

**“Education for Knowledge, Science and Culture”**

**... Shikshanmaharshi Dr. Bapuji Salunkhe**

**VIVEKANAND COLLEGE (AUTONOMOUS), KOLHAPUR**



**Department of Mathematics**

**M.Sc.-II**

**Semester III & IV**

**CBCS Revised Syllabus to be implemented from  
June-2022 Onwards**

**Vivekanand College, Kolhapur (Autonomous)**  
**Department of Mathematics**

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**POs:**

- Pursue research in reputed institutions and solve the existing mathematical problems using the knowledge of pure and applied mathematics.
- Acquire the strong foundation of basic concepts which will benefit them to become good academicians. The students will be eligible to pursue higher studies abroad.
- Critically interpret data, write reports and apply the basics of rules of evidence.
- Provide a systematic understanding of the concepts and theories of mathematics and their application in the real world – to an advanced level, and enhance career prospects in a huge array of fields.

**PSOs:**

- Handle the advanced technique in algebra, analysis, computational techniques, optimization, differential equations, engineering, finance and actuarial science to analyse and design algorithm solving variety of problems related to real life problems
- Adopt changing scientific environment in the process of sustainable development by using mathematical tools
- Have necessary skills and expertise in the field of research and developments through seminar and dissertation
- A student be able to apply their skills and knowledge, that is, translate information presented verbally into mathematical form, select and use appropriate mathematical formulae or technique in order to process the information and draw the relevant conclusion

## **M.Sc. Part-II Mathematics Semester-III**

### **Compulsory Course**

**CC-1180C : Functional Analysis**

**CC-1181C : Advanced Discrete Mathematics**

### **Choice Based Course**

**CBC-1182C : Number Theory**

**CBC-1183C : Fuzzy Mathematics I**

**CBC-1184C : Operational Research -I**

**CBC-1185C : Lattice Theory - I**

**CBC-1186C : Dynamical System - I**

**CBC-1187C : Commutative Algebra**

## SEMESTER III

**Course Code: CC-1180C**

**Credit - 05**

### **I) Title of Paper: Functional Analysis**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** Functional Analysis plays an important role in the applied sciences as well as in mathematics itself. Consequently, it becomes more and more desirable to introduce the student to the field at an early stage of study.

#### **Course Outcomes:**

- 1 To familiarize the students with the fundamental topics, principles and methods of functional analysis
- 2 Understand and apply fundamental theorems from the theory of normed and Banach spaces, including the Hahn-Banach theorem, the open mapping theorem, the closed graph theorem.
- 3 Able to understand Hilbert space and its application and acquire knowledge of orthogonal sets and operators
- 4 Understand Adjoint of an operator on a Hilbert space and concept of projection, self – adjoint, normal and unitary operator.

### **IV) Syllabus:**

**UNIT I:** Normed linear spaces, Banach spaces, Quotient spaces, Continuous linear transformations, Equivalent norms, Finite dimensional normed spaces and properties, Conjugate space and separability, The Hahn-Banach theorem and its consequences. **15 Lectures**

**UNIT II:** Second conjugate space, the natural embedding of the normed linear space in its second conjugate space, Reflexivity of normed spaces, Weak \* topology on the conjugate space. The open mapping theorem, Projection on Banach space, the closed graph theorem, the conjugate of an operator, the uniform boundedness principle. **15 Lectures**

**UNIT III:** Hilbert spaces: examples and elementary properties, Orthogonal complements, The projection theorem, Orthogonal sets, The Bessel's inequality, Fourier expansion and Parseval's equation, separable Hilbert spaces, The conjugate of Hilbert space, Riesz's theorem, The adjoint of an operator. **15 Lectures**

**UNIT IV:** Self adjoint operators, Normal and Unitary operators, Projections, Eigen values and eigenvectors of an operator on a Hilbert space, The determinants and spectrum of an operator, The spectral theorem on a finite dimensional Hilbert space. **15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

### **V) Recommended Reading:**

#### **Recommended Book(s):**

1. G. F. Simmons: Introduction to Topology and Modern Analysis, Tata McGraw Hill, 1963.

#### **Reference Books:**

1. Erwin Kreyszig: Introductory Functional Analysis with Applications, John Wiley and Sons, 1978
2. G. Bachman and L. Narici: Functional Analysis, Academic Press, 1972.
3. A. E. Taylor: Introduction to Functional analysis, John Wiley and sons, 1958.
4. J. B. Conway, A course in Functional Analysis, Springer-Verlag, 1985.
5. B. V. Limaye: Functional Analysis, New age international, 1996

## Course Code: - CC-1181C

### I) Title of Paper: Advanced Discrete Mathematics

Credit - 05

II) A Brief Note: Theorems and proofs are expected to be Prepared from recommended books

III) Specific Objectives: To learn about proof techniques, to learn about combinatorics and graph theory, to provide a background of mathematics that will be used in theoretical computer science.

Course Outcomes: Upon Successful completion of these course students will be able to

1. Solve discrete probability problems and use set to solve problems in combinatorics and probability theory.
2. Determine if a given graph is simple or a multigraph, directed or undirected graph, cyclic or acyclic, and determine the connectivity of a graph.
3. To determine if graph has a Euler or a Hamiltonian path or circuit, Define Pigeonhole principle and solve problems related to this.
4. Identify the types of Lattices and find supremum and infimum.

