STATISTICAL INFERENCE

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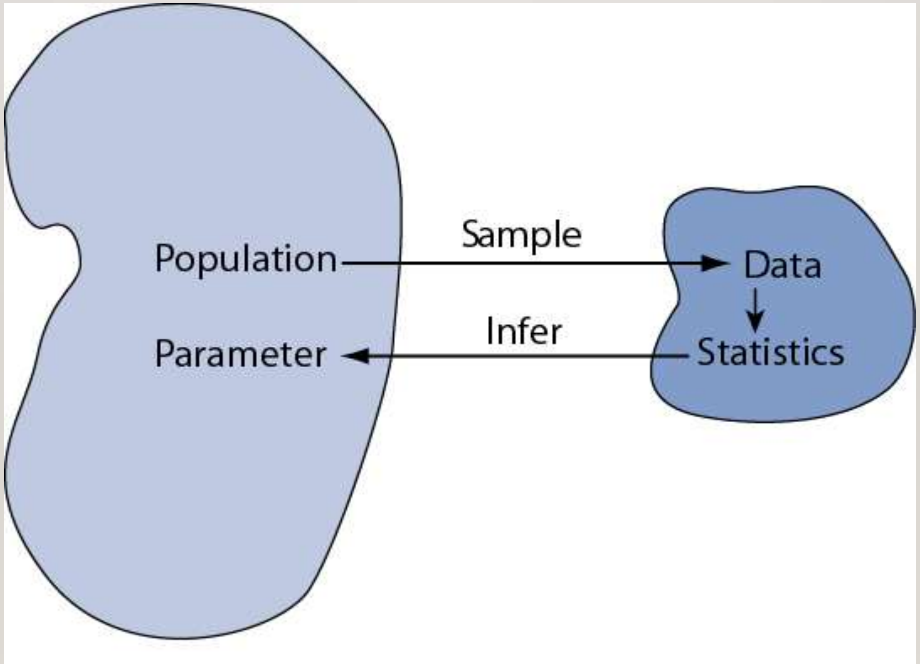
STATISTICAL INFERENCE

Statistical inference is the act of generalizing from a **sample** to a **population** with calculated degree of certainty.

We want to learn about population *parameters* …



using statistics calculated in the *sample*

PARAMETERS AND STATISTICS

We MUST draw distinctions between parameters and statistics

	Parameters	**Statistics**
Source	Population	Sample
Calculated?	No	Yes
Constants?	Yes	No
Examples	μ, σ, *p*	$\bar{x}, s, \hat{p}$

STATISTICAL INFERENCE

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Statistical inference provides methods for drawing conclusions about a population from sample data.

There are two forms of statistical inference:

- **Hypothesis (“significance”) tests**
- **Confidence intervals**

EXAMPLE: NEAP MATH SCORES

- Young people have a better chance of good jobs and wages if they are good with numbers.
- NAEP math scores
  - Range from 0 to 500
  - Have a Normal distribution
  - Population standard deviation σ is known to be 60
  - Population mean μ *not* known
- We sample n = 840 young mean
- Sample mean (“x-bar”) = 272
- Population mean μ unknown
- We want to estimate population mean NEAP score μ

Reference: Rivera-Batiz, F. L. (1992). Quantitative literacy and the likelihood of employment among young adults. *Journal of Human Resources*, **27,** 313-328.

CONDITIONS FOR INFERENCE

1. Data acquired by **S**imple **R**andom **S**ample
2. Population distribution is **Normal** or large sample
3. The value of **σ is known**
4. The value of **μ is NOT known**

THE DISTRIBUTION OF POTENTIAL SAMPLE MEANS: **THE SAMPLING DISTRIBUTION OF THE MEAN**

- Sample means will vary from sample to sample
- In theory, the sample means form a **sampling distribution**
- The sampling distribution of means is Normal with mean μ and **standard deviation** equal to population standard deviation σ divided by the square root of *n*:

