Linear Programming

By

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Definition-Linear programming deals with optimization of a function of variables known as objective function subject to a set of linear equalities and/or inequalities known as constraints.

Linear programming is a mathematical technique which involves the allocation of limited resources in an optimal manner.

Formulation of LPP

- Write down the decision variables of the problem.
- Formulate objective function to be optimized(maximized or minimized) as a linear function of the decision variables.
- Formulate the other conditions of problems such as linear equations or inequations in terms of the decision variables.
- Add non-negativity constraints from the considerations.

General formulation of LPP

$Z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots\ldots\ldots c_nx_n$

s.t.

$a_{11}x_1 + a_{12}x_2 \ldots\ldots\ldots a_{1n}x_n (\leq or \geq or =) b_1$

$a_{21}x_1 + a_{22}x_2 \ldots\ldots\ldots a_{2n}x_n (\leq or \geq or =) b_2$

$a_{31}x_1 + a_{32}x_2 \ldots\ldots\ldots a_{3n}x_n (\leq or \geq or =) b_3$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$a_{i1}x_1 + a_{i2}x_2 \ldots\ldots\ldots a_{in}x_n (\leq or \geq or =) b_i$

$a_{m1}x_1 + a_{m2}x_2 \ldots\ldots\ldots a_{mn}x_n (\leq or \geq or =) b_m$

The standard weight of a special purpose brick is 5 kg and it contains two ingredients B_1 and B_2. B_1 costs Rs. 5 per kg and B_2 costs Rs. 8 per kg .Strength considerations dictate that the brick contains not more than 4 kg of B_1 and minimum of 2 kg of B_2 since the demand for the product is likely to be related to the price of the brick. Formulate above problem as a L.P. model.

Let x_1 and x_2 be the weight in kg of B_1 and B_2 ingredients

Min $Z=5x_1+8x_2$

Subject to

$x_1+x_2=5$

$x_1 \leq 4$

$x_2 \geq 2$

$x_1,x_2 \geq 0$

Egg contains 6 units of vitamin A and 7 units of vitamin B per gram and cost 12 paise per gram .Milk contains 8 units of vitamin A and 12 units of vitamin B per gram and cost 20 paise per gram .The daily minimum requirement of vitamin A and vitamin B are 100 units and 120 units respectively. Find the optimal product mix.

	Egg	Milk	
A	6	8	100
B	7	12	120

x_1 and x_2 be the units of Egg and Milk in gram respectively

Min $Z=12x_1+20x_2$

Subject to

$6x_1+8x_2 \geq 100$

$7x_1+12x_2 \geq 120$

A manufacturer has 3 machines A,B,C with which he produces 3 different articles P,Q,R. The different machine time required per article ,the amount of time available in any week on each machine and the estimated profits per article are furnished in the following table.

Article	Machine time (in hrs) A	B	C	profit per article in Rs.
P	8	4	2	20
Q	2	3	0	6
R	3	0	1	8
Available Machine(hr)	250	150	50	

Formulate the problem as linear programing problem.

Let x_1 be the number of articles of type P

x_2 be the number of articles of type Q

x_3 be the number of articles of type R

Since profit on type P is Rs. 20/- ,$20x_1$ be the profit on selling x_1 units of type P.

Similarly $6x_2$ and $8x_3$

Therefore the profit on selling x_1 articles of P, x_2 articles of Q and x_3 articles of R is given by

$Z=20x_1+6x_2+8x_3$

Since machine A takes 8 hrs of time for article P , 2 hrs for article Q and 3 hrs for article R

Total units of time required on machine A is $8x_1+2x_2+3x_3 \leq 250$

Similarly $4x_1+3x_2+0x_3 \leq 150$

$2x_1+0x_2+1x_3 \leq 50$

The general formulation of L.P.P is

Max $Z = 20x_1 + 6x_2 + 8x_3$

S.t.

$$8x_1 + 2x_2 + 3x_3 \leq 250$$
$$4x_1 + 3x_2 + 0x_3 \leq 150$$
$$2x_1 + 0x_2 + 1x_3 \leq 50$$

Since it is not possible to produce negative articles $x_1, x_2, x_3 \geq 0$

Thank You