### IV) Syllabus:

**UNIT I:** Graph Theory: Definition, examples and properties, Simple graph, Graph isomorphism, Bipartite graphs, Complete Bipartite graph, regular graph, sub-graphs spanning sub-graph, Edge deleted sub-graph, Vertex deleted sub-graph, Union and intersection of two graphs, complements of a graph, self-complementary graph, paths and cycles in a graph, Eccentricity, radius and diameter of a connected graph, Peterson graph, Wheel graph. Isomorphism of Graphs. First theorem of graph theory. **15 Lectures**

**UNIT II:** The Matrix representation of a graph, Adjacency matrix and Incidence matrix of a graph, Definition and simple properties of a tree, bridges, spanning trees, Inclusion exclusion principle. Simple examples on Inclusion exclusion principle Pigeonhole principle, examples on Pigeonhole principle. **15 Lectures**

**UNIT III:** Discrete numeric functions and sum and product of two numeric functions, generating functions, Linear recurrence relations with constant coefficients Particular solutions of linear recurrence relations, Total solutions. **15 Lectures**

**UNIT IV:** Ordered sets and lattices Hasse diagrams of posets, Supremum and infimum, Isomorphic ordered sets, well-ordered sets, Lattices, Bounded lattices, Distributive lattices, Complements complemented lattices, Boolean algebra, Basic definitions, Basic theorems, duality, Boolean algebras as lattices **15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

### V) Recommended Reading:

#### Recommend Books:

1. Seymour Lipschitz and Mark Lipson: Discrete Mathematics (second edition) , Tata McGraw Hill Publishing Company Ltd. New Delhi

#### Reference Books-:

1. Gorrett Birkhoff : Lattice Theory
2. Rich and Brualdi : Combinatoric
3. John Clark and Derek Holton: A first book at Graph Theory Applied Publishers Ltd.
3. C. T. Liu: Discrete Mathematics

**Course Code: - CBC-1182C**

**Credit - 05**

**I) Title of Paper: Number Theory**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** To show Fundamental theorem of Arithmetic and Euler's Generalization of Fermat's theorem

**Course Outcomes:** Upon Successful completion of these course students will be able to

1. learn more advanced properties of primes and pseudo primes.
2. Able to apply Mobius Inversion formula to number theoretic functions.
3. Able to explore basic idea of cryptography.
4. understand concept of primitive roots and index of an integer relative to a given primitive root.

**IV) Syllabus:**

**UNIT I:** Review of Divisibility: The division algorithm, G.C.D., Euclidean algorithm, Diophantine equation  $ax + by = c$ , Primes and their distribution: Fundamental theorem of arithmetic, The Goldbach Conjecture.

**15 Lectures**

**UNIT II:** Congruences: Properties of congruences, Linear congruences, Chinese Remainder Theorem, Special divisibility tests, Fermat's theorem, Wilson's theorem and applications.

**15 Lectures**

**UNIT III:** Number Theoretic Functions: Euler's phi function, Euler's theorem, Greatest integer function, the functions  $\tau$  and  $\sigma$ , Mobius function and Mobius inversion formula, Properties of these functions and their inter relations.

**15 Lectures**

**UNIT IV:** Primitive roots: The order of an integer modulo  $n$ , Primitive roots of primes, composite numbers having primitive roots, The theory of indices, The quadratic reciprocity law: Eulerian criteria, The Legendre symbol and its properties, quadratic reciprocity, quadratic reciprocity with composite moduli.

**15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

**V) Recommended Reading:**

**Recommended Book(s):**

- 1 D. M. Burton: Elementary Number Theory, Universal book stall, New Delhi.

**Reference Books:**

- 1 S. B. Malik: Basic Number theory Vikas publishing House.
- 2 George E. Andrews: Number theory, Hindustan Pub. Corp. (1972)
- 3 Niven, Zuckerman: An Introduction to theory of numbers. John Wiley & Sons
- 4 S. G. Telang, Number Theory, Tata Mc. Graw-Hill Publishing Co.,  
New Delhi

**Course Code: - CBC-1183C**

**Credit - 05**

**I) Title of Paper: Fuzzy Mathematics I**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** To teach the students the need of fuzzy sets, fuzzy relations and its applications and to Construct the appropriate fuzzy theory corresponding to uncertain and imprecise collected data

**Course Outcomes:** Upon Successful completion of these course students will be able to

1. Acquire the knowledge of notion of crisp sets and fuzzy sets
2. Understand the basic concepts of crisp set and fuzzy sets
3. Develop the skill of operation on fuzzy sets and fuzzy arithmetic's
4. Demonstrate the technologies of fuzzy sets and fuzzy numbers

**IV) Syllabus:**

**UNIT I:** Fuzzy sets and crisp sets, examples of fuzzy sets, types of fuzzy sets, standard operations, cardinality, degree of subset hood, level cuts and its properties, representation of fuzzy sets, decomposition theorems, extension principle, properties of direct and inverse images of fuzzy sets

**15 Lectures**

**UNIT II:** Operations on fuzzy sets, types of operations, fuzzy complement, equilibrium and dual point Increasing and decreasing generators, fuzzy intersection: t-norms.

**15 Lectures**

**UNIT III:** Fuzzy union t-conorms, characterization theorem of t-conorm, combination of operators, aggregation operations, ordered weighted averaging operations.

