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A variable sampling interval sign chart for variability based on deciles

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Chakraborti and Eryilmaz (2007) and Chakraborti and Van der Wiel (2008) have developed nonparametric quality control charts based on the nonparametric test statistic like sign, signed-rank and Mann-Whitney. Pawar and Shirke (2010) proposed synthetic control chart for monitoring the process median based on signed-rank statistic. Liu et al. (2015) provided an adaptive Phase II nonparametric exponentially weighted moving average control chart using variable sampling interval for location parameter. Mukherjee and Chakraborti (2012) provided a distribution-free control chart for the joint monitoring of location and scale. Zombade and Ghute (2014) developed nonparametric control charts based on run rules for variability. Chowdhury, Mukherjee, and Chakraborti (2015) proposed a distribution-free CUSUM chart using Ansari-Bradly's test statistic for monitoring process location and dispersion simultaneously. Liu, Tsung, and Zhang (2014) have developed a sequential rank based adaptive nonparametric cumulative sum control chart for process location.

The parametric charts like R chart, S chart and S^2 chart are usually used to monitor the process variability, these traditional charts require the assumption of normality. R chart is less efficient than S^2 chart, when the process distribution is normal. Amin, Reynolds, and Bakir (1995) discussed effect of non-normality on $\bar{X}$ control chart. Since the false alarm probabilities of the traditional parametric control charts depend on the process distribution, it may vary as process distribution changes. It is not the case in the nonparametric setup because the false alarm probability does not depend on process distribution. Average run length (ARL) is used to measure performance of control charts. It is the average number of sub-group samples required to get a signal. There can be rather large differences between the actual ARLs when the process distribution is non-normal and ARL calculated under the normality assumption. While designing variable sampling interval chart, current value of charting statistic decides time to next sample. If the process starts to deviate from the target then sample should be taken rapidly, otherwise samples should be taken in usual manner. The average time to signal (ATS) is reasonable measure to study the performance of chart with variable sampling interval policy. Reynolds et al. (1988) have suggested adjusted ATS (AATS) instead of ATS to measure performance of charts with variable sampling interval. Das (2008) proposed a nonparametric control chart for controlling variability based on squared rank test. Khilare and Shirke (2010) developed a nonparametric synthetic chart for monitoring process variability based on sign statistic. Amin and Widmaier (1999) proposed two sign control charts with variable sampling interval policy for process median and process variability. Pawar and Shirke (2014) provided nonparametric moving average control chart for process variability, which is based on sign statistic. Zhou, Zhou, and Geng (2016) developed a nonparametric control chart for monitoring variability based on the Ansari-Bradley's nonparametric test. Gou and Wang (2016) provided the variable sampling interval S^2 chart with known or unknown in-control variance. Haq (2017) proposed nonparametric EWMA control chart for process variability. Villanueva-Guerra et al. (2017) developed control chart for variance based on squared ranks. The idea behind variable sampling interval scheme is that, divide the in-control region of chart into two regions and the length of sampling interval for next sample is decided based on these regions. The shorter sampling interval is taken, if the charting statistic falls close to the control limits. It will increase sensitivity of chart for quick detection of the

