Generating Functions

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Introduction:

The measure of central tendency (location) and measure of dispersion (variation) both are useful to describe a data set but both of them fail to tell anything about the shape of the distribution. We need some other certain measure called the moments to identify the shape of the distribution known as skewness and kurtosis.

Introduction:

Moments are popularly used to describe the characteristic of a distribution. They represent a convenient and unifying method for summarizing many of the most commonly used statistical measures such as measures of tendency, variation, skewness and kurtosis. Moments are statistical measures that give certain characteristics of the distribution. Moments can be raw moments, central moments and moments about any arbitrary point.

Introduction:

For example, the first raw moment gives mean and the second central moment gives variance. Although direct formulae exist for central moments even then they can be easily calculated with the help of raw moments. The rth central moment of the variable x is hr times the rth central moment of u where $u = (x - A)/h$ is a new variable obtained by subjecting x to a change of origin and scale. Since A does not come into the scene so there is no effect of change of origin on moments.

Moments:

Moment about origin(Raw moment)

Let X be random variable then its r th Moment about origin is denoted by $\mu_r^{'}$ and defined as

$$\mu_r^{'} = E(x^r) = \begin{cases} \sum_x x^r P[X = x] & forXisD.R.V. \\ \int_x x^r f(x)dx & forXisC.R.V. \end{cases}$$

provided that right hand series and right hand integral is absolutely convergent

Moments:

Moment about mean(Central moment)

Let X be random variable then its r th Moment about origin is denoted by μ_r and defined as

$$\mu_r = E((x-\mu)^r) = \begin{cases} \sum_x (x-\mu)^r P[X=x] & forXisD.R.V. \\ \int_x (x-\mu)^r f(x)dx & forXisC.R.V. \end{cases}$$

provided that right hand series and right hand integral is absolutely convergent

Moment Generating Function:

Moment Generating Function

Let X be random variable then its Moment Generating Function(M.G.F.) is denoted by $M_X(t)$ of denoted by and defined as

$$M_X(t) = E(e^{tx}) = \begin{cases} \sum_x e^{tx} P[X = x] & forXisD.R.V. \\ \int_x e^{tx} f(x) dx & forXisC.R.V. \end{cases}$$

provided that right hand series and right hand integral is absolutely convergent

Remark:

The integration or summation being extended to the entire range of x, t being the real parameter and it is being assumed that RHS of above equation is absolutely convergent for some positive number h such that $-h < t < h$

Some limitations of MGF

1. A random variable X may have no moments although m.g.f. exists.
2. A random variable X can have mgf and some(or all) moments, yet the mgf does not generate the moments.
3. A random varible X can have all or some moments, but mgf does not exist except perhaps at one point.

Moment Generating Function: Property2

MGF of the sum of number of independent random variables is equal to the product of their respective mgf's.

$$M_{X_1+X_2+...+X_n}(t) = M_{X_1}(t)M_{X_2}(t)..M_{X_n}(t)$$

$$M_{\sum_{i=1}^{n} X_i(t)} = \prod_{i=1}^{n} M_{X\,i}(t)$$

Property3:Effect of change of origin and scale on MGF

If X is random variable then

$$U = \frac{X-a}{h} then M_U(t) = e^{\frac{-at}{h}} M_X(\frac{t}{h})$$

Uniqueness Theorem of M.G.F.:

The moment generating function of a distribution, if it exists, then it uniquely determines the distribution.


thank you