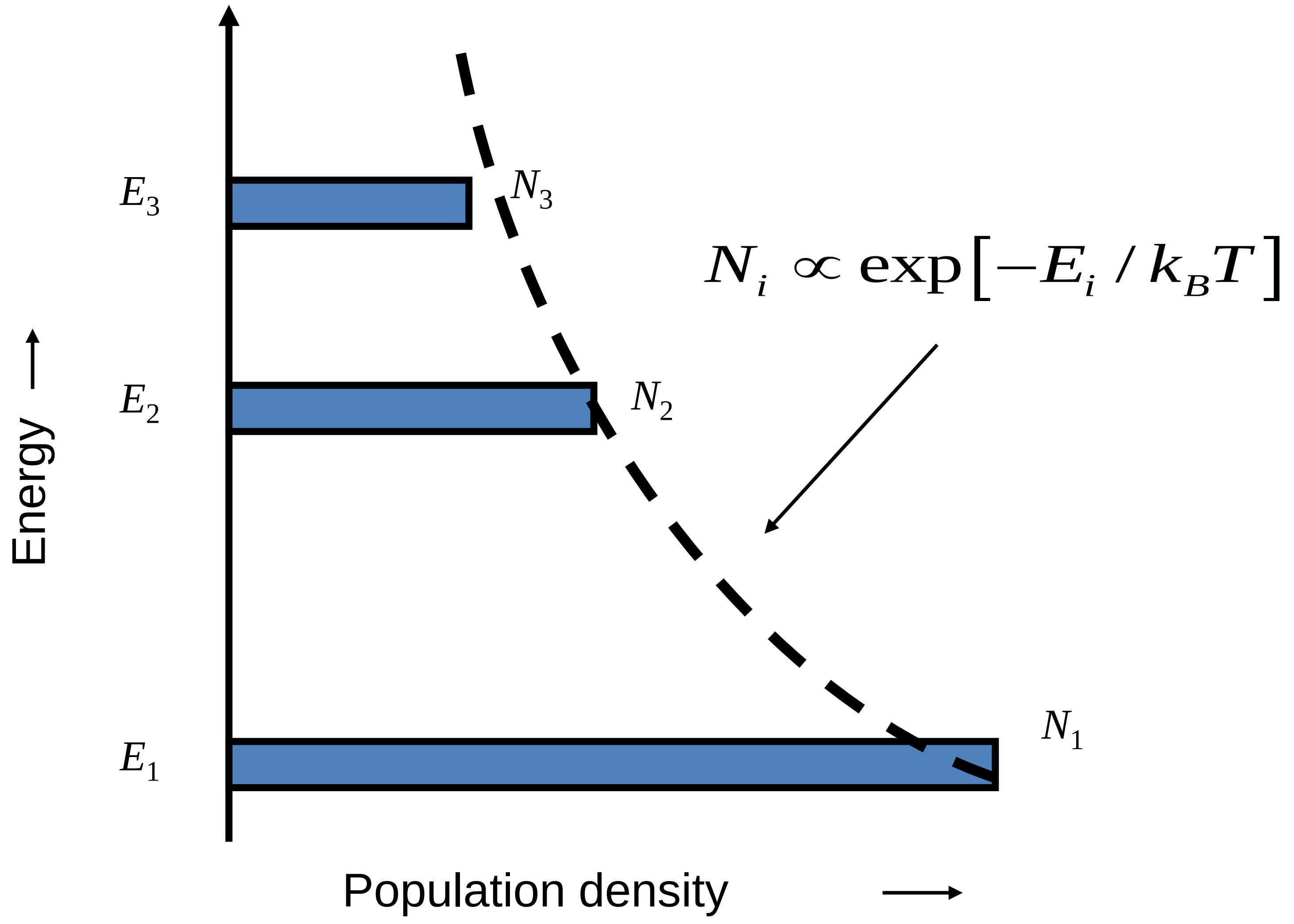
- This equation agrees with experimental measurements for long wavelengths (low frequencies) and failure at short wavelengths.

* It predicts an energy output that diverges towards infinity as wavelengths grow smaller.

* The failure at short wavelengths has become known as the ***ultraviolet catastrophe***.



Boltzmann Population Factors



N_i is the number density of molecules in state i (i.e., the number of molecules per cm^3).

T is the temperature, and k_B is Boltzmann's constant.