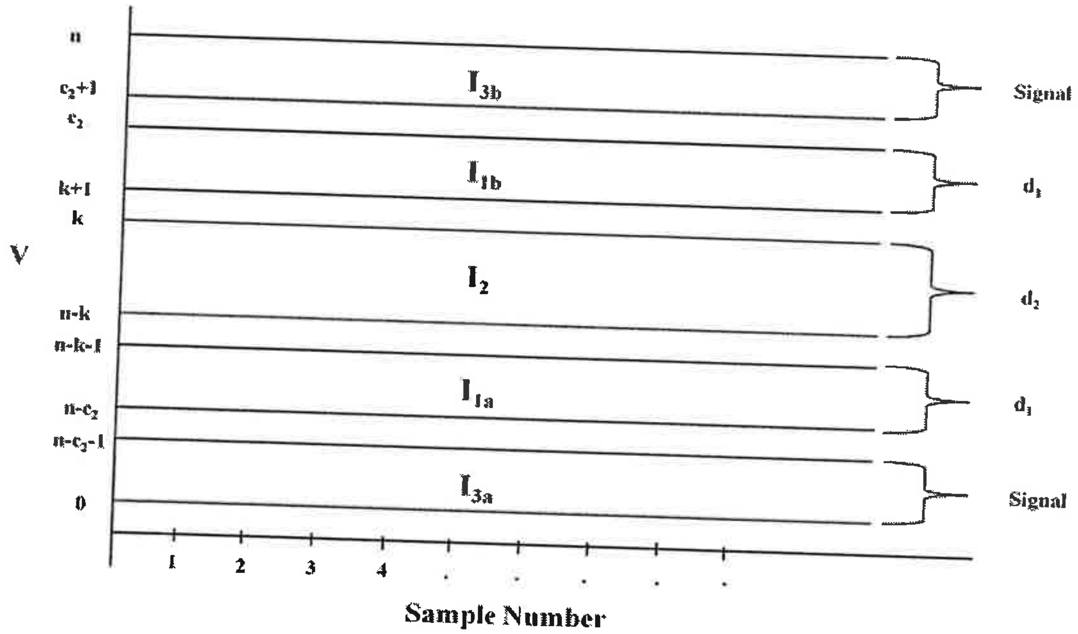

Figure 1. Outline of two-sided variable sampling interval sign chart based on deciles.

Widmaier (1999) have used sign statistic as charting statistic, which is used by Amin, Reynolds, and Bakir (1995).

3. A variable sampling interval sign control chart based on deciles

Shirke, Pawar, and Chakraborti (2016) developed a sign chart for variability based on in-control deciles, which is a modification of sign chart based on in-control quartiles given by Amin, Reynolds, and Bakir (1995). They used the fact that, if the standard deviation increases, tail probabilities also increase. This motivates to think about charts based on deciles of the in-control process distribution.

We use charting statistic proposed by Shirke, Pawar, and Chakraborti (2016) for designing a variable sampling interval chart. Consider D_2 and D_8 respectively be the 2^{nd} and 8^{th} deciles, when the process is in-control. We assume that such D_2 and D_8 are known apriori. Let

$$W_{ij} = \begin{cases} 1 & X_{ij} \le D_2 \text{ or } X_{ij} \ge D_8 \\ -1 & D_2 < X_{ij} < D_8 \end{cases} \tag{4}$$

and $W_i = \sum_{j=1}^{n} W_{ij}$, where W_i be sign statistic. Define a random variable $V_i = (W_i + n)/2$, which has a binomial distribution with parameters n and p, where $p = P\{X_{ij} \le D_2 \text{ or } X_{ij} \ge D_8 | \sigma_1 = \delta\sigma_0\}$ and $\delta = \sigma_1/\sigma_0$. Here $\delta < 1$ or $\delta > 1$ indicates change in the process standard deviation. Moreover, when the process is in the state of control, $p = 0.4$. The chart gives signal, if $V_i > c_2$ or $V_i < n - c_2$ and c_2 is chosen such that,

$$\alpha_0 \approx \sum_{j=0}^{n-c_2-1} \binom{n}{j} p^j (1-p)^{n-j} + \sum_{j=c_2+1}^{n} \binom{n}{j} p^j (1-p)^{n-j}, \tag{5}$$

where α_0 is false alarm probability when the process is in-control. In one-sided case (upper shift) c_2 is chosen such that,

Table 1. One-sided AATS and d_2 values for various values of k when $n=15$.

k	1	2	3	4	5
d_2	101.38	27.65	9.66	4.28	2.36
AATS	315.07	281.90	273.80	271.38	270.52
k	6	7	8	9	10
d_2	1.58	1.24	1.09	1.03	1.00
AATS	270.17	270.01	269.94	269.91	269.90

where $\alpha_1 = P(V_i<n-c_2 \text{ or } V_i>c_2|\sigma_1 = \delta\sigma_0)$ and $p_{1j} = P(V_i \in I_j|\sigma_1 = \delta\sigma_0)$ such that $p_{1j} = p_{0j}$ when $\delta=1$ for $j=1,2$.

