24/09/2018

Vivekanand College, Kolhapur (Autonomous)

Department of Mathematics

M. Sc. I Sem. I and M.Sc. II Sem III

Internal Examination 2018-19

All the students of M.Sc. I and M.Sc. II are hereby informed that their Internal Examination of Mathematics will be conducted on **as given below timetable.** The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus and timetable for examination will be as mentioned in following table.

Syllabus for M. Sc. I Sem. I

Sr. No.	Name of Paper	Topics
1	CP-1170A : Algebra	Unit I
2	CP-1171A: Advanced Calculus	Unit I
3	CP-1172A: Complex analysis	Unit I
4	CP-1173A: Ordinary Differential Equation	Unit I
5	CP-1174A: Classical Mechanics	Unit I

Syllabus for M. Sc. II Sem. III

Sr. No.	Name of Paper	Topics
1	CC-1180C: Functional Analysis	Unit I
2	CC-1181C: Advanced Discrete Mathematics	Unit I
3	CBC-1182C : Lattice Theory	Unit I
4	CBC-1183C: Number theory	Unit I
5	CBC-1184C : Operational Research -I	Unit I

Timetable

Date	Time	Class	Subject
08/10/2018	03:00 PM to 04: 00 PM	M.Sc. I	Algebra
	03:00 PM to 04: 00 PM	M.Sc. II	Functional Analysis
09/10/2018	03:00 PM to 04: 00 PM	M.Sc. I	Advanced Calculus
	03:00 PM to 04: 00 PM	M.Sc. II	Advanced Discrete Mathematics
1010/2018	03:00 PM to 04: 00 PM	M.Sc. I	Complex analysis
	03:00 PM to 04: 00 PM	M.Sc. II	Lattice Theory
11/10/2018	03:00 PM to 04: 00 PM	M.Sc. I	Ordinary Differential Equation
	03:00 PM to 04: 00 PM	M.Sc. II	Number theory
12/10/2018	03:00 PM to 04: 00 PM	M.Sc. I	Classical Mechanics
	03:00 PM to 04: 00 PM	M.Sc. II	Operational Research -I

Nature of question paper

Time:-1 Hours **Total Marks: 30**

Q.1) Choose the correct alternative for the following question. [05]

Five questions

Q.2) Attempt any three [15]

Four questions

Q.3) Attempt any One [10]

Two questions

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(Prof. S. P. Patankar)
HEAD
Department of Mathematics
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Vivekanand College, Kolhapur (Autonomous)
M.Sc. I Semester-I Internal Examination : 2018-19
MATHEMATICS

Sub: Algebra (CP-1170A) Time: 03:00 PM – 04:00PM
Date: 08/10/2018 Total Marks:30

Q1) Select the correct alternatives **(5)**

1] *In a group of order* 15 *the number of subgroups of order* 3 *is* ...
 a) 3 *b*) 5 *c*) 1 *d*) 2

2] *If G is an arbitrary group of even order* 2*n then* ...
 a) *G has a proper normal subgroup which is non trivial.*
 b) *G admits a quotient group of order n*
 c) *G has a subgroup of order* 2
 d) *G admits a quotient group of order* 2.

3] *For* $n \geq 2$ *let* $\left(\frac{\mathbb{Z}}{n\mathbb{Z}}\right)^*$ *be the groups of units of* $\frac{\mathbb{Z}}{n\mathbb{Z}}$ *which one of the following is cyclic ?*
 a) $\left(\frac{\mathbb{Z}}{8\mathbb{Z}}\right)^*$ *b*) $\left(\frac{\mathbb{Z}}{15\mathbb{Z}}\right)^*$ *c*) $\left(\frac{\mathbb{Z}}{10\mathbb{Z}}\right)^*$ *d*) $\left(\frac{\mathbb{Z}}{35\mathbb{Z}}\right)^*$

4] *Let* $G = \{1, -1\}$ *then the group* $< G \ . >$ *is*
 a) *isomorhic to* $< \mathbb{Z}, + >$ *b*) *homomorphic image of* $< \mathbb{Z}_5, + >$
 c) *isomorhic to* $< \mathbb{Z}_5, + >$ *d*) *a homomorphic image of* $< \mathbb{Z}, + >$

5] Order of A_5 is...
 a) 60 b) 120 c) 5 d) 5!

Q2) Solve any THREE of the following. **(15)**

1] Prove that every permutation σ of a finite set A is a product of disjoint cycles.

2] Define commutator subgroup of group G. Show that G is abelian if and only if commutator subgroup is {e}.

3] Show that for $n \geq 3$, Subgroup generated by 3 – cycle of A_n is A_n.

4] Define index of subgroup . Find index of A_n in S_n . Show that A_n is normal in S_n.

Q3) Solve any ONE of the following. **(10)**

1] State and prove Schreier theorem.

2] State and prove Caley's theorem.