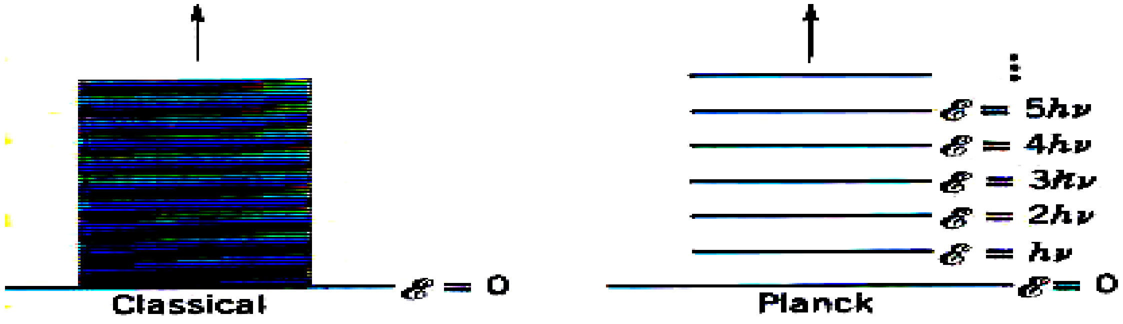
Quantum mechanics of the black body radiation

Planck's Postulate and its implication

Planck's postulate: **Planck assumed that the radiation emitted by the body was due to the oscillations of the electrons in the constituent particles of the material body.**

(i.e., simple harmonic oscillation can posses only total energy

$$\varepsilon = nh\nu \qquad n = 1,2,3 \text{---}$$



Quantum mechanics of the black body radiation

Planck's solution

EM energy cannot be radiated or absorbed in any arbitrary amounts, but only in discrete "quantum" amounts.

The energy of a "quantum" depends on frequency as

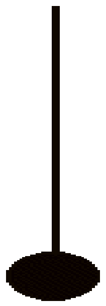
$$E_{quantum} = h\,\upsilon$$

$h = 6.6 \times 10^{-34}$ Js

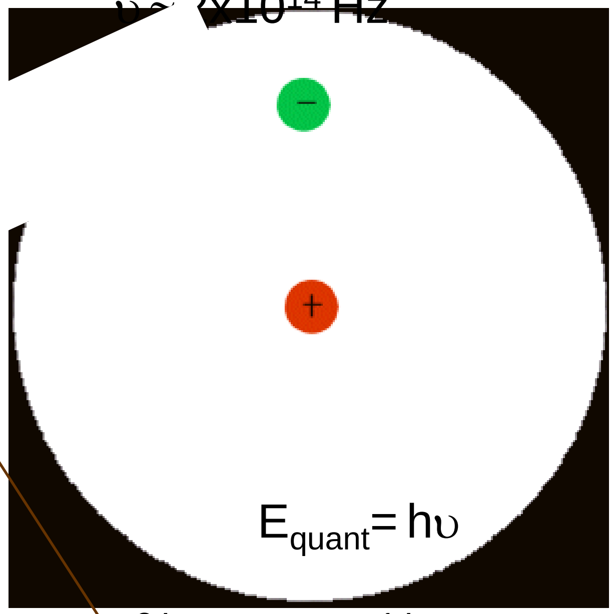
"Planck's constant"

Planck's quantum is small for "ordinary-sized" objects but large for atoms etc

"ordinary"
pendulum
$\upsilon = 1$ Hz



Hydrogen atom
$\upsilon \sim 2\times10^{14}$ Hz



about the same as the electron's KE

$E_{quant} = h\upsilon$ $= 6.6\times10^{-34}$ Js x 1Hz

$= 6.6\times10^{-34}$ J

very tiny

$= (6.6\times10^{-34}\text{Js})\times(2\times10^{14}\text{Hz})$

$= (6.6 \times 2) \times 10^{-34+14}$ J

$= 1.3 \times 10^{-19}$ J

Classical vs ***Quantum*** world

In everyday life,
quantum effects
can be safely
ignored

At atomic & subatomic scales,
quantum effects
are dominant & must be considered

This is because
Planck’s constant is so small

Laws of nature
developed without
consideration of
quantum effects do not work for atoms