The charting statistic proposed by Shirke, Pawar, and Chakraborti (2016) is discrete in nature. Therefore it is not possible to get an upper control limit c_2 that gives the exact desired in-control AATS. One can choose control limit c_2, which will give AATS close to the required in-control AATS. It is clear that, statistic V has binomial distribution. In order to compute probabilities p_{0j}, p_{1j} $(j=1, 2)$ and α_1 based on V statistic, we use normal approximation for binomial distribution to compute binomial probabilities, otherwise it will not be possible to compare these charts because of discrete nature of charting statistic. The statistic V has approximate normal distribution with mean np and variance $np(1-p)$. Box, Hunter, and Hunter (1978) have given following rough rules for normal approximation; (1) n is large and p is not near to extreme of 0 and 1. (2) $n>5$ and absolute value of $(1/\sqrt{n})(\sqrt{(1-p)/p}-\sqrt{p/(1-p)})$ is less than 0.3. For example, approximate one-sided false alarm probability can be obtained as

$$P(V>c_2) = \alpha \Rightarrow P\left(\frac{V-np}{\sqrt{np(1-p)}} > \frac{c_2-np}{\sqrt{np(1-p)}}\right) \approx \alpha. \tag{9}$$

Moreover, better normal approximation for $P(V > C_2)$ can be obtained by adding 1/2 in c_2, which is popularly known as Yates adjustment or continuity correction. Due to normal approximation c_2 may be fraction while comparing the charts. While implementing the control chart in practice control limit $\lceil c_2 \rceil$ should be used or use c_2 as it is but transform the charting statistic as given in expression (9).

Table 1 shows the values of d_2 and in-control AATS of decile based chart for different values of k. It can be observed that AATS and d_2 values are large for smaller values of k. Since, for smaller value of k, the probability of interval I_2 becomes very small when sample size n is large. In particular for $n=15$ and $k\geq 4$ it gives AATS$\approx$270, which is near to the AATS of fixed sampling intervals chart. Therefore select the values of k which yields in-control AATS near to the corresponding fixed sampling intervals chart while comparing the charts. Table 2 gives one-sided in-control AATS values of the sign chart based on deciles with their design parameters, where d_1 is taken as 0.1 time unit and d_2 is computed using an expression (7).

Amin, Reynolds, and Bakir (1995) have showed that, the nonparametric control chart based on in-control quartiles performs better as compared to the parametric S^2 chart when the process distribution is gamma and double exponential. The performance of chart based on in-control deciles reported by Shirke, Pawar, and Chakraborti (2016) is found to be better than the chart based on in-control quartiles. In the present study, a sign chart based on in-control deciles and sign chart based on in-control quartiles with variable sampling interval are compared. In most of the situations, an increase in the

Table 3. Comparison of AATS values for normal distribution.

			VSI AATS					
			$k=2$		$k=3$		$k=4$	
$\frac{\sigma_1}{\sigma_0}$	$n=9$, $c_1=8$ FSI-SCQ	$n=9$, $c_2=7.34$ FSI-SCD	SCQ	SCD	SCQ	SCD	SCQ	SCD
1	512.0	510.9	512.0	511.8	513.0	510.8	512.8	510.5
1.1	257.1	201.0	174.5	**147.1**	193.1	161.9	213.0	176.9
1.2	147.7	100.8	72.3	**55.3**	84.9	65.7	101.4	77.7
1.3	93.6	59.4	35.8	**25.4**	42.5	31.6	53.7	40.0
1.4	64.0	39.0	20.8	**13.7**	23.8	17.3	31.1	23.0
1.5	46.3	27.7	14.0	**8.5**	14.7	10.5	19.4	14.4
1.6	35.1	20.9	10.5	**5.9**	9.9	6.9	12.8	9.7
1.7	27.6	16.5	8.6	**4.5**	7.2	4.9	9.0	6.8
1.8	22.4	13.4	7.5	**3.7**	5.6	**3.7**	6.7	5.0
1.9	18.6	11.3	6.8	3.3	4.6	**2.9**	5.1	3.9
2	15.8	9.7	6.3	3.0	3.9	**2.4**	4.1	3.1