Vivekanand College, Kolhapur (Autonomous)

M.Sc. (Part-I) Semester-I Internal Examination:2018-19

Subject :Advanced Calculus **Time: 03:oopm-04:00 pm**

Total Marks: 30 **Date:09/10/2018**

Q. 1 Select the correct alternative for each of the following: **[5]**

i. *he series* $\sum_{n=1}^{\infty} a_n sin(nx)$ *converges uniformly on* $\mathbb{R}$ *if*

A) $\sum_{n=1}^{\infty} a_n$ *converges* B) $\sum_{n=1}^{\infty} |a_n|$ *converges*

C) $\sum_{n=1}^{\infty} sin(nx)$ *converges* D) $\sum_{n=1}^{\infty} |sin(nx)|$ *converges.*

ii. *Radius of convergence for the seris* $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$ *is* ______

A) 2 B) 1/2

C) 1 D) *series always diverges.*

iii. *For a vector field* $\bar{f}, \bar{f}'(\bar{c}; \bar{0}) =$ ______

A) 0 B) $\bar{c}$

C) $\bar{0}$ D) $\|\bar{c}\|$

iv. *If* $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ & $S: \mathbb{R}^m \longrightarrow \mathbb{R}^p$ *are linear then order of matrix of* $(\bar{S}.\bar{T}) =$ ______

A) $n \times p$ B) $m \times p$ C) $p \times n$ D) $m \times n$

v] If α_1v1+α_2v2+ …+$\alpha_n$vn=0, where v1, v2, … ,vn are linearly independent vectors in a vector space V(F), then ________.

i) i=0 for all i=1,2, …, n ii) i≠0 for all i=1,2, …, n

iii) i=0 for at least one i iv) i≠0 for at least one i

Q.2. Attempt any three of the following: **[15]**

1) Prove that the sequence $\{f_n\}_{n=1}^{\infty}$ *converges pointwise but not uniformly where* $f_n(x) = \frac{1}{nx+1}, 0 < x < 1$

2) *State & prove Green's theorem for plane region bounded by piecewise smooth Jordon curve.*

3) *If* $\sum_n a_n$ *converges absolutely then prove that every subseris* $\sum_n b_n$ *also converges absolutely.*

4) Let $\bar{f}$ *be a vector field given by* $\bar{f}(x, y) = \sqrt{y}i + (x^3 + y)j$ *where* $(x, y) \in \mathbb{R}^2$.

Q.3. Attempt any one of the following: **[10]**

1. *If* $\{f_n\}$ & $\{g_n\}$ *be sequences of Riemann integrable functions defined on* $[a, b]$ &

$$\underset{n\to\infty}{\text{l.i.m}} f_n = f \ \& \ \underset{n\to\infty}{\text{l.i.m}} g_n = g \ on \ [a, b]. Let \ h_n(x) = \int_a^x f_n(t) g_n(t) dt \ \&$$

$$h(x) = \int_a^x f(t)\, g(t) dt$$

prove that $h_n \to h$ *uniformely on* $[a, b]$

2. *If* $\{M_n\}$ *be a sequence of non − negative real numbers such that* $0 \le |f_n(x)| \le M_n$ $\forall n \in \mathbb{N}$ & $\forall x \in S$. *Prove that* $\sum f_n$ *converges uniformely on S if* $\sum M_n$ *converges.*

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-I Internal Examination :2018-19

Subject : Complex Analysis **Time,Date: 03:00pm onwards , 10/10/2018**
Total Marks: 30

Q. 1 Select the correct alternative for each of the following: **[5]**

i) If $f(z) = x^2y^2 + i2xy$ and $g(z) = 2xy + i(x^2 - y^2)$ $\forall z \in c$, then in the complex plane C.

A) f is analytic and g is not analytic B) f is not analytic and g is analytic
C) f is analytic and g is analytic D) f is not analytic and g is not

ii) If C is the circle |z - a | = r then $\int_C \frac{dz}{(z-a)^n} = 2\pi i$, when ......

iii) If C is the circle of radius 2 with center at the origin in the complex plane, oriented in the anti - clockwise direction. Then the integral $\oint \frac{dz}{(z-1)^2}$ is equal to.....

A) 1 B) $2\pi i$ C) 0 D) $1/2\pi i$

iv) For the function f(z) = $\frac{z-sinz}{z^3}$, at the point z = 0 is.....

A) Pole of order 3 B) Pole of order 2
C) Essential singularity D) Removable singularity

v) For the function f(z) = $\frac{z-sinz}{z^3}$, at the point z = 0 is.....

A) Pole of order 3 B) Pole of order 2
C) Essential singularity D) Removable singularity

Q.2. Attempt any three of the following: **[15]**

1) If radius of convergence of f(z) = $\sum_{n=1}^{\infty} a_n z^n$ is 0, then f(z) is.........analytic (no where)
2) The radius of convergence of $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$ is..... (e)
3) If $u(x, y) = x^3 + ax^2y + bxy^2 + 2y^3$ is harmonic function and $v(x, y)$ its harmonic conjugate.
4) If γ is a contour with parameter interval [a, b] and $f(z) = u(x, y) + iv(x, y)$ is continuous function on the contour γ with $|f(z)| \leq M, \forall z \in \gamma$, then prove that $|\int_C f(z)dz| \leq ML$ where L is the length of contour given by $\int_a^b |\gamma'(t)|dt$

Q.3. Attempt any one of the following: **[10]**

1) $0 \leq R < \infty$ called the radius of convergence with the following properties
   i) $\sum a_n z^n$ converges absolutely for every z with |z| < R.
   ii) If | z| > R, the terms of power series become unbounded and so the series diverges. find the radius of convergence of the power series $\sum_{n=0}^{\infty} (\frac{1}{n})^n z^n$.
2) Define harmonic conjugate and prove that the function $u = x^2 - y^2 + xy$ satisfies Laplace's equation and find the corresponding analytic function f(z)

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-I
Internal Examination(2018-19)
Ordinary Differential Equations

Time: 03:00PM to 04:00PM **Total Marks: 30**
Date: 11/10/2018

Q.1) Choose the correct alternative for the following question. [05]

i) Wronskian of the two solutions of the differential equation $y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I is

A) Identically Zero B) Never Zero

C) Either identically zero or never zero D) Always Constant

ii) Which of the following is not solution of $y''' - 3r_1y'' + 3{r_1}^2y' - {r_1}^3y = 0$, where r_1 is constant

A) $\emptyset(x) = e^{r_1x}$ B) $\emptyset(x) = x^2e^{r_1x}$

C) $\emptyset(x) = xe^{r_1x}$ D) $\emptyset(x) = x^3e^{r_1x}$

iii) If $p_n(x)$ and $p_m(x)$ are n^{th} and m^{th} Legendary polynomials respectively, then $\int_{-1}^{1} p_n(x)p_m(x) = 0$ is possible when ......