This relationship is known as the **square root law**

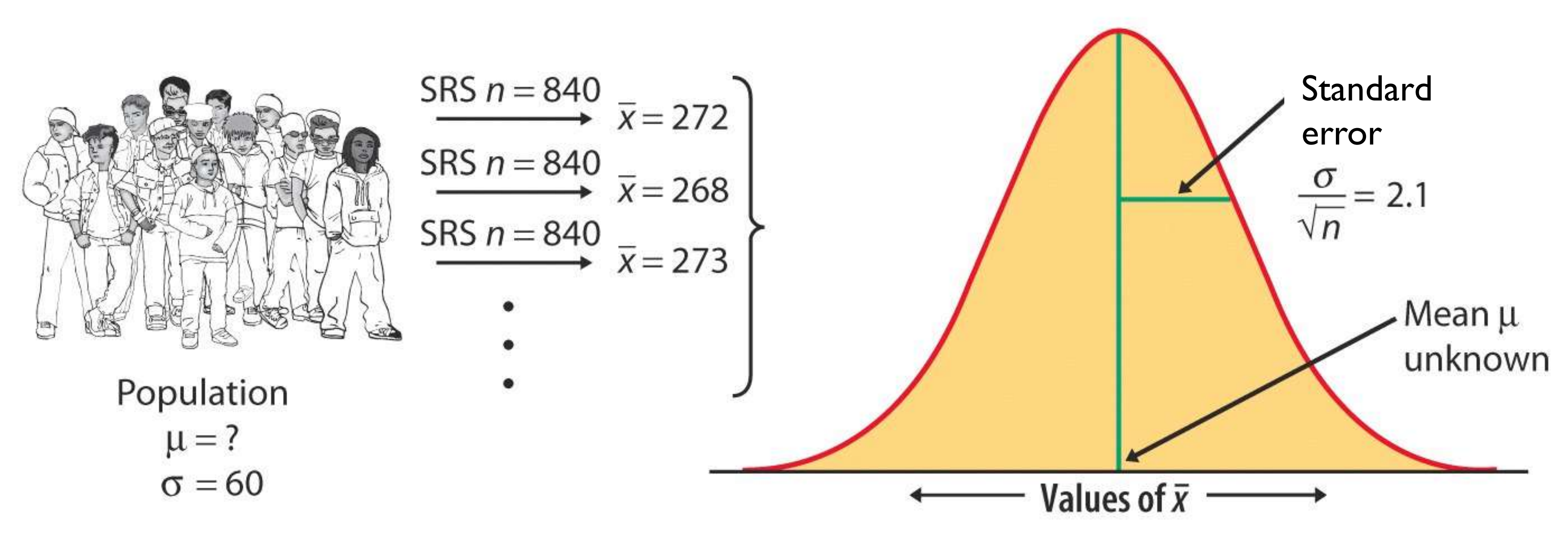
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = SE_{\bar{x}}$$

This statistic is known as the **standard error**

STANDARD ERROR OF THE MEAN

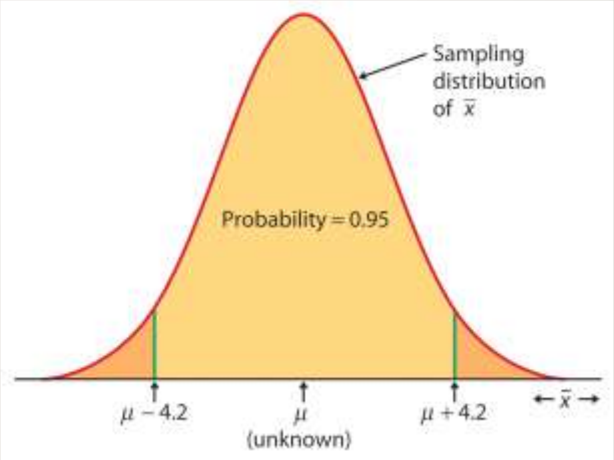
For our example, the population is Normal with σ = 60 (given). Since n = 840,