[a]FSI-fixed sampling interval.
[b]VSI-variable sampling interval.
[c]SCQ- sign chart based on in-control quartiles.
[d]SCD- sign chart based on in-control deciles.

Table 4. Comparison of AATS values for double exponential distribution.

			VSI AATS					
			$k=2$		$k=3$		$k=4$	
$\frac{\sigma_1}{\sigma_0}$	$n=9$, $c_1=8$ FSI-SCQ	$n=9$, $c_2=7.34$ FSI-SCD	SCQ	SCD	SCQ	SCD	SCQ	SCD
1.0	512.0	512.0	512.0	512.9	512.7	512.0	511.9	511.7
1.1	290.4	242.3	211.6	**189.9**	230.1	204.9	249.2	219.7
1.2	181.0	135.1	99.9	**84.1**	115.1	96.8	133.5	110.6
1.3	121.4	84.5	53.1	**42.7**	63.0	51.6	77.2	62.4
1.4	86.1	57.5	31.6	**24.2**	37.4	30.2	47.6	38.3
1.5	64.0	41.7	20.8	**15.0**	23.7	19.0	31.0	25.1
1.6	49.4	31.7	15.0	**10.2**	16.1	12.7	21.2	17.3
1.7	39.2	25.1	11.7	**7.4**	11.5	9.0	15.1	12.5
1.8	32.0	20.4	9.7	**5.7**	8.7	6.6	11.2	9.3
1.9	26.7	17.1	8.4	**4.7**	6.9	5.1	8.5	7.2
2.0	22.6	14.6	7.5	**4.0**	5.6	4.1	6.7	5.7

Table 5. Comparison of AATS values for Cauchy distribution.

			VSI AATS					
			$k=2$		$k=3$		$k=4$	
$\frac{\sigma_1}{\sigma_0}$	$n=9$, $c_1=8$ FSI-SCQ	$n=9$, $c_2=7.34$ FSI-SCD	SCQ	SCD	SCQ	SCD	SCQ	SCD
1.0	512.0	512.0	515.6	512.9	512.7	511.9	511.9	511.7
1.1	301.6	270.7	226.1	**220.8**	243.0	235.4	261.6	249.5
1.2	191.6	160.1	110.0	**107.0**	125.2	120.8	144.0	135.4
1.3	129.4	103.1	59.1	**57.1**	69.5	67.7	84.4	79.8
1.4	91.9	71.0	35.0	**33.0**	41.3	40.6	52.2	50.2
1.5	68.1	51.6	22.8	**20.6**	26.1	25.9	33.9	33.3
1.6	52.2	39.1	16.1	**13.7**	17.5	17.3	23.0	23.0
1.7	41.2	30.6	12.4	**9.7**	12.4	12.1	16.3	16.6
1.8	33.4	24.7	10.1	**7.3**	9.2	8.8	11.9	12.3
1.9	27.6	20.5	8.7	**5.8**	7.2	6.7	9.0	9.4
2.0	23.3	17.3	7.7	**4.8**	5.8	5.2	7.0	7.3

Table 9. Comparison of AATS values for Cauchy distribution.