A) $m = n$ B) $m \leq n$ C) $m \geq n$ D) $m \neq n$

iv) If $f(x,y) = y^{\frac{2}{3}}$, $R = \{(x,y)||x| \leq 1, |y| \leq 1\}$ and K is Lipschitz constant then ......

A) F satisfies Lipschitz Condition on R with $k = \frac{1}{2}$

B) F satisfies Lipschitz Condition on R with $k = 0$

C) F satisfies Lipschitz Condition on R with $k = 1$

D) F do not satisfy Lipschitz Condition on R

v) The General solution of $y'' + y' - 2y = 0$ is .....

A) $c_1e^{-x} + c_2e^{-2x}$ B) $c_1e^{x} + c_2e^{-2x}$

C) $c_1e^{x} + c_2xe^{x}$ D) $c_1e^{-x} + c_2e^{2x}$

Q.2) Attempt any three [15]

i) Determine all complex numbers m of the given problem $-y'' = my$, $y(0) = 0$, $y(1) = 0$ as a non-trivial solution and compute such a solution for each of this m.

ii) Compute the Second lineally independent solution of equation $x^2y'' - xy' + y = 0$, $\emptyset_1(x) = x, (x > 0)$

iii) If $\emptyset_1, \emptyset_2, \ldots\ldots \emptyset_n$ are the 'n' solutions of $L(y) = 0$ on an interval I containing point x_0, satisfying ${\emptyset_i}^{(i-1)}(x_0) = 1$, ${\emptyset_i}^{(j-1)}(x_0) = 0$, $j \neq i$, i,j=1,2,3.....n ,and $\emptyset$ is any solution of $L(y) = 0$ on I, then show that there are 'n' constants $c_1, c_2 \ldots\ldots c_n$ such that $\emptyset = c_1\emptyset_1 + c_2\emptyset_2 + \cdots + c_n\emptyset_n$

iv) Show that $\int_{-1}^{1} p_n(x)\, p_m(x) = 0$ (n$\neq m$)

Q.3) Attempt any One **[10]**

i) If $\emptyset(x)$ is any solution of L(y) = $y'' + a_1y' + a_2y = 0$ on an interval I containing an point x_0, than show that $\forall\ x_0$ in I, $\|\emptyset(x_0)\|\, e^{-k|x-x_0|} \leq \|\emptyset(x)\| \leq \|\emptyset(x_0)\|\, e^{k|x-x_0|}$ where, $\|\emptyset(x)\| = [|\emptyset(x)|^2 + |\emptyset'(x)|^2]^{\frac{1}{2}}$ and $k = 1 + |a_1| + |a_2|$

ii) If $\emptyset_1$ is a solution of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I and $\emptyset_1(x) \neq 0$ on an interval I , then show that the second solution $\emptyset_2$ of $L(y) = 0$ on I given by, $\emptyset_2(x) = \emptyset_1(x) \int_{x_0}^{x} \frac{1}{[\emptyset_1(s)]^2}\, exp[-\int_{x_0}^{s} a_1(t)dt]ds$ and function $\emptyset_1, \emptyset_2$ form a basis for the solution of $L(y) = 0$ on I.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-I
Internal Examination(2018-19)
Classical Mechanics

Time: 3:00PM–4:00PM **Total Marks: 30**

Date:12/10/2019

Q.1) Choose the correct alternative for the following question. [05]

1) Kinetic energy of a particle of mass m and position vector $\bar{r}$ in polar form is ....

A) $T= m(\dot{r}^2 + r^2\dot{\theta}^2)$ B) $T=\frac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2)$

C) $T= 2m(\dot{r}^2 + r^2\dot{\theta}^2)$ D) $T= \dot{r}^2 + r^2\dot{\theta}^2$

2) Mathematical expression for D Alembert's Principle is ....

A) $\sum(\bar{F}_i - \bar{P}_i)\, \delta\bar{r}_i=0$ B) $\sum(\bar{F}_i - \bar{P}_i)\, \delta\bar{r}_i\neq0$

C) $\sum(\bar{F}_i - \dot{\bar{p}}_i)\, \delta\bar{r}_i=0$ D) $\sum(\bar{F}_i - \dot{\bar{p}}_i)\, \delta\bar{r}_i \neq0$

3) Expression for the Rayleigh's dissipation function is ....

A) $R=\Sigma\lambda_i(\dot{\bar{r}}_i)^2$ B) $R=2\Sigma\lambda_i(\dot{\bar{r}}_i)^2$

c) $R=\frac{1}{2}\Sigma\lambda_i(\dot{\bar{r}}_i)^2$ D) $R= \lambda_i(\dot{\bar{r}}_i)^2$

4) A body continues in it's state of rest or uniform motion, unless no external force is applied to it is ....

A) LAW OF INERTIA B) LAW OF FORCE

C) LAW OF ACTION AND REACTION D) NONE OF THESE

5) The time rate of change of momentum is proportional to impressed force is ....