$$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{60}{\sqrt{840}} = 2.1$$



MARGIN OF ERROR *M* FOR 95% CONFIDENCE

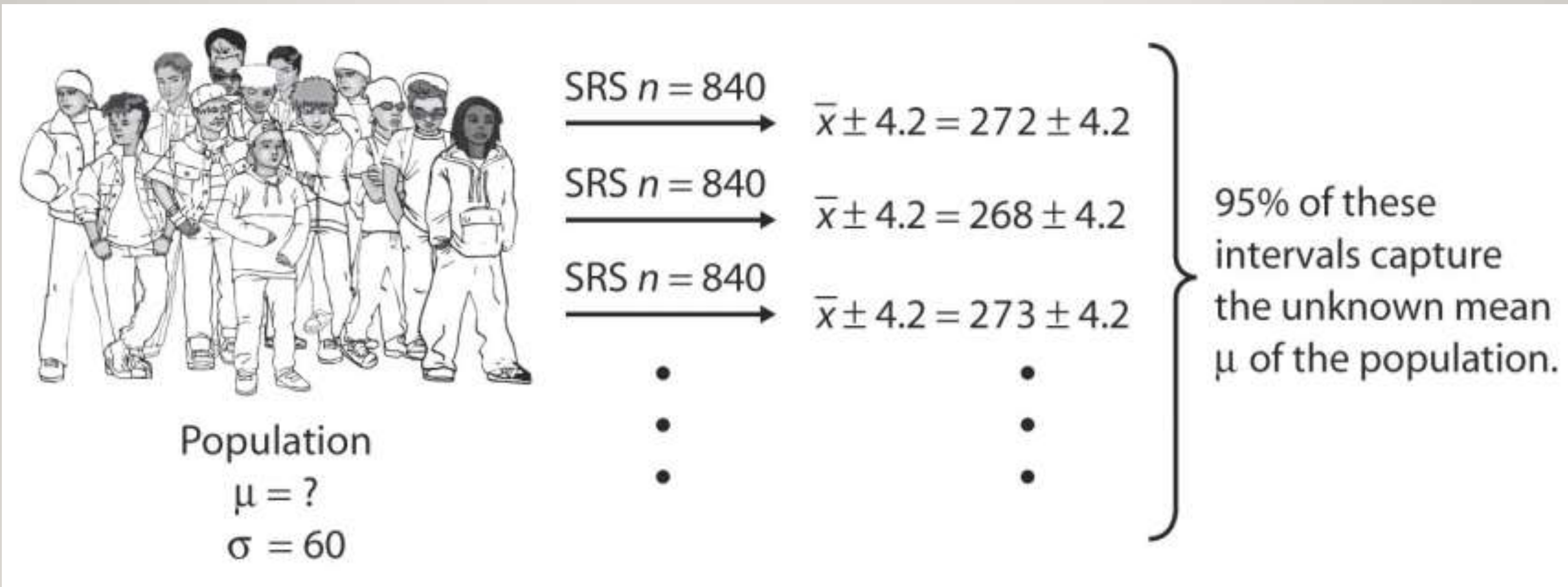
- The 68-95-99.7 rule says 95% of x-bars will fall in the interval
  $\mu \pm 2 \cdot SE_{xbar}$
- More accurately, 95% will fall in
  $\mu \pm 1.96 \cdot SE_{xbar}$
- $1.96 \cdot SE_{xbar}$ is the **margin of error *m*** for 95% confidence



For the data example

$$m = 1.96 \cdot SE_{\bar{x}} = 1.96 \cdot 2.1 = 4.2$$

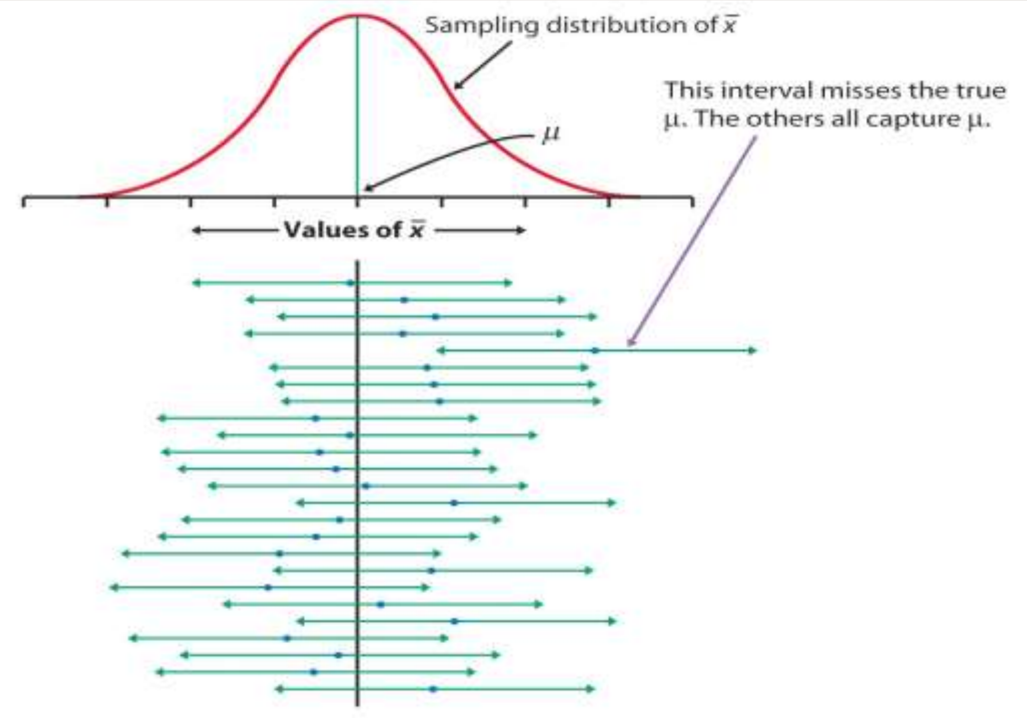
IN REPEATED INDEPENDENT SAMPLES:



We call these intervals

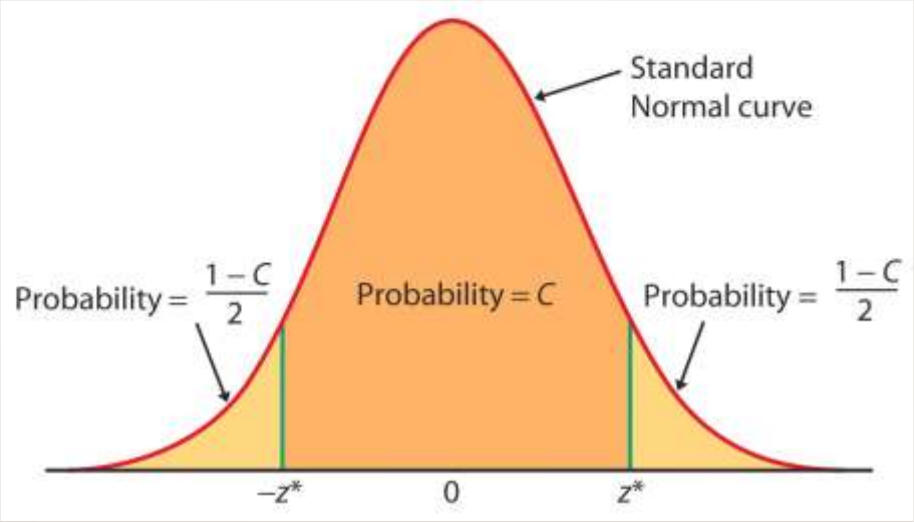
Confidence Intervals (CIs)

HOW CONFIDENCE INTERVALS BEHAVE



OTHER LEVELS OF CONFIDENCE

Confidence intervals can be calculated at various levels of confidence by altering coefficient $\mathbf{z_{1-\alpha/2}}$



α (“lack of confidence level”)	.10	.05	.01
Confidence level (1–α)100%	90%	95%	99%
$z_{1-\alpha/2}$	1.645	1.960	2.576

(1–A)100% CONFIDENCE INTERVAL FOR

FOR M WHEN Σ KNOWN

$$\bar{x} \pm z_{1-\frac{\alpha}{2}} \cdot SE_{\bar{x}}$$

$$\text{where } SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

MARGIN OF ERROR (*M*)

Margin of error (*m*) quantifies the precision of the sample mean as an estimator of μ. The direct formula for *m* is:

$$m = z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$$

Note that *m* is a function of

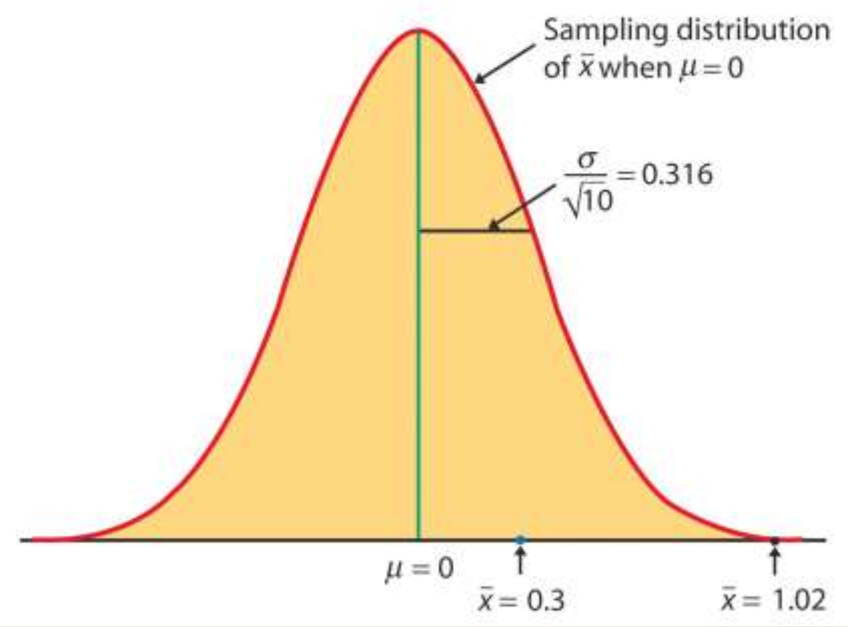
- confidence level 1 – α (as confidence goes up, z increase and m increases)
- population standard deviation σ (this is a function of the variable and cannot be altered)
- sample size *n* (as *n* goes up, *m* decreases; to decrease *m*, increase *n*!)

TESTS OF SIGNIFICANCE

- Recall: two forms of statistical inference
  - Confidence intervals
  - Hypothesis tests of statistical significance
- Objective of confidence intervals: to estimate a population parameter
- Objective of a test of significance: to weight the evidence against a “claim”

TESTS OF SIGNIFICANCE: REASONING

- As with confidence intervals, we ask what would happen if we repeated the sample or experiment many times
- Let X ≡ weight gain
  - Assume population standard deviation $\sigma = 1$
  - Take an SRS of $n = 10$,
  - $SE_{xbar} = 1 / \sqrt{10} = 0.316$
  - Ask: Has there weight gain in the population?



TESTS OF STATISTICAL SIGNIFICANCE: PROCEDURE

A. The claim is stated as a **null hypothesis** H_0 and **alternative hypothesis** H_a

B. A **test statistic** is calculated from the data

C. The test statistic is converted to a probability statement called a ***P*-value**

D. The *P*-value is **interpreted**

TEST FOR A POPULATION MEAN – NULL HYPOTHESIS

- ***Example***: We want to test whether data in a sample provides reliable evidence for a population weight gain
- The **null hypothesis H_0** is a statement of "no weight gain"
- In our the **null hypothesis** is H_0: $\mu = 0$

ALTERNATIVE HYPOTHESIS

- The **alternative hypothesis H_a** is a statement that contradicts the null.
- In our weight gain example, the **alternative hypothesis** can be stated in one of two ways
  - **One-sided alternative** H_a: $\mu > 0$
    ("positive weight change in population")
  - **Two-sided alternative** H_a: $\mu \neq 0$
    ("weight change in the population")

TEST STATISTIC

$$z_{\text{stat}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}}$$

where

$$\bar{x} \equiv \text{the sample mean}$$

$$\mu_0 \equiv \text{the value of the parameter under the null hypothesis}$$

$$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

TEST STATISTIC, EXAMPLE

$$\text{Given} : \sigma = 1$$

$$\text{Data} : \bar{x} = 1.02; n = 10$$

$$SE_{\bar{x}} = \frac{1}{\sqrt{10}} = 0.3126$$

$$z_{\text{stat}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}} = \frac{1.02 - 0}{0.3162} = 3.23$$

P-VALUE FROM Z TABLE

Convert z statistics to a *P*-value:

- For H_a: $\mu > \mu_0$
  P-value = $\Pr(Z > z_{stat})$ = right-tail beyond z_{stat}
- For H_a: $\mu < \mu_0$
  P-value = $\Pr(Z < z_{stat})$ = left tail beyond z_{stat}
- For H_a: $\mu \neq \mu_0$
  P-value = 2 × one-tailed *P*-value

P-VALUE: INTERPRETATION

- ***P*-value (definition)** ≡ the probability the sample mean would take a value as extreme or more extreme than observed test statistic *when* H_0 is true
- ***P*-value (interpretation)** Smaller-and-smaller *P*-values → stronger-and-stronger evidence *against* H_0
- ***P*-value** (**conventions)**