			VSI AATS					
			$k=4$		$k=5$		$k=6$	
$\frac{\sigma_1}{\sigma_0}$	$n=15$, $c_1=12$ FSI-SCQ	$n=15$, $c_2=10.582$ FSI-SCD	SCQ	SCD	SCQ	SCD	SCQ	SCD
1.0	270.8	270.8	276.7	271.8	272.6	270.9	271.2	270.6
1.1	140.3	129.8	**97.6**	99.5	101.7	105.8	108.9	112.5
1.2	80.6	71.1	**41.8**	42.2	43.5	47.2	48.9	53.0
1.3	50.4	43.2	22.2	**20.5**	21.5	23.5	24.4	27.6
1.4	33.7	28.4	14.4	**11.3**	12.2	12.9	13.5	15.6
1.5	23.9	19.8	11.1	**7.1**	8.0	7.8	8.2	9.5
1.6	17.7	14.6	9.5	**5.0**	5.9	5.1	5.5	6.2
1.7	13.6	11.2	8.6	**3.9**	4.8	3.6	4.1	4.3
1.8	10.8	8.9	8.1	3.2	4.2	**2.8**	3.2	3.1
1.9	8.8	7.3	7.8	2.9	3.8	**2.3**	2.7	2.4
2.0	7.4	6.1	7.6	2.6	3.5	**1.9**	2.4	**1.9**

Table 10. Comparison of AATS values for gamma distribution.

			VSI AATS					
			$k=4$		$k=5$		$k=6$	
$\frac{\sigma_1}{\sigma_0}$	$n=15$, $c_1=12$ FSI-SCQ	$n=15$, $c_2=10.582$ FSI-SCD	SCQ	SCD	SCQ	SCD	SCQ	SCD
1.0	270.8	270.4	276.7	271.4	272.6	270.5	271.2	270.2
1.1	199.8	182.6	171.0	**159.6**	173.6	164.6	178.9	169.9
1.2	146.7	123.8	104.8	**93.1**	108.9	99.4	116.1	106.2
1.3	108.5	85.6	65.4	**55.2**	68.7	60.7	75.4	67.1
1.4	81.4	60.8	42.4	**33.7**	44.2	38.0	49.6	43.4
1.5	62.1	44.4	28.9	**21.4**	29.2	24.5	33.2	28.7
1.6	48.2	33.4	21.0	**14.2**	20.1	16.3	22.8	19.6
1.7	38.1	25.8	16.2	**9.9**	14.4	11.3	16.1	13.7
1.8	30.7	20.4	13.3	**7.3**	10.8	8.1	11.7	9.9
1.9	25.1	16.5	11.4	**5.7**	8.5	6.0	8.8	7.3
2.0	20.8	13.6	10.2	**4.6**	6.9	**4.6**	6.8	5.6

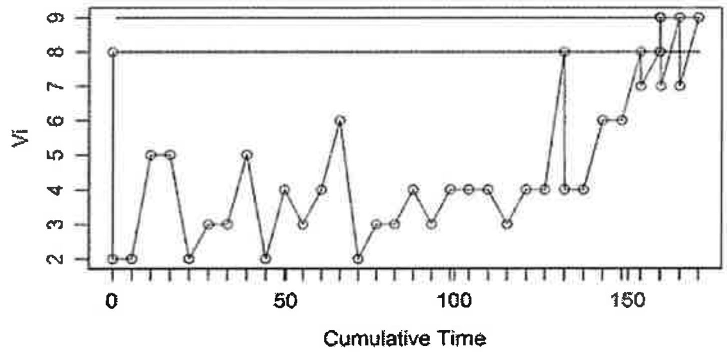

Figure 2. Variable sampling interval sign chart for variability.

shows that the proposed chart has better AATS performance as compared to variable sampling interval control charts due to Amin and Widmaier (1999) and fixed sampling interval control charts due to Shirke, Pawar, and Chakraborti (2016). The proposed chart performs better when process has normal, double exponential, gamma distributions for sample sizes $n=9$ and 15. When the process distribution is Cauchy and sample size $n=15$ proposed chart does not perform as good as for smaller shifts, however it performs better for moderate shift greater than 1.4. The AATS performance of the

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