A) LAW OF INERTIA B) LAW OF FORCE

C) LAW OF ACTION AND REACTION D) NONE OF THESE

Q.2) Attempt any three [15]

1) Explain how the generalized co-ordinates of a rigid body with N particles reduces to six for its description.
2) Explain Atwood machine and discuss it's motion.
3) Find the differential equation of the geodesic on the surface of an inverted cone with semi-vertical angle θ.
4) Obtain the Lagrangian L from the Hamiltonian H and show that it satisfies Lagrange's equations of motion.

Q.3) Attempt any One [10]

1) Define the following terms.

a) Holonomic Constraints
b) D'Alembert's Principle
c) Brachistochrone Problem
d) Hamiltonian Function

2) Find the expression for kinetic energy as the quadratic function of generalized velocities.Further show that

I) When the constraints are scleronomic, the kinetic energy is homogeneous function of generalized velocities and

$$\sum_j \dot{q}_j \frac{\partial T}{\partial \dot{q}_j} = 2T$$

II) When the constraints arerheonomic , then

$\sum_j \dot{q}_j \frac{\partial T}{\partial \dot{q}_j} = 2T_2 + T_1$; where$T_1$ and T_2 have usual meaning

Vivekanand College, Kolhapur (Autonomous)
M.Sc. II Semester-III Internal Examination: 2018-19
MATHEMATICS

Sub: Functional Analysis | **Time: 03:00pm -04:00pm**
Date: 08/10/2018 | **Total Marks:30**

Q.1 . Choose correct Alternative for the following. **(5)**

i) Consider following two statements;

I) Every normed linear space is a metric space.

II) Every metric space is normed linear space.

A) Only II is true. B) I is true and II is false

C) Only I is false D) II is true and I is false.

ii) Every projection on a Banach space B is___.

A) linear, bounded, idempotent B) linear, idempotent, continuous

C) linear, norm preserving, nilpotent D) Both A and B

iii) In Hilbert space every sequence is ____.

A) convergent B) not convergent C) oscillatory D) none of these

iv) If A and B are self-adjoint operators on H then their product AB is ___ if and only if___.

A) $A^2 = B^2$ B) $AB = BA$

C) $A = B$ D) $AB \neq BA$

v) Consider following two statements

I)Every Banach space is reflexive norm linear space

II)Every reflexive norm linear space is Banach Space

A) Only II is true. B) I is true and II is false C)Only I is false D) II is true and I is false

Q2) Solve any THREE of the following. (15)

1)If N' is Banach space then prove B(N , N') is Banach space with respect to norm

$\|T\| = \sup\{\|T(x)\|, x \text{ in } N \ \|x\|\leq 1\}$

2)If N is a normed linear space and x_0 is non zero vector in N then show that there exist a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$

3)Define Banach space. Show that l_∞ (space of all bounded sequences of scalars) which is normed linear space with $\|\ \|_\infty$ given by $\|x\|_\infty = \sup|x_i|$ for all x in l_∞ is banach space.

4)If N is a normed linear space and x_0 is non zero vector in N then show that there exist a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$

Q3) Solve any ONE of the following. (10)

1)Prove that a normed linear space N is finite dimensional if and only if $S = \{x \text{ in } N /\|x\|\leq 1\}$ is compact.

2)State and prove Hahn Banach theorem.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III
Internal Examination(2018-19)
Advanced Discrete Mathematics

Time:3:00PM–4:00PM **Total Marks: 30**
Date: 09/10/2018

Q.1) Choose the correct alternative for the following question. [05]

i) A k-cycle has ----- number of edges.

A) k B) $k-1$ C) $k+1$ D) $\frac{k}{2}$

ii) If G is connected graph, then G is tree iff every edge of G is -----

A) loop B) bridge C) not bridge D) none of these

iii) The order of recurrence relation $a_r - 4a_{r-2} + 3a_{r-3} = 5r + 2$ is -----

A) 0 B) 1 C) 2 D) 3

iv) For a bounded distributive lattice an element can have ----- complement if they exist.

A) only one B) exactly two C) more than two D) zero

v) The homogeneous solution to recurrence relation $a_r + 2a_{r-1} - 8a_{r-2} = 0$ is -----

A) $A_1(2)^r + B_1(4)^r$ B) $A_1(2)^r + B_1(-4)^r$

C) $A_1(-2)^r + B_1(4)^r$ D) $A_1(-2)^r + B_1(-4)^r$

Q.2) Attempt any three [15]

i) If G is a simple graph and G is not connected, then show that $\overline{G}$ is connected.

ii) Define regular graph. If G is a k-regular graph where 'k' is odd number, then prove that number of edges in G is multiple of 'k'.

iii) Prove that an edge e of graph G is a bridge iff e is not part of any cycle in G.

iv) If G is a graph with n-vertices, q-edges and w(G) number of connected components, then show that G has at-least n-w(G) number of edges.

Q.3) Attempt any One [10]

i) If G is a non-empty graph with at-least two vertices, then prove that G is bipartite iff G has no odd cycles.

ii) Define Radius and Diameter of graph. Prove that in any connected graph G,
$\text{rad}G \leq \text{diam}G \leq 2\,\text{rad}G$

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III Internal Examination: 2018-19
MATHEMATICS