**15 Lectures**

**UNIT IV** Fuzzy numbers, characterization theorem, linguistic variables, arithmetic operations on intervals, arithmetic operations on fuzzy numbers, lattice of fuzzy numbers, fuzzy equations

**15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

**V) Recommended Reading:**

**Recommended Books:**

1. George J. Klir, Bo Yuan, Fuzzy sets and Fuzzy Logic. Theory and Applications, PHI, Ltd.2000

**Reference Books: -**

1. M. Grabish, Sugeno, and Murofushi Fuzzy Measures and Integrals: Theory and Applications, PHI, 1999.
2. H.J.Zimmerermann, Fuzzy Set Theory and its Applications, Kluwer, 1984.
3. M. Hanss, Applied Fuzzy Arithmetic, An Introduction with Engineering Applications, Springer- Verlag Berlin Heidelberg 2005.
4. M. Ganesh, Introduction to Fuzzy Sets & Fuzzy Logic; PHI Learning Private Limited, New Delhi 2011.
5. Bojadev and M. Bojadev, Fuzzy Logic and Application, World Scientific Publication Pvt.Ltd. 2007.

**Course Code: - CBC-1184C**

**Credit - 05**

**I) Title of Paper: Operational Research –I**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** To introduce the students to the basic methodology for the solution of linear programs and Kuhn Tucker, Wolfe's method and Beale's method for solving Non-linear programming

**Course Objectives:** Upon Successful completion of these course students will be able to

1. Able to identify convex set and convex functions and construct linear integer programming models discuss the solution techniques
2. Able to solve multi –level decision problems using dynamic programming method.
3. able to identify the appropriate methods to solve the different kinds of Optimization Problems.
4. Formulate the nonlinear programming models and able to find solution methods for solving the nonlinear and linear optimization problems.

**IV) Syllabus:**

**UNIT I:** Convex sets and their properties. Lines and hyper planes convex set Important Theorems, polyhedral convex set, convex combination of vectors, convex hull, convex polyhedron, convex cone, simplex and convex function, General formulation of linear programming Matrix form of LP problem, definitions of standard LPP., Fundamental Theorem of linear programming.

**15 Lectures**

**UNIT II:** Simplex method, computational procedure of simplex method, problem of degeneracy and method to resolve degeneracy. Revised simplex method in standard form I, Duality in linear programming duality theorems, Integer linear programming, Gomory's cutting plane method, Branch and Bound method.

**15 Lectures**

**UNIT III:** Dynamic programming. Bellman's principle of Optimality, solution of problems with a finite number of stages. Application of dynamic programming in production, inventory control and linear programming.

**15 Lectures**

**UNIT IV:** Nonlinear programming unconstrained problems of maximum and minimum Lagrangian method Kuhn Tucker necessary and sufficient conditions, Wolfe's method, Beale's method.

**15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

**V) Recommended Reading:**

**Recommended Books:**

1. S. D. Sharma: Operations Research, Kedar Nath Ram Noth and co

**Reference Books: -**

1. Kanti Swarup, P. K. Gupta and Manmohan : Operations research, S. Chand & Co.
2. Hamady Taha: Operations Research: Mac Millan Co.
3. S. D. Sharma: Nonlinear and Dynamic programming Kedar Nath Ram Nath and Co. Meerut
4. R. K. Gupta: Operations Research Krishna Prakashan Mandir, Meeru
5. G. Hadley: Linear programming, Oxford and IBH Publishing Co.



**Course Code: - CBC-1185C**

**Credit - 05**

**I) Title of Paper: Lattice Theory - I**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** Student will gain the knowledge of basic theories of lattices and their equivalence and apply the methods for characterizing distributive lattices

**Course Objectives:** Upon Successful completion of these course students will be able to

1. Understand the relation between posets and lattices
2. Study the basic properties and characterization of lattice
3. Understand and apply the distributive complemented lattice
4. Design analyse and implement the concepts of stone's theorem and its consequence, pseudo complemented lattices and it's dual

**IV) Syllabus:**

**UNIT I:** Basic concepts. Posets, Definition and examples of posets. Two definitions of lattices and their equivalence, examples of lattices. Description of Lattices, some algebraic concepts. Duality principle, Special elements. Homomorphism, Isomorphism and isotone maps. **15 Lectures**

**UNIT II:** Special types of Lattices. Distributive lattices – Properties and characterizations. Modular lattices – Properties and characterizations. Congruence relations. Boolean algebras – Properties and characterizations. **15 Lectures**

**UNIT III:** Ideal theory, Ideals and filters in lattices. Lattice of all ideals  $I(L)$ . Properties and characterizations of  $I(L)$ . Stone's theorem and its consequences. **15 Lectures**

**UNIT IV:** Stone algebra, Pseudo complemented lattices.  $S(L)$  and  $D(L)$  – special subsets of pseudo complemented lattices. Distributive pseudo complemented lattice. Stone lattices – properties and characterizations **15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

**V) Recommended Reading:**

**Recommended Books:**

1. George Gratzer : General lattice theory, Academic Prress publications, 1978
2. B. V. Davey and H. A. Priestley: Introduction to Lattices and Order, Cambridge University Press, Second edition, 2002.

**Reference Books:**

1. Lattice theory by G. Birkhoff, Amer. Math. Soc. Coll. Publications, Third Edition 1973

**Course Code: - CBC-1186C**

**Credit - 05**

**I) Title of Paper: Dynamical System I**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** This course aims to provide insight and practice in how dynamical system (i.e., differential equation and difference equation) models can be used to better understand biology, physics, biochemistry, economics and sociology.

**Course Outcomes:** Upon successful completion of this course, the student will be able to:

1. Classify equilibrium points of the dynamical system
2. Construct bifurcation diagrams and analyze the system for different values of parameter.
3. Relate the qualitative properties of the system with the eigen values of coefficient matrix.
4. Construct the exponential of a matrix and apply it to solve the dynamical system.