  $.10 < P < 1.0 \Rightarrow$ evidence against H_0 not significant

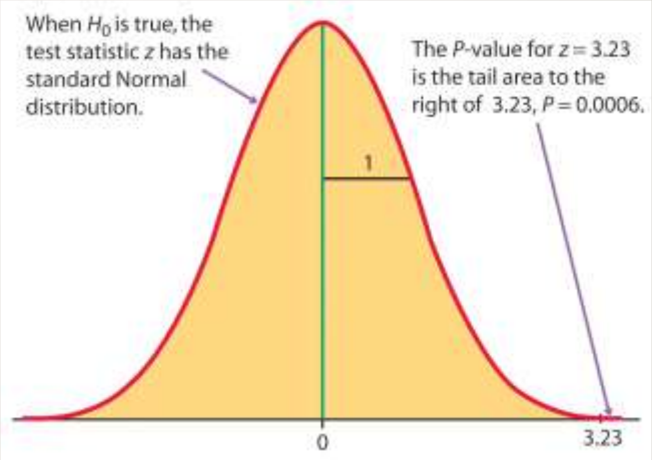
  $.05 < P \leq .10 \Rightarrow$ marginally significant

  $.01 < P \leq .05 \Rightarrow$ significant

  $0 < P \leq .01 \Rightarrow$ highly significant

P-VALUE: EXAMPLE

- $z_{stat} = 3.23$
- **One-sided *P*-value**
  = Pr(Z > 3.23)
  = 1 − 0.9994
  = 0.0006
- Two-sided *P-value*
  = 2 × one-sided *P*
  = 2 × 0.0006
  = 0.0012



Conclude: *P* = .0012
Thus, data provide highly significant evidence against H_0

SIGNIFICANCE LEVEL

- **α** ≡ threshold for "significance"
- *If* we choose $\alpha = 0.05$, we require evidence so strong that it would occur no more than 5% of the time when H_0 is true
- **Decision rule**

  P-value $\leq \alpha \Rightarrow$ evidence is significant

  P-value $> \alpha \Rightarrow$ evidence *not* significant

- For example, let $\alpha = 0.01$

  P-value = 0.0006

  Thus, $P < \alpha \Rightarrow$ evidence is significant

SUMMARY

z TEST FOR A POPULATION MEAN

Draw an SRS of size n from a Normal population that has unknown mean μ and known standard deviation σ. To test the null hypothesis that μ has a specified value,

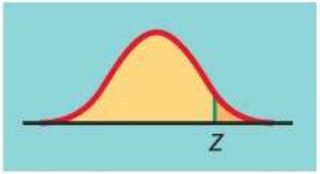
$$H_0: \mu = \mu_0$$

calculate the **one-sample z statistic**

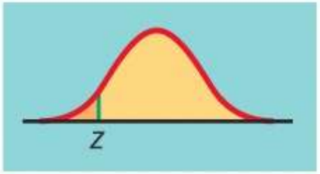
$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

In terms of a variable Z having the standard Normal distribution, the P-value for a test of H_0 against

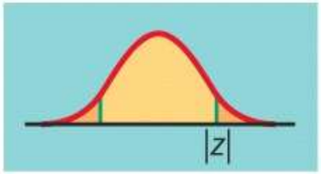
$H_a: \mu > \mu_0$ is $P(Z \geq z)$



$H_a: \mu < \mu_0$ is $P(Z \leq z)$



$H_a: \mu \neq \mu_0$ is $2P(Z \geq |z|)$



RELATION BETWEEN TESTS AND CIS

- The value of μ under the null hypothesis is denoted μ_0
- Results are significant at the α-level of when μ_0 falls outside the $(1-\alpha)100\%$ CI
- For example, when $\alpha = .05 \Rightarrow (1-\alpha)100\% = (1-.05)100\% = 95\%$ confidence
- When we tested H_0: $\mu = 0$, two-sided $P = 0.0012$. Since this is significant at $\alpha = .05$, we expect "0" to fall outside that 95% confidence interval

RELATION BETWEEN TESTS AND CIS

Data: x-bar = 1.02, n = 10, σ = 1

$$95\%\ \text{CI for}\ \mu = \bar{x} \pm z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 1.02 \pm 1.96 \frac{1}{\sqrt{10}}$$

$$= 1.02 \pm 0.62 = 0.40\ \text{to}\ 1.64$$

Notice that 0 falls outside the 95% CI, showing that the test of H_0: μ = 0 will be significant at α = .05

THANK YOU…!