Subject: Lattice Theory **Time: 03: 00 PM-04:00pm**
Date: 10/10/2028 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i. *Consider the following statements*
Statement − 1) *Every ideal is heriditory subset.*
Statement − 2) *Every heriditory subset is ideal.*
A) Only 1) *true* *B) Only* 2) *true* *C) Both* 1)&2) *true* *D) Both* 1)&2) *false.*

ii. *Consider the following statements*
Statement − 1) *Every ideal is heriditory subset.*
Statement − 2) *Every heriditory subset is ideal.*
A) Only 1) *true* *B) Only* 2) *true* *C) Both* 1)&2) *true* *D) Both* 1)&2) *false*

iii. *Consider the following statements*
Statement − 1) M_3 *is modular lattice.*
Statement − 2) *Every chain need not be modular lattice.*
A) Only 1) *true* *B) Only* 2) *true* *C) Both* 1)&2) *true* *D) none of these*

iv. *Which of the following is incorrect regarding lattice?*
A) $[\{1,2,3,6,9,18\} \,|]$ *is bounded lattice* *B)* $[\mathbb{Z} \ \leq]$ *is not bounded lattice*
C) $[(0,1) \ \leq]$ *is bounded lattice* *D)* $[[0,1] \leq]$ *is bounded lattice*

v. *Consider the following statements*
Statement − 1) $J(L)$ *is not ring of set.*
Statement − 2) $H(J(L))$ *is ring of set.*
A) Only 1) *true* *B) Only* 2) *true* *C) Both* 1)&2) *true* *D) none of these*

Q.2. Attempt any three of the following: [15]

1) *If* θ *is a congruence relation on lattice L then for every* $a \in L$ *prove that* $[a]_\theta$ *is convex sublattice of L.*

2) *Prove that the lattice L is distributive lattice iff* $\exists$*median* $\forall\, a, b, c \in L$.

3) *If* θ *be a congruence relation on lattice L then show that* $\forall\, a \in L, [a]_\theta$ *is convex sublattice of L.*

4) *Show that set of all ideals of lattice L forms a lattice under set inclusion.*

Q.3. Attempt any one of the following: [10]

1) *In any lattice L prove the following conditions always holds.*

i) $(x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z)$

ii) $x \vee (y \wedge z) \ \leq (x \vee y) \wedge (x \vee z) \quad \forall x, y, z \in L$

2) *State and prove Stones theorem.*

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III
Internal Examination(2018-19)
Number Theory

Time:03:00PM to 04:00PM **Total Marks: 30**

Date:11/10/2018

Q.1) Choose the correct alternative for the following question. **[05]**

1] If GCD of two number is 8 and LCM is 144, then what is the second number if first number is 72?

$a)$ 24 $b)$ 2

$c)$ 3 $d)$ 16

2] The linear combination of GCD (252, 198) is......

a) $252 * 4 - 198 * 5$ b) $252 * 5 - 198 * 4$

c) $252 * 4 + 198 * 5$ d) $252 * 5 + 198 * 54$

3] Square of any odd integer is of the form........

$a)$ $2k$ $b)$ $2k + 1$

$c)$ $8k + 1$ $d)$ $4k$

4] What is GCD of 48, 18, 0.......

$a)$ 24 $b)$ 3

$c)$ 2 $d)$ 6

5] The prime factorization of 7007 is ....

a) $7^3.11.13$ b) $7^2.11.13$

c) $11^3.7.13$ d) $13^3.7.13$

Q.2) Attempt any three **[15]**

1)Prove that the expression $\frac{a(a^2+2)}{3}$ is an integer for a>1.

2)Prove that for given integers a and b not both zero there exists integers x and y such that $\gcd(a, b) = ax + by$.

3)Prove that for any positive integer n and a, $\gcd(a, b)\ /n$ and hence prove that $\gcd(a, a + 1) = 1$

4)State and Prove Euclid's Lemma.

Q.3) Attempt any One **[10]**

1) State and Prove Division Algorithm.
2) Prove that for given integers a and b not both zero, $lcm(a, b) \times \gcd(a, b) = ab$.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III
Internal Examination(2018-19)
Operational Research-I

Time: 03:00PM to 04:00 PM **Total Marks: 30**
Date: 12/10/2018

Q.1) Choose the correct alternative for the following question. [05]

i) The set $x = \{(x_1, x_2): 2x_1 + 3x_2 = 7\}$ is …
A) a convex set B) concave set C) not a convex set D) none of these

ii) The point at which $\nabla f(x) = 0$ are called …
A) boundary points B) interior points C) extreme points D) convex point

iii) The solution of Dynamic Programming Problem is based upon…
A) Bellman's principle of calculus B) Principle of Optimality
C) Bellman's principle of optimality D) None of these

iv) The general NLPP with inequality constraints….
A) Can be solved by using Kuhn –Tucker conditions
B) Can be solved by Lagrange's method
C) Can be solved only if the constraints are of $\leq$ type

v) If an optimal solution is degenerate, then
A) it has an alternate optimal solution B) solution is infeasible
C) solution is unbounded D) None of these

Q.2) Attempt any three [15]

i) Define convex set. Show that the set S = $\{(x_1, x_2): 3x_1^2 + 2x_2^2 \leq 6\}$ *is convex set*

ii) Prove that the set of all convex combinations of a finite number of points of set

S= $\{x: x = \sum_{i=1}^{m} x_i\lambda_i, \lambda_i \geq 0, \sum_{i=1}^{m} \lambda_i = 1\}$ is a convex set

iii) Solve the following problem by dynamic programming Min $Z = x_1^2 + x_2^2 + x_3^2$

$subject\ to\ x_1 + x_2 + x_3 \geq 100,$
$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

iv) Prove that the set of feasible solution to an LPP is a convex set

Q.3) Attempt any One [10]

i) Prove that a basic feasible solution to an LPP must correspondence to an extreme point of the set of all feasible solution and conversely (Extreme point correspondence theorem)

ii) Explain the computational procedure of the simplex method in detail.

Date : 25/04/2019

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
M. Sc. I Sem II and M.Sc. II Sem IV
Internal Examination 2018-19

All the students of M.Sc. I and M.Sc. II are hereby informed that their Internal Examination of Mathematics will be conducted on **as given below timetable.** The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus and timetable for examination will be as mentioned in following table.