**IV) Syllabus:**

**UNIT I:** First order systems- Qualitative Analysis: Introduction: First order linear systems, equilibrium points- classification, stability, bifurcation, phase portraits, Scalar autonomous non-linear systems, Stability (linearization, equilibrium points), phase portraits- slope fields, Examples, two-parameter family. **15**

**Lectures**

**UNIT II:** Planer systems- Qualitative Analysis, Second order linear ODE as a system of first order ODEs, preliminaries from algebra, eigenvalues and eigenvectors, solution of planar linear systems, Phase portraits for planar systems: Real distinct eigenvalues, complex eigenvalues, repeated eigenvalues, changing co-ordinates, Classification of planar systems: the trace-determinant plane **15 Lectures**

**UNIT III:** Higher order systems: Preliminaries from linear algebra, Higher order ODEs as a vector differential equation, real distinct, complex and repeated eigenvalues, The Exponential of a Matrix, Solving a system of first order differential equations by using exponential of a matrix, Non-autonomous systems of the form  $X'(t) = AX(t) + G(t)$ , Variation of parameters. **15 Lectures**

**UNIT IV:** Discrete dynamical systems: Introduction to the discrete maps (iterative maps), orbit, periodic points, cobweb plots, Fixed points of a map, stability analysis of a fixed point (sink, source, saddle), Bifurcation and chaos: Standard examples (Logistic map, tent map, doubling map). **15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

**V) Recommended Reading:**

**Recommended Book:**

1. M. Hirsch, S. Smale and R. L. Devaney : Differential equations, dynamical systems, and an introduction to chaos , Elsevier Academic Press, USA, 2004.

**Course Code: - CBC-1187C**

**Credit - 05**

**I) Title of Paper: Commutative Algebra**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** The course develops the theory of commutative rings. These rings are of fundamental significance since geometric and number theoretic ideas is described algebraically by commutative rings.

**Course Outcomes:** Upon successful completion of this course, the student will be able to:

1. classify the ideals to solve the related problems.
2. understand various radicals and know Hilbert basis theorem and apply it to other development.
3. use Nakayama Lemma for further development in Noetherion Rings.
4. Derive the Krull intersection theorem

**IV)Syllabus:**

**UNIT I:** Rings and ring homomorphism, Ideals. Quotient rings, Zero divisors. Nilpotent elements. Units, Prime ideals and Maximal ideals, Nilradicals and Jacobson radical, Operations on ideals, Extension and contraction. **15 Lectures**

**UNIT II:** Modules and modules homomorphisms, Submodules and quotient modules , Operations On submodules ,Direct sum and product ,Finitely generated modules ,Exact sequences. **15 Lectures**

**UNIT III:** Tensor product of modules , Restriction and extension of scalars , Exactness properties Of the tensor product , Algebras of tensor products. **15 Lectures**

**UNIT IV:** Rings and modules of fractions, Local properties ,Extended and contracted ideals in rings of fractions , primary decomposition. **15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

**V) Recommended Reading:**

**Recommended Books:**

1. M. F. Atiyah and I. G. MacDonald – Introduction to commutative Algebra, Addison Wesley publishing company.

**Reference Books:**

1. M.D. Larsen and P. J. McCarthy ; Multiplicative theory of ideals, Academic press,1971
2. D.G. Nortcot Ideal theory, Cambridge University press,1953

# **M.Sc. Part-II Mathematics**

## **Semester-IV**

### **Compulsory Course**

**CC-1190D : Field Theory**

**CC-1191D : Measures and Integrations**

### **Choice Based Course**

**CBC-1192D : Algebraic Number Theory**

**CBC-1193D : Fuzzy Mathematics II**

**CBC-1194D : Operational Research -II**

**CBC-1195D : Introduction to Data Science**

**CBC-1196D : Dynamical System II**

**CBC-1197D : Combinatorics**

**CBC-1198D : Fractional Calculus**

## SEMESTER IV

**Course Code: - CC-1190D**

**Credit - 05**

**I) Title of Paper: Field Theory**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** The course develops the theory of commutative rings. These rings are of fundamental significance since geometric and number theoretic ideas is described algebraically by commutative rings.

**Course Outcomes:** Upon successful completion of this course, the student will be able to:

1. Apply the knowledge of algebra to attain a good mathematical maturity and enables to build mathematical thinking and reasoning
2. Identify and analyse different types of algebraic structures such as algebraically closed fields, splitting fields, finite field extension to understand and use the fundamental results in Algebra
3. Design analyse and implement the concepts of Gauss lemma, separable extension etc.
4. Identify the challenging problems in advanced algebra to pursue further research.