The Planck Function

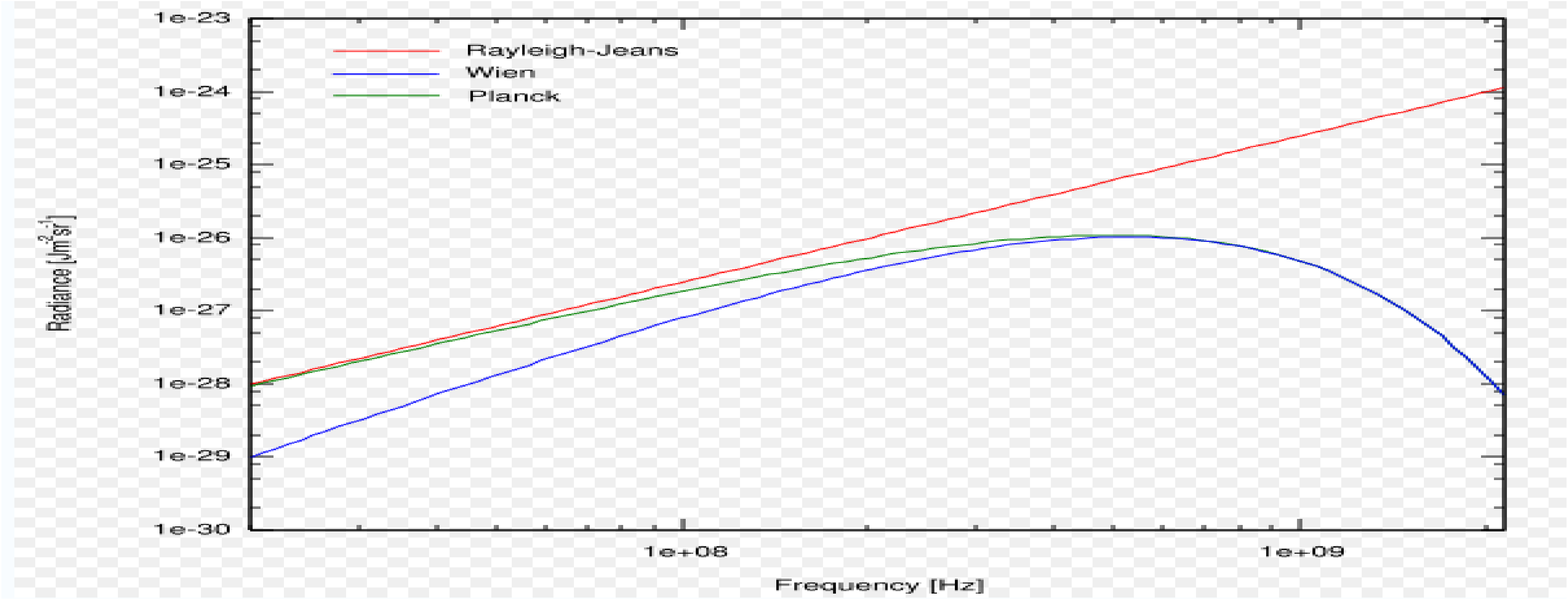
$$E_\lambda \, d\lambda = \frac{c_1}{\lambda^5} \frac{d\lambda}{e^{c_2/\lambda T} - 1}$$

$C_2 = h\,C/\,k$, $C_1 = 8\pi\, h\, C$

$h = 6.6262 \times 10^{-34}$ joule sec
$k = 1.3806 \times 10^{-23}$ joule deg^{-1}
$c = 2.99793 \times 10^{+8}$ m/s
T = object temperature in Kelvins

- **Blackbody radiation follows the Planck function at all the wavelength range**

Black-Body Radiation Laws



Comparison of Rayleigh-Jeans law with Wien's law and Planck's law, for a body of 8 mK temperature.

- Planck equation was applicable with practical results at short wavelengths and long wavelengths. It is important to note that:
- *when λ is small*, Planck equation can be reduced to Wien equation by eliminating (-1) compared with the high value of the exponential in the denominator in Planck equation,
- Thus *While at large λ,* Planck equation $E_\lambda \, d\lambda = \frac{C_1}{\lambda^5} \frac{d\lambda}{e^{C_2/\lambda T} - 1} \longrightarrow$
- can be devolve into Rayleigh-Jeans equation $e^{C2/\lambda T}$ when compensated by the amount $(1 + C_2/\lambda T)$
- Thus

$$E_\lambda \, d\lambda = \frac{C_1}{\lambda^5} \, e^{-C2/\lambda T} \, d\lambda$$

$$E_\lambda \, d\lambda = \frac{C_1}{\lambda^5} \frac{d\lambda}{e^{C_2/\lambda T} - 1} \longrightarrow E_\lambda \, d\lambda = \frac{C_1}{C_2} \frac{T}{\lambda^4} \, d\lambda$$

Photoelectric effect

- It was found that if two metallic plates were kept in a high vacuum chamber, and one was illuminated, an electro static potential developed between them.