Syllabus for M. Sc. I Sem. II

Sr. No.	Name of Paper	Topics
1	Linear Algebra (CP-1175B)	Unit I
2	Measure And Integration (CP-1176B)	Unit I
3	General Topology (CP-1177B)	Unit I
4	Partial Differential Equations (CP-1178B)	Unit I
5	Numerical Analysis (CP-1179B)	Unit I

Syllabus for M. Sc. II Sem. IV

Sr. No.	Name of Paper	Topics
1	Field Theory (CP-1190D)	Unit I
2	Integral Equation (CP-1191D)	Unit I
3	Algebraic Number Theory (CP-1192D)	Unit I
4	Operational Research II(CP-1194D)	Unit I
5	~~Fractional Differential Equations (CP-1178B)~~ Combinatorics (CP-1198D)	Unit I

Timetable

Date	Time	Class	Subject
08/04/2019	03:00 PM to 04: 00 PM	M.Sc. I	Linear Algebra (CP-1175B)
	03:00 PM to 04: 00 PM	M.Sc. II	Field Theory (CP-1190D)
09/04/2019	03:00 PM to 04: 00 PM	M.Sc. I	Measure And Integration (CP-1176B)
	03:00 PM to 04: 00 PM	M.Sc. II	Integral Equation (CP-1191D)
10/04/2019	03:00 PM to 04: 00 PM	M.Sc. I	General Topology (CP-1177B)
	03:00 PM to 04: 00 PM	M.Sc. II	Algebraic Number Theory (CP-1192D)
11/04/2019	03:00 PM to 04: 00 PM	M.Sc. I	Partial Differential Equations (CP-1178B)
	03:00 PM to 04: 00 PM	M.Sc. II	Operational Research II(CP-1194D)
12/04/2019	03:00 PM to 04: 00 PM	M.Sc. I	Numerical Analysis (CP-1179B)
	03:00 PM to 04: 00 PM	M.Sc. II	Combinatorics (CP-1198D)

Nature of question paper

Time:-1 Hours **Total Marks: 30**

Q.1) Choose the correct alternative for the following question. [05]

Five questions

Q.2) Attempt any three [15]

Four questions

Q.3) Attempt any One [10]

Two questions

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(Prof. S. P. Patankar)
HEAD
Department of Mathematics
Vivekanand College, Kolhapur

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II Internal Examination: 2018-19
MATHEMATICS

Subject: Linear Algebra
Date: 08/04/2019

Time: 03: 00 PM-04:00pm
Total Marks: 30

Q. 1 Select the correct alternative for each of the following: **[5]**

i) Let V denote the vector space of $n \times n$ Skew symmetric matrices, over R. Then dim V as a vector space over R is

A) n^2 B) $(n^2 + n)/2$ C) $n^2 + n$ D) $(n^2 - n)/2$

ii) If S is subset of V, A(S) =

A) A(L(S)) B) (L(S)) C) A(S) D) S

iii) T1 , T2 and T3 are three maps defined on R^3 ,

$T1\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}x+1\\y\\z\end{bmatrix}$, $T2\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}xy\\z\end{bmatrix}$, $T3\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}x+y\\y+z\\z+x\end{bmatrix}$ as Which of these maps are linear ?

A) T1 , T2 , T3 B) T1 , T2 C) T3 D) T1 , T3

iv) dim(Hom(R,R)(R)) =

A) 2 B) 1 C) 4 D) None

v] If T is a linear operator on $\mathbb{R}^2$ defined by $T(x_1, x_2) = (0, 0)$ then rank of T= ___ .

B) i) 3 ii) 0
C) iii) 2 iv) 1

Q.2. Attempt any three of the following: **[15]**

1. V is finite dimensional vector space of V then $A(A(W)) \approx W$
2. Show that A (A(S)) is subspace of $\hat{V}$ for any subset S of V .
3. Show that A(S) = A(L(S)) for any non empty subset of V.
4. Let W_1 , W_2, be subspaces of V which is finite dimensional. Describe $A(W_1 + W_2)$ in terms of $A(W_1)$ and $A(W_2)$
5. Let W_1 , W_2, be subspaces of V which is finite dimensional. Describe $A(W_1 \cap W_2)$ in terms of $A(W_1)$ and $A(W_2)$

Q.3. Attempt any one of the following: **[10]**

1.If T is homomorphism from U onto V with kernel W , Then V is isomorphic to U\W
Conversely, If U is vector space and W is subspace of U then there is homomorphism of U onto U\W

2.If V is internal direct sum of U 1 , U 2 ,…U n ,Then prove that V is isomorphic to external direct sum of U 1 , U 2 ,…U n

Vivekanand College, Kolhapur (Autonomous)

M.Sc. (Part-I) Semester-II Internal Examination :2018-19

Subject :Measure and Integration
Date: 09/04/2019
Time: 03: 00 PM
Total Marks: 30

Q. 1 Select the correct alternative for each of the following: **[5]**

i. For $1 < p < \infty, q$ the conjugate of p, & any two positive numbers a & b

A) $ab \geq \frac{a^p}{p} + \frac{b^q}{q}$ B) $ab = \frac{a^p}{p} + \frac{b^q}{q}$