**IV) Syllabus:**

**UNIT I:** Field Extensions Extension of a field, Algebraic extensions, algebraically closed fields, Derivatives and multiple roots, Finite Fields **15 Lectures**

**UNIT II:** Galois Theory Separable and normal extensions, Automorphism groups and fixed fields, Fundamental theorem of Galois theory **15 Lectures**

**UNIT III:** Finite Fields Prime fields, Fundamental theorem of algebra, Cyclic extensions, Cyclotomic extensions **15 Lectures**

**UNIT IV:** Applications of Galois theory Constructions by ruler and compass, Solvable groups, Polynomials solvable by radicals. **15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

**V) Recommended Reading:**

**Recommended Books:**

1. U. M. Swamy, A. V. S. N. Murthy, Algebra: Abstract and Modern, Pearson Education, 2012.

**Reference Books:**

1. Nathan Jacobson ,Basic Algebra I, second edition, W. H. Freeman and company, New York.
2. M. Artin, Algebra, PHI, 1996.
3. N. Herstein, Topics in Algebra, Wiley Eastern Ltd.
4. Bhattacharya, Jain and Nagpal, Basic Abstract Algebra, 2nd edition, Narosa Publishing House, New Delhi.
5. John Fraleigh : A first course in Abstract Algebra (3rd edition) Narosa publishing house, New Delhi

**Course Code: - CC-1191D**

**Credit - 05**

**I) Title of Paper: Measure and Integration**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** The objective of this course is to introduce the concepts of measure and integral with respect to a measure, to show their basic properties, and to provide a basis for further studies in Analysis, Probability, and Dynamical Systems.

**Course Outcomes:** Upon successful completion of this course, the student will be able to:

1. Understand algebra of sets, open and closed sets of real number and outer measure and measurable sets
2. understand the abstract measure theory and definition and main properties of the integral
3. Able to construct Lebesgue's measure on the real line and in n-dimensional Euclidean space.
4. Able to use the concept of measure theory to solve the problems related to probability theory, stochastic calculus and functional analysis

**IV) Syllabus:**

**UNIT I:** Open Sets, Closed Sets and Borel Sets, Lebesgue Outer Measure, The sigma algebra of Lebesgue Measurable Sets, Countable Additivity, Continuity and Borel-Cantelli Lemma, non-measurable set. **15 Lectures**

**UNIT II:** Sums, Product and Composition of Measurable Functions, Sequential Pointwise limits and Simple Approximation. Littlewood's Three Principles, Egoroff's Theorem and Lusin's Theorem, Lebesgue Integration of a Bounded Measurable Function, Lebesgue Integration of a Non-negative Measurable Function. **15 Lectures**

**UNIT III:** The General Lebesgue Integral, Characterization of Riemann and Lebesgue Integrability, Differentiability of Monotone Functions, Lebesgue's Theorem, Functions of Bounded Variations: Jordan's Theorem. **15 Lectures**

**UNIT IV:** Absolutely Continuous Functions, Integrating Derivatives: Differentiating Indefinite Integrals, Normed Linear Spaces, Inequalities of Young, Holder and Minkowski, The Riesz-Fischer Theorem. **15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

**V) Recommended Reading:**

**Recommended Books:**

1. Royden H. L., Fitzpateick P.M., Real Analysis. (2009) 4th edition. Prentice Hall of India, New Delhi

**Reference Books: -**

1. G. deBarra. Measure Theory and Integration. (1981) Wiley Eastern Ltd.
2. Rana, I. K. An Introduction to Measure and Integration. (1997) Narosa Book Company.
3. Berberian, S. K. Measure and Integration. (1965) McMillan, New York.
4. Jain P. K. and Gupta V. P. ,Lebesgue measure and Integration, (1986), Wiley Easter Limited.
5. Rudin W., Principles of Mathematical Analysis, (1964) McGraw-Hill Book Co

**Course Code: - CBC-1192D**

**Credit - 05**

**I) Title of Paper: Algebraic Number Theory**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** This course is an introduction to algebraic number theory and the structure of the integers and algebraic numbers, combining methods from commutative algebra, complex analysis, and Galois theory.

**Course Outcomes:** Upon successful completion of this course, the student will be able to:

1. Understand The concept (definition and significance) of algebraic numbers and algebraic integers.
2. Understand and clearly define number fields and their ring of integers, in particular quadratic number fields and cyclotomic number fields.
3. Able to factorise an algebraic integer into irreducible and find the ideals of an algebraic number ring.
4. Able to compute the class groups and the group of units of a number field.

**IV) Syllabus:**

**UNIT I:** Revision of rings, polynomial rings and fields, Field extensions, Symmetric polynomials, Modules, Free Abelian groups. **15 Lectures**

**UNIT II:** Algebraic Numbers, Algebraic number fields, Conjugates and Discriminants, Algebraic integers, Integral Bases, Norms and Traces, Ring of integers, Quadratic fields, Cyclotomic fields. **15 Lectures**

**UNIT III:** Factorization into irreducible, Noetherian rings, Dedekind rings, Examples of Non- Unique factorization into irreducible, Prime factorization, Euclidean Domains, Euclidean quadratic fields. **15 Lectures**

**UNIT IV:** Ideals, Prime factorization of ideals, Norm of an ideal, Nonunique factorization in cyclotomic fields, Two-squares theorem, Four-squares theorem, class groups and class numbers, Finiteness of the Class groups . **15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

**V) Recommended Reading:**

**Recommended Books:**

1. I.N. Stewart & D.O. Tall, Algebraic Number Theory , Academic press.

**Reference Books:**

1. N. Jacobson, Basic Algebra - I, Hindustan Publishing Corporation (India), Delhi.
2. P. Samuel, Algebraic Theory of Numbers, Hermann, Paris (1970).
3. Mathematical Pamphlet, Algebraic Number Theory, TIFR, Bombay.
4. Paulo Ribenboim, Classical Theory of Algebraic Numbers, Springer, New York (2001).
5. N.S. Gopalkrishnan, University Algebra, New Age International(P) Ltd. Publishers.

**Course Code: - CBC-1193D**

**Credit - 05**

**I) Title of Paper: Fuzzy Mathematics II**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** Provide an emphasis on the differences and similarities between fuzzy sets and classical sets.