- This was shown to be due to the emission of electrons from the plate which was illuminated with light.

- For a given metal, it was found that there was one frequency below which no electrons were emitted.

Photoelectric effect

- For frequencies above this cutoff, however weak the light, electrons were always emitted. The number of photoelectrons emitted being proportional to the light intensity.

- If the frequency were further increased, the kinetic energy of the individual electrons emitted increased.

- Einstein explained this by saying that the quanta were not only units of exchange, Light always consisted of particles; these quanta he called Photons.

Problems with atoms

- According to Dalton and others each material consists of very small basic constituents called atoms which is electrically neutral.

- Rutherford performed a Scattering experiment by bombarding a gold foil by α particles to understand the structure of such atom.

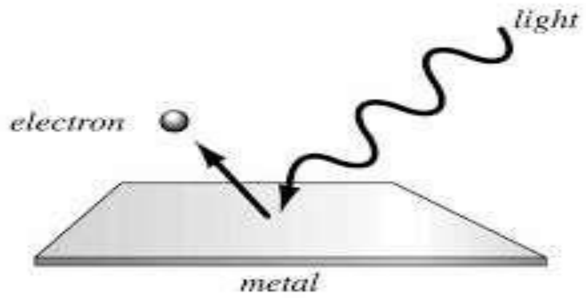
- The experiment concluded that atom consist of a very heavy and charged center around which much lighter particles of opposite charges are moving.

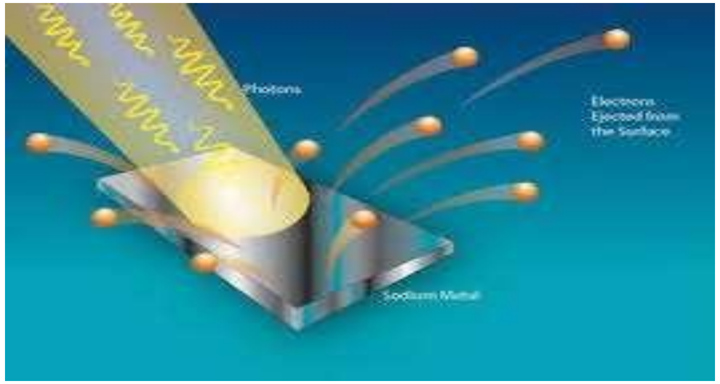
- Classical electromagnetic theory predicts that such a structure is going to be energetically unstable and the lighter particles around the heavy center can have energy as low as $-\infty$. However experiment shows that such energy always has a lower bound which is -13.6ev.

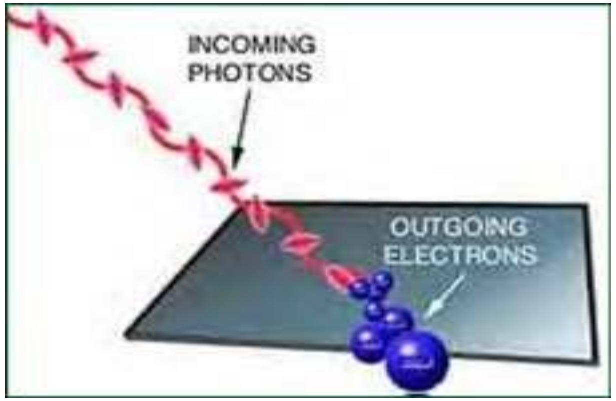
- Moreover classical electromagnetic theory predicts that when such lighter charged particle will emit or absorb electromagnetic energy due to their accelerated motion they will form a continuous spectrum. But it was shown from experiments that such lines form discrete structures.

PHOTON (QUANTUM OF LIGHT)

- In 1905, Einstein proposed thatelectromagnetic radiation or light is made up of photons. Thus the photon is the elementary element of light or light is made up of photons.
- Einstein show that- light energy is not emitted continuously but it is emitted by individual amount of energy called as quantum of energy.