C) $ab > \frac{a^p}{p} + \frac{b^q}{q}$ D) $ab \leq \frac{a^p}{p} + \frac{b^q}{q}$

ii. If $A_n = \left(\frac{-1}{n+1}, \frac{1}{n+1}\right)$ then $\bigcap_{n=1}^{\infty} A_n$ is ________

iii. A set F is F_σ set if it is ______

a) Countable union of open sets b) Countable intersection of open sets

c) Countable union of closed sets d) Countable intersection of closed sets

iv. Let $f(x) = |x|, x \in [-1,1]$ then________

a) $D^+f(0) = 1$ b) $D^-f(0) = 0$

c) $D^+f(0) = 0$ d) $D^-f(0) = 1$

v. If $A_n = \left(\frac{-1}{n+1}, \frac{1}{n+1}\right)$ then $\bigcap_{n=1}^{\infty} A_n$ is ________

a) 1 b) 0 c) ∞ d) $\frac{2}{n}$

Q.2. Attempt any three of the following: **[15]**

1) Prove that $L^1(E)$ is normed linear space

2) If a function f is measurable then prove that the set $\{x|f(x) = c\}$ is measurable. for all c in $\mathbb{R}$

3) If E_1 is measurable set & $m^*(E_1 \Delta E_2) = 0$ then show that E_2 is measurable.

4) If a function f is measurable then prove that the set $\{x|f(x) = c\}$ is measurable. for all c in $\mathbb{R}$.

Q.3. Attempt any one of the following: **[10]**

1) State & prove Holders inequality.

2) State & prove Lebesgue convergence theorem.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II
Internal Examination(2018-19)

Subject: General Topology **Total Marks: 30**
Date: 10/04/2019 **Time:03:00 PM to 04:00 PM**

Q.1) Choose the correct alternative for the following question. [05]

1) Out of the following ....defines a topology on X = {a, b}
   a) $\{\emptyset, \{a\}, \{b\}\}$ b) {X, {a}, {b}} c) $\{\emptyset, \{a\}\}$ d) $\{\emptyset, X\}$
2) In a topology, every open set can be expressed as.................
   a) union of some member of subbases
   b) intersection of bases
   c) union of intersection of some member of subbases
   d) intersection of union of some member of subbase
3) Limit point of a subset of $\{1, \frac{1}{2}, \frac{1}{3}, ...\}$ of R is.......
   a) 1 b) ∞ c) 0 d) 2
4) A is closed if and only if ....
   a) $\overline{A} = A$ b) $\overline{A} = \emptyset$ c) $\overline{A} \subset A$ d) none of these
5) Boundary point set of a set of integer Z is
   a) N b) Z c) R d) Q

Q.2) Attempt any three **[15]**

1) If X be an infinite set and $\tau = \{\emptyset\} \cup \{A \subseteq X | A^C \text{ is countable}\}$ then show that τ is topology on X.
2) Define the following terms:
   a) Limit point b) closure of set c) interior set d) neighbourhood
3) Show that A ∪ D(A) is closed set
4) Let (X, τ) be a topological space and A, B$\subseteq X$, then show that
   a) $D(A \cup B) = D(A) \cup D(B)$
   b) if A $\subseteq B$, then D(A) $\subseteq D(B)$

Q.3) Attempt any One **[10**

1) Prove that let X be any non – empty set and B be family of some subset of τ. Then B is base for τ on X if and only if
   a) $X = \cup \{B_i : B_i \in B\}$
   b) $\forall B_1, B_2 \in B, \forall x \in B_1 \cap B_2 \; \exists B_3 \in B$ such that $x \in B_3 \subseteq B_1 \cap B_2$
2) Consider the topology τ = { ∅, {a}, {b, c}, X} on X ={a, b, c} and V = { ∅, {r}, {p}, {q}, Y} on Y = { p, q, r}.
   Which of the following mapping is a) continuous b) open c) closed d)homeomorphism
   i) $f(a) \to r, f(b) \to r, f(c) \to r$
   ii) $g(a) \to p, g(b) \to q, g(c) \to r$
   iii) $h(a) \to r, h(b) \to p, h(c) \to q$

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II
Internal Examination(2018-19)
Partial Differential Equations

Time: 03:00 PM to 04:00 PM **Total Marks: 30**
Date :11/04/2019

Q.1) Choose the correct alternative for the following question. [05]

1) I. *The normals to the two surfaces represented by the equations* $Pdx+Qdy+Rdz=0$ $\&Pp+Qq=R$ *are…*
   a) collinear b) Orthogonal c) Parellel d) intersects at acute angle
2) *The equation* $Ruxx+Suxy+Tuyy+g=0$ *is parabolic if…*
   a) $S^2-4RT<0$ b) $S^2-4RT>0$ c) $S^2-4RT=0$ d) None of these
3) *The complete integral of* $z=px+qy+\sqrt{pq}$ *is*
   a) $z=a+b+ab$ b) $z=ax+by+\sqrt{pq}$ c) $z=c$ d) none of these
4) *Which of the following is not a Pffafian differential equation*
   a) $tdx+xdt=0$ b) $xdx+ydy-zdz=0$ c) $xdx+zdy=0$ d) $xdx+ydy+zdz=0$
5) *The complete integral of* $z=px+qy+pq$ *is*
   a) $z=a+b+ab$ b) $z=ax+by+ab$ c) $z=c$ d) none of these

Q.2) Attempt any three [15]

1) *Find the general solution of* $z(xp-yq)=y^2-x^2$.

2) *Form partial differential equation from* $z^2(1+a^3)=8(x+ay+b)^3$

3) *Find the general solution of* $p+q=2\sqrt{z}$.