**Course Outcomes:** Upon successful completion of this course, the student will be able to:

1. Acquire the concept of fuzzy relations
2. Understand the basic concepts of fuzzy logic and fuzzy algebra
3. Construct approximate solutions of fuzzy relation equations
4. Solve problems in Engineering and medicine

**IV) Syllabus:**

**UNIT I:** Projections and cylindrical extensions, binary fuzzy relations on single set, fuzzy equivalence relations, fuzzy compatibility relations, fuzzy ordering relations, fuzzy morphisms sup-i composition and inf-wi composition. **15 Lectures**

**UNIT II:** Fuzzy relation equations, problem partitioning, solution methods, fuzzy relational equations based on sup-i and inf-wi compositions, approximate solutions. **15 Lectures**

**UNIT III:** Fuzzy propositions, fuzzy quantifiers, linguistic hedges, inference from conditional fuzzy propositions, qualified and quantified propositions. **15 Lectures**

**UNIT IV:** Nonspecificity of Crisp Sets, Nonspecificity of Fussy Sets, Fuzziness of Fuzzy sets.

**15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

**V) Recommended Reading:**

**Recommended Books:**

1. George J Klir, Bo Yuan, Fuzzy Sets and Fuzzy Logic. Theory and applications, PHI.Ltd. (2000)

**Reference Books:**

1. M. Grabish, Sugeno, and Murofushi, Fuzzy Measures and Integrals: Theory and Applications  
PHI, 1999.
2. John Mordeson, Fuzzy Mathematics, Springer, 2001
3. H.J. Zimmermann, Fuzzy set: Theory and its Applications, Kluwer, 1984.
4. M. Ganesh : Introduction to Fuzzy sets & Fuzzy Logic; PHI Learning Private Limited, New Delhi, 2011.



**Course Code: - CBC-1194D**

**Credit - 05**

**I) Title of Paper: Operation Research II**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:**

To introduce the students to inventory models and queuing system.

**Course Outcomes:** Upon successful completion of this course, the student will be able to:

1. Decide an optimal replacement period/policy for a given item/equipment/machine
2. Understand the various selective inventory control techniques and its applications. Capability to develop deterministic inventory models
3. Understand the mathematical modeling of queuing systems To apply and extend queueing models to analyze real world systems.
4. Understand application of PERT and CPM techniques and able to construct network diagrams.

**IV) Syllabus:**

**UNIT I:** Replacement Problems, Failure mechanism of items, Replacement policy for items whose maintenance cost increases with time and money values is constant, Group replacement of items that fail completely **15 Lectures**

**UNIT II:** Inventory – Cost involved in inventory problems, variables in inventory problem, symbols in inventory concept of EOQ, Methods with calculus method ,Model I (a) The economic lot size system with uniform demand, Model I (b) Economic lot size with different rates of demand in different cycles., Model I (c) Economic lot size with finite Rate of Replenishment.,(EOQ production model) EOQ model with shortages , Model II(a) The EOQ with constant rate of demand, scheduling, time constant. **15 Lectures**

**UNIT III:** Queuing Theory, Queuing systems, Queuing Problems: transient and steady states, traffic intensity, Probability distributions in Queuing systems Poisson process, Properties, Exponential process, Classification of Queuing Models, Model I:(M/M/I) : ( $\infty$ /FCFS), Model II (a) : General Erlang queuing model. **15 Lectures**

**UNIT IV:** Information Theory: Communication process, Quantitative measure of information, A binary unit of information, measure of uncertainty of entropy, basic properties of entropy function (H) Joint and conditional entropies, Uniqueness theorem, Chanel capacity, efficiency and redundancy Encoding, Shannon Fano encoding procedure, PERT / CPM: Applications of PERT /CPM techniques, Network diagram, representations. Rules for constructing the Network diagram, determination of the critical path. **15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

**V) Recommended Reading:**

**Recommended Books:**

1. S.D.Sharma : Operations Research Kedarnath and co. 1999.

**Reference Books:**

1. KantiSwarup ,P.K.Gupta and Manmohan : Operations research, S.Chand& Co.
2. HamadyTaha : Operations Research :Mac Millan Co.
3. R.K.Gupta : Operations Research Krishna PrakashanMandir, Meerut

**Course Code: - CBC-1195D**

**Credit - 05**

**I) Title of Paper: Introduction to Data Science**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** Building the fundamentals of data science and Imparting design thinking capability to build big-data and developing design skills of models for big data problems and gaining practical experience in programming tools for data sciences

**Course Outcomes:** Upon successful completion of this course, the student will be able to:

1. Having an ability to apply mathematics and science in AI and machine learning applications
2. Having computational thinking (Ability to translate vast data into abstract concepts and to understand database reasoning)
3. Having problem-solving ability- solving social issues and engineering problems
4. Having an ability to use techniques, skills and modern engineering tools necessary for engineering practice

**IV) Syllabus:**

**UNIT I:** Data science in a big data world : Benefits and uses of data science and big data, Facts of data, The data science process, The big data ecosystem and data science

The data science process : Overview of the data science process, Retrieving data, Cleansing, integrating, and transforming data, Exploratory data analysis

**15 Lectures**

**UNIT II:** Machine learning: What is the machine learning, The modelling process, Types of machines learning, Semi-supervised learning

**15 Lectures**

**UNIT III:** Handling large data: General technique for handling large volume data, General programming tips for dealing with large data sets, Case study predicting malicious URLs