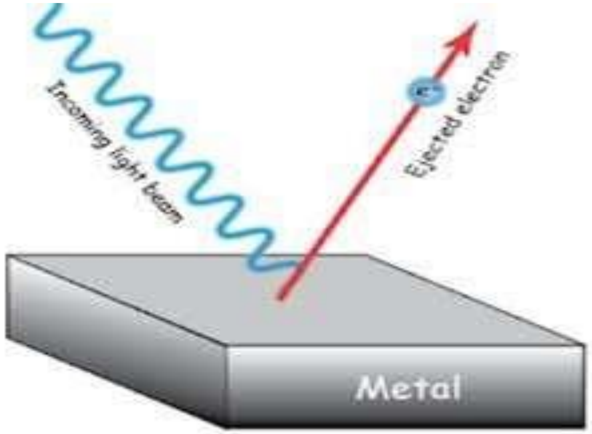
Energy of photon

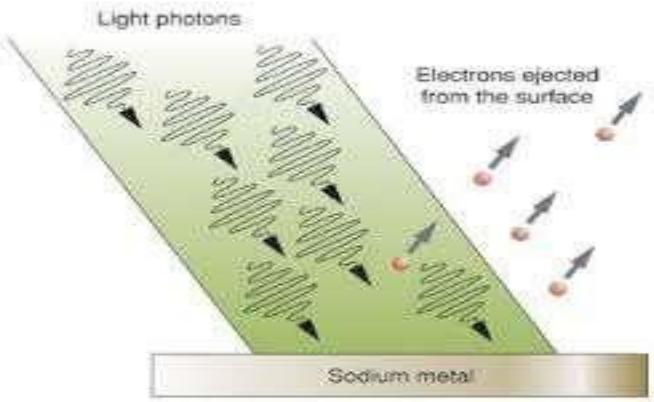
- According to Einstein, each photon of a light wave of frequency has the energy E is given by, $E = hv$ 1

- where E= energy of photon(joule)

  h= planks constant-6.626×10^{-34} J.s

  v= frequency of photon(Hz)

