Expectation and Variance

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Definition:

Expectation of D.R.V:

Let X be discrete random variable then its expectation of denoted by $E(X)$ and defined as

$$E(X) = \sum_{i=1} xP[X = x_i]$$

provided that right hand series is absolutely convergent

Definition:

Expectation of C.R.V:

Let X be continous random variable then its expectation of denoted by $E(X)$ and defined as

E(X)= $\int_x xf(x)dx$provided that right hand integral is absolutely convergent

Remark:Absolute Convergence

$$\sum_{i=1} |xP[X = x_i]| = \sum_{i=1} |x|P[X = x_i] < \infty$$

$$\int_{-\infty}^{\infty} |xf(x)|dx = \int_{-\infty}^{\infty} |x|f(x)dx < \infty$$

Remarks

1. $E(X)$ exist if$E(|X|)$ exist

Expectation of function of random variable

Expectation of function of D.R.V:

Let $g(X)$ be function of discrete random variable then its expectation of denoted by $E[g(X)]$ and defined as

$$E[g(x)] = \sum_{i=1} g(x_i)P[X = x_i]$$

Expectation of function of random variable

Expectation of function of C.R.V:

Let X be continuous random variable then its expectation of denoted by $E(X)$ and defined as

$$E[g(X)] = \int_x g(x)f(x)dx$$

Properties of Expectation:

Addition Theorem of Expectation

$$E(X+Y) = E(X) + E(Y)$$

Addition Theorem of Expectation:Generalisation

$$E(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} E(X_i)$$

Properties of Expectation:

Multiplication Theorem of Expectation

If X and Y are independent random variables then

$$E(XY) = E(X)E(Y)$$

Multiplication Theorem of Expectation:Generalisation

$$E\left(\prod_{i=1}^{n} X_i\right) = \prod_{i=1}^{n} E(X_i)$$

Property 3

If X is random variable and a is constant then

$$E(\psi(X) + a)) = E(\psi(X)) + a$$

$$E(a(\psi(X)) = a(E(\psi(X))$$

Property4

If X is random variable a and b are constants then
$$E(aX + b) = aE(X) + b$$

Expectation of linear combination of random variables

Let $X_1, X_2, ..., X_n$ be any n random variables and if $a_1, a_2, ..., a_n$ are any n constants, then

$$E(\sum_{i=1}^{n} a_i X_i) = \sum_{i=1}^{n} E(a_i)E(X_i)$$

provided all the expectations exist

Remaining Properties:

1. If $X \geq 0$ then $E(X) \geq 0$ If X and Y are two random variables such that $X \leq Y$ then
   $E(X) \leq E(Y)$
2. $|E(X)| \leq E(|X|)$
3. If $\mu_r^{'}$ exists then $\mu_s^{'}$ exists for all $1 \leq s \leq r$
4. If X and Y are independent random variables then

$$E[h(x)k(y)] = E[h(x)]E[k(y)]$$

Definition:Variance

Variance:

Let X be random variable then its Variance is denoted by $V(X)$ and defined as

$$V(X) = E[X - E(X)]^2 = E(X^2) - [E(X)^2]$$

1. If X is a random variable, then

$$V(aX + b) = a^2 V(X)$$

Corollary:

1. If $b = 0$, then
$$V(aX) = a^2 V(X)$$
i.e. Variance is not independent by change of scale
2. If $a = 0$, then
$$V(b) = 0$$
i.e. Variance of constant is zero
3. If $a = 1$, then
$$V(X + b) = V(X)$$
i.e. Variance is independent of change of origin.

Covariance:

Definition:Covariance

If X and Y are two random variables , then cavariance between them is defined as

$$Cov(X, Y) = E[X - E(X)Y - E(Y)] = E(XY) - E(X)E(Y)$$

Results related to Co variance:

1. If X and Y are independent then

$$Cov(X, Y) = 0$$

2.

$$Cov(aX, bY) = abCov(X, Y)$$

Results related to Covariance:

3.

$$Cov(X + a, Y + b) = Cov(X, Y)$$

4.

$$Cov(\frac{X - \bar{X}}{\sigma_X}, \frac{Y - \bar{Y}}{\sigma_Y}) = \frac{1}{\sigma_X \sigma_Y} Cov(X, Y)$$

Result related to Covariance:

If X and Y are independent, $Cov(X, Y) = 0$, however converse is not true.

Variance of Linear Combination of Random Variables

let $X_1, X_2, ..., X_n$ be n random variables then

$$V(\sum_{i=1}^{n} a_i X_i) = \sum_{i=1}^{n} (a_i)^2 V(X_i) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j Cov(X_i, X_j)$$

Important results related to Variance:

1. If $X_1, X_2, ..., X_n$ are independent random variables then

$$V(\sum_{i=1}^{n}(X_i)) = \sum_{i=1}^{n} V(X_i)$$

2. $$V(X_1 + X_2) = V(X_1) + V(X_2) + 2Cov(X_1, X_2)$$

3. If X_1 and X_2 are random variables then

$$V(X_1 \pm X_2) = V(X_1) + V(X_2) \pm 2Cov(X_1, X_2)$$

4. If X_1 and X_2 are independent random variables then

$$V(X_1 \pm X_2) = V(X_1) + V(X_2)$$


thank
you