**15 Lectures**

**UNIT IV:** First step in big data: Distributing data storage and processing with frameworks, Case study: Assessing risk

**15 Lectures**

**UNIT V:** Examples, Seminars, Group Discussion on above four units

**V) Recommended Reading:**

**Recommended Books:**

1. Davy Cielen, Arno D. B. Meysman, Mohamed Ali, Introduction to data science, Manning Publications Co., 1<sup>st</sup> edition, 2016

**Reference Books:**

1. Trevor Hastie, Robert Tibshirani An Introduction to Statistical Learning: with Applications in R, Gareth James , Springer, 1st edition, 2013
2. Ethem Alpaydin, Introduction to Machine Learning, Third Edition 2018 PHI Learning Private Limited

**Course Code: - CBC-1196D**

**Credit - 05**

**I) Title of Paper: Dynamical System II**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** This course aims to provide insight and practice in how dynamical system (i.e. differential equation and difference equation) models can be used to better understand biology, physics, biochemistry, economics and sociology.

**Course Outcomes:** Upon successful completion of this course, the student will be able to:

1. test for the existence and uniqueness of solution of nonlinear system.
2. relate the stability of the system with its linearization.
3. distinguish between stable and unstable sets corresponding to the given system.
4. identify the chaotic behaviour in the system by using Lyapunov exponents.

**IV) Syllabus:**

**UNIT I:** Basic concepts of nonlinear dynamics: Introduction, Historical developments, Autonomous system of nonlinear ODEs: fundamental existence and uniqueness of solution, dependence of solution on initial conditions and parameters, The maximal interval of existence. **15 Lectures**

**UNIT II:** Stability analysis: The flow defined by a differential equation, Linearization, Stable manifold theorem, Hartman Grobman theorem, Stability and Lyapunov functions, Bifurcation. **15 Lectures**

**UNIT III:** II Chaos: Concept, properties, Limit sets and attractors, Poincare-Bendixson theorem, The Poincare map, Lyapunov exponents in flows, Numerical computation of Lyapunov exponents, Examples: Lorenz system, Chua circuit, Rossler attractor, Forced oscillators, Chaos synchronization. **15 Lectures**

**UNIT IV:** Applications and computer experiments: Application of chaos to secure communication, Introduction to fractals, Use of computer softwares to solve problems in Dynamical Systems: Solving linear and nonlinear systems, data visualization-2D and 3D plots, vector field plots, chaotic phase portraits, solving discrete systems- cobweb plots. **15 Lectures**

**UNIT V:** Examples, seminars, group discussions on above four units.

**V) Recommended Reading:**

**Recommended Book(s):**

1. Perko Differential Equations and Dynamical Systems, Springer, New York.
2. Alligood, Sauer and Yorke Chaos - an introduction to dynamical systems, Springer, New York.

**Reference Books:**

1. M. Hirsch, Smale and R.L. Devaney Differential equations, dynamical systems, and an introduction to chaos, Elsevier Academic Press, USA, 2004.
2. Strogatz, Nonlinear dynamics and chaos, Perseus Books, New York.
3. Wiggins, Introduction to applied nonlinear dynamics and chaos, Springer, New York.
4. Arrowsmith and Place Dynamical systems: differential equations, maps and chaotic behaviour, Chapman and Hall, London. (Applications)

**Course Code: - CBC-1197D**

**Credit - 05**

**I) Title of Paper: Combinatorics**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** To introduce a large variety of application and through some of them, Ramsey Numbers, Catalan Numbers and Stirling Numbers and develop combinatorial approach for solving the practical approach

**Course Outcomes:** Upon successful completion of this course, the student will be able to:

1. Students will familiar with fundamental combinatorial structures than naturally appears in various other field of mathematics
2. Learn how to use those structure to represent mathematical applied questions
3. Able to use generating function to solve a variety of combinatorial problems
4. Identify the challenging problems in arrangement and selections.

**IV) Syllabus:**

**UNIT I:** The sum Rule and the product Rule, Permutations and combinations, The Pigeonhole Principle, Ramsey Numbers, Catalan Numbers, Stirling Numbers. **15 Lectures**

**UNIT II:** Generalized Permutations and combinations, Multinomial Theorem, The Inclusion – Exclusion principle, Sieve’s formula, Derangements, System of Distinct Representatives (SDR), Combinatorial Number theory. **15 Lectures**

**UNIT III:** Rook- Polynomial, Ordinary and Exponential generating functions, Partitions of a positive integer, Recurrence Relations, Fibonacci sequence. **15 Lectures**

**UNIT IV:** Group Theory in Combinatorics, The Burnside Frobenius Theorem, Permutation Groups and Their Cycle Indices, Polya’s Enumeration Theorems. **15 Lectures**

**UNIT V:** Examples, seminars, group discussions on above four units.

**V) Recommended Reading:**

**Recommended Book(s):**

1. V.K. Balakrishnan Schum’s Outline of Theory and problems of combinatorics. Schum’s Outline Series Mc. Grew Hill INC

**Reference Books:**

1. Alan Tucker – Applied Combinatorics. – John Willey Sons.
2. Richard A Broadly, Introductory combinatorics New Holland.
3. Sharad Sane- Combinatorial Techniques-Hindustan Book Agency

**Course Code: - CBC-1198D**

**Credit - 05**

**I) Title of Paper: Fractional Calculus**

**II) A Brief Note:** Theorems and proofs are expected to be Prepared from recommended books

**III) Specific Objectives:** Identify the types of Fractional Differential Equations and select and apply the appropriate analytical technique for finding the solution