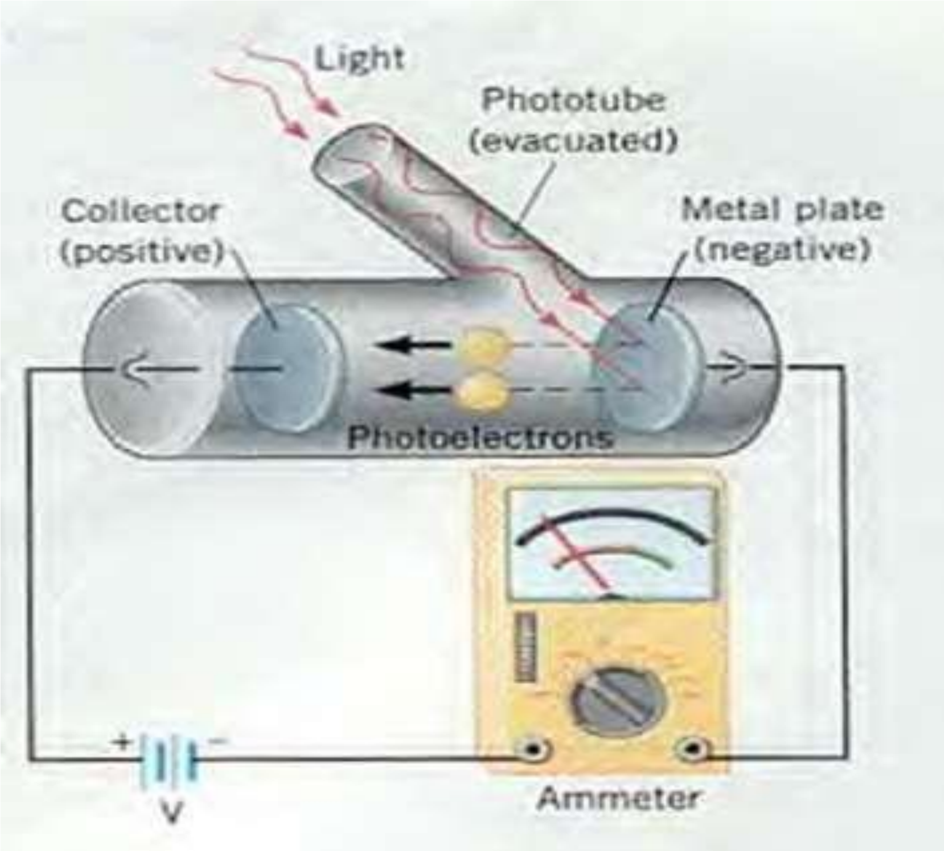




Properties of photon

- A photon does not have any mass.
- A photon does not have any charge and are not deflected in electric field or magnetic field.
- All the quantum numbers are zero for a photon
- In empty space, the photon moves at speed of light.
- In the interaction of radiation with matter, radiation behaves as if it is made up of particles called photons.
- The energy and momentum of a photon are related as follows E= p.c where p- magnitude of momentum and c is the speed of light.
- Photon is called as a virtual particles.
- The energy of a photon is directly proportional to frequency and inversly proportional to its wavelength.

Photo electric effect



- When a beam of light of sufficiently high frequency onto a clean metal surface then the light will cause electrons to leave the surface.
- Definition : the phenomenon of emission of electrons by the metals when they are exposed to light of suitable frequency is called as the **photo electric effect** and emitted electrons is called as **photoelectrons**.

Bohr model

- Neil's' Bohr proposed a set of postulates with which he was able to explain the stability of the structure of atoms and the related experimental results of the atomic spectrum.

- These postulates say that these lighter particles which we henceforth electrons are only allowed to travel in orbits around the heavy positively charged center that we shall henceforth call as nucleus, such that their angular momentum and hence energy is quantized and does not decay. As result such electrons does not decay energetically and when they make transition from one orbit to another, the absorbed and emitted electromagnetic radiation also comes in discrete quanta.

- Again such postulates cannot be justified by starting from the classical motion of a charged particle.

Matter Wave

- The previous set of examples shows that a number of experiments were done at the beginning of 20th century whose results cannot be explained within classical mechanics.

- Each of these experiments were described by a set of postulates and again none of these postulates cannot be justified by the classical theory.

- We shall end this discussion by pointing out another experiment.
- The previous set of experiments pointed out that energy associated with the light or electromagnetic radiation is emitted or absorbed as quantas called photons giving $E=h\nu=\hbar\omega$

- On the other hand it is known that for light the energy and momentum are related as $E=cp$

- Since $\omega=ck$, this gives $p=\hbar k=h/\lambda$. Thus for such light particles momentum and the wavelength are interrelated.

Matter wave

- Typically the momentum is associated with the motion of a particle where as the wavelength is associated with an wave. As we have discussed in classical physics these two are completely different entities. But the interrelation for light quanta "photons" thus indicates particle nature of radiation.

- This corpuscular nature of photons obtained a great experimental support from the experiments of Compton. He allowed a beam of x rays (photon) with a well defined wavelength to to fall on a graphite target. The intensity of the scattered rays are measured as a function of the scattering angle. The experimental result was well explained by treating the X-rays as collection of relativistic particle photon with energy and momentum given by $E=h\nu$ and $p=\hbar k$ and then analyzing their elastic collisions with electrons.

- Compton experiment prompted people to think the same way the momentum of photon is related to its wavelength, can we write down a similar relation for every material particle such as electron by associating a wavelength with their momentum. As you know this was done by Louis de Broglie

DeBroglie's hypothesis

- Note now the theory said that light was a wave (Maxwell).
- Light consists of particles called photons (Einstein).
- At this point, de Broglie pointed out that such photons also carried momenta namely.
- He then went on to postulate that all matter too should consist of waves given by the same two relations for energy and momentum.
- At that time there was no experimental justification to suggest such wave character associated with matter. So it was a very bold assumption.

Experimental Justification

- The idea of the de Broglie that wave like properties can be attached with each material particle was confirmed by experiments by Davidson and Germer in United States and by Thompson in Scotland.

- High speed electrons generated from a heated filament are accelerated through a potential difference and are allowed to fall on a single crystal on nickel. The atoms in crystal forms a structure like a three dimensional grating. As we know if electrons behave like classical particles then the scattered electrons should accumulate in the detectors almost uniformly as a function of scattering angle. However it turns out the that the intensity of the scattered electron beams form a diffraction pattern with consecutive maxima and minima.

- Subsequent experiments by other experimentalist also show interference and diffraction pattern for the slow neutrons

THANK YOU