**Course Outcomes:** Upon successful completion of this course, the student will be able to:

1. Understand G-L and RL-fractional integral and evaluate fractional integrals of some common functions
2. RL and Caputo-fractional derivatives and evaluate fractional derivatives of some common functions
3. To Solve Linear Fractional Differential Equation using the Laplace and Mellin transform.
4. The study of fractional differential

**IV) Syllabus:**

**UNIT I:** Brief review of Special Functions of the Fractional Calculus: Gamma Function, Mittag-Leffler Function, Wright Function, Fractional Derivative and Integrals: Grünwald-Letnikov (GL) Fractional Derivatives-Unification of integer order derivatives and integrals, GL Derivatives of arbitrary order, GL fractional derivative of , Composition of GL derivative with integer order derivatives, Composition of two GL derivatives of different orders. Riemann-Liouville (RL) fractional derivatives-Unification of integer order derivatives and integrals, Integrals of arbitrary order, RL derivatives of arbitrary order **15 Lectures**

**UNIT II:** Composition of RL derivative with integer order derivatives and fractional derivatives, Link of RL derivative to Grünwald-Letnikov approach, Caputo's fractional derivative, generalized functions approach, Left and right fractional derivatives. Properties of fractional derivatives: Linearity, The Leibnitz rule for fractional derivatives, Fractional derivative for composite function, Riemann-Liouville fractional differentiation of an integral depending on a parameter, Behaviour near the lower terminal, Behaviour far from the lower theory. **15 Lectures**

**UNIT III:** Laplace transforms of fractional derivatives- Laplace transform of the RiemannLiouville fractional derivative, Caputo derivative and Grünwald-Letnikov fractional derivative. Fourier transforms of fractional integrals and derivatives. Mellin transforms of fractional derivatives-Mellin transforms of the Riemann-Liouville fractional integrals and fractional derivative, Mellin transforms of Caputo derivative. **15 Lectures**

**UNIT IV:** Existence and uniqueness theorem: Linear fractional differential equations (FDE), Fractional differential equation of a general form, Existence and uniqueness theorem as a method of solution. Dependence of a solution on initial conditions. Methods of solving FDE's: The Laplace transform method. The Mellin transform method, Power series method. **15 Lectures**

**UNIT V:** Examples, seminars, group discussions on above four units.

**V) Recommended Reading:**

**Recommended Book(s):**

1. Igor Podlubny, Fractional differential equations. San Diego: Academic Press; 1999.

**Reference Books:**

1. A. Kilbas, H.M. Srivastava, J.J. Trujillo, Theory and Applications of Fractional Differential Equations, Elsevier, Amsterdam, 2006.
2. Kai Diethelm, The Analysis of Fractional Differential Equations, Springer, 2010.  
L. Debnath, D. Bhatta, Integral Transforms and Their Applications, CRC Press

## Nature of Question Paper

### Instructions :

- 1) Question No. 1 is compulsory.
- 2) Attempt any four questions from Que. No. 2 to Que. No. 7.
- 3) All questions carry equal mark.
- 4) Figure to right indicates full marks
- 5) Use of log table or calculator is allowed.

Time: 3 Hours

Total Marks: 90

**Q.1 A) Choose correct alternative. (2 Marks Each) [08]**

i)

A)                      B)                      C)                      D)

ii)

A)                      B)                      C)                      D)

iii)

A)                      B)                      C)                      D)

iv)

A)                      B)                      C)                      D)

**B) Fill in the blanks./ True or False/ Defination (2 Marks Each) [10]**

i)

ii)

iii)

iv)

v)

**Q.2 [18]**

A)

B)

C)

**Q.3 [18]**

A)

B)

C)

**Q.4 [18]**

A)

B)

C)

**Q.5** **[18]**

A)

B)

C)

**Q.6** **[18]**

A)

B)

C)

**Q.7** **[18]**

A)

B)

C)

Remark:

Note that the distribution of marks for A, B, C or A, B (Q.N.2 To Q.N. 7) may vary according to the nature of question.

## SCHEME OF MARKING (THEORY)

<b>Sem.</b>	<b>CC and CBC</b>	<b>Marks</b>	<b>Evaluation</b>	<b>Answer Books</b>	<b>Standard of Passing</b>
III	CC -1180C CC -1181C	90	Semester Wise	As per Instructions	40% (36 Marks)
III	CBC -1182C TO CBC -1187C (Choose any three)	90	Semester Wise	As per Instructions	40% (36 Marks)
IV	CC -1190D CC -1191D	90	Semester Wise	As per Instructions	40% (36 Marks)
IV	CBC -1192D TO CBC -1198D (Choose any three)	90	Semester Wise	As per Instructions	40% (36 Marks)

## SCHEME OF MARKING (CIE) Continuous Internal Evaluation

<b>Sem.</b>	<b>CC and CBC</b>	<b>Marks</b>	<b>Evaluation</b>	<b>Answer Books</b>	<b>Standard of Passing</b>
III	CC -1180C CC -1181C	30	Concurrent	As per Instructions	40% (12 Marks)
III	CBC -1182C TO CBC -1187C (Choose any three)	30	Concurrent	As per Instructions	40% (12 Marks)
IV	CC -1190D CC -1191D	30	Concurrent	As per Instructions	40% (12 Marks)
IV	CBC -1192D TO CBC -1198D (Choose any three)	30	Concurrent	As per Instructions	40% (12 Marks)

**\*A Separate passing is mandatory**





