4) *Form partial differential equation from the relation* $F(x-y, x-\sqrt{z})=0$

Q.3) Attempt any One [10]

1) *Solve Pfaffian differntial equation*

$$(6x+yz)dx+(xz-2y)dy+(xy+2z)dz=0$$

2) *If* $\overline{X}=(P,Q,R)$ *is a vector such that* $\overline{X}.\text{ curl }\overline{X}=0$ & μ *is an arbitrary differentiable of* x, y, z *then prove that* $\mu\overline{X}.\text{ curl }\mu\overline{X}=0$

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II
Internal Examination(2018-19)

Subject : Numerical Analysis **Total Marks: 30**

Date: 12/04/2019 **Time: 03:00PM to 04:00PM**

Q.1) Choose the correct alternative for the following question. **[05]**

1) The solution of system of equations $x_1 + x_2 + x_3 = 1$, $4x_1 + 3x_2 - x_3 = 6$, $3x_1 + 5x_2 + 3x_3 = 4$ by using Crout's method are ----

A) $x_1 = 1$, $x_2 = \frac{3}{2}$, $x_3 = \frac{1}{2}$ B) $x_1 = 1$, $x_2 = \frac{1}{2}$, $x_3 = \frac{-1}{2}$

C) $x_1 = \frac{1}{2}$, $x_2 = 1$, $x_3 = \frac{-1}{2}$ D) $x_1 = \frac{-1}{2}$, $x_2 = \frac{1}{2}$, $x_3 = 1$

2) The Jacobi 's method is a method of solving matrix equation on a matrix that has no zeros along --------

A) Leading diagonal
B) Last column
C) Last row
D) Non – leading diagonal

3) The quadratic factor of polynomial $x^4 + x^3 + 2x^2 + x + 1$, where $p_0 = 0.5$ and $q_0 = 0.5$ is ------

A) $x^2 + 2x + 9$ B) $x^2 + (1.0375)x + (0.5563)$

C) $x^2 + (0.5563)x + (1.0375)$ D) $x^2 + (1.9413)x + (1.9542)$

4) In Bairstow method Δp = ----------

A) $\Delta p = \frac{b_n c_{n-2} + b_{n-1}(b_{n-1} - c_{n-1})}{c^2_{n-2} + c_{n-3}(b_{n-1} - c_{n-1})}$ B) $\Delta p = \frac{b_{n-2} c_n + b_{n-1}(b_n - c_n)}{c^2_{n-2} + c_{n-3}(b_n - c_n)}$

C) $\Delta p = \frac{b_n c_n + b_{n-1}(b_{n-2} - c_{n-2})}{c^2_n + c_{n-3}(b_{n-1} - c_{n-1})}$ D) $\Delta p = \frac{b_{n-1} c_{n-2} - b_n c_{n-3}}{c^2_{n-2} + c_{n-3}(b_{n-1} - c_{n-1})}$

5) . An iterative method is said to be of order p , if p is largest positive real number for which there exist a finite constant $C \neq 0$ such that ------

A) $|\epsilon_{k+1}| \leq c\,|\epsilon_k|^p$ B) $|\epsilon_k| \leq c\,|\epsilon_{k+1}|^p$

C) $|\epsilon_k| \leq c\,|\epsilon_k|^p$ D) $|\epsilon_{k+1}| \leq c\,|\epsilon_k|^{p-1}$

Q.2) Attempt any three **[15]**

1) Determine the rate of convergence of Regula Falsi method.
2) Derive Gauss Legendre integration method for n=1
3) Estimate the eigen value of the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ by using Gerschgorin theorem and Brauer theorem.
4) Explain General form of linear multistep method

Q.3) Attempt any One **[10]**

1) State and prove Brauer theorem.
2) Derive Trapezoidal rule by using Newton cotes formula. Find error term also.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV Internal Examination: 2018-19
MATHEMATICS

Subject : Field Theory **Time: 03: 00 PM-04:00pm**
Date:08/04/2019 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i. e and π are .... elements over Q

A) Transcendental B) Algebraic C) Irreducible D) Reducible

ii. Polynomial of degree one is always ......

A) Inseparable B) Separable C) Monic D) Simple

iii. If $f(x)$ is of degree 3 , Then $f(x)$ has .... Root.

A) Complex B) Unique C) Distinct D) Real

iv. Any subgroup and any quotient group of a .... group is solvable.

A)Solvable B) Normal C) Separable D) None

v. If F ⊆ K ⊆ L are fields. If a ∈ L be algebraic over K and K is an algebraic extension of F. Then, a is ....

A) Algebraic over K B) Algebraic Over F C) Algebraic D) Separable

Q.2. Attempt any three of the following: **[15]**

1.If F ⊆ K be fields , $p(x)$ a monic irreducible polynomial over F and a and b ∈ K be roots of p(x). Then prove that , there exists an isomorphism j : F(a) → F(b) such that

j(a) = b and j (s) = s for all s ∈ F.

2.Prove that an algebraic extension E of F is finitely generated over F if and only iff E is a finite extension of F.

3. Find the smallest extension of Q having a root of $x^4 - 2 \in Q[x]$
4. If F ⊆ E be fields such that [E : F] is prime number. Prove that there are no fields property between F and E.

Q.3. Attempt any one of the following: **[10]**

1.If F ⊆ K ⊆ L are field and [L : F] is finite , Then prove that [L : K] and [K : F] are divisors of [L : F]

2.A. Let F ⊆ K be field and a ∈ K be algebraic over F. Then prove that there exist a unique monic irreducible polynomial p(x) over F such that p(a) = 0. Also ,for any f(x) ∈ F[x] , f(a) = 0 if and only if p(x) divides f(x) in F[x]

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV
Internal Examination : 2018-19

Sub: Integral Equations **Date : 09/04/2019**

Total Marks: 30 **Time : 03:00pm-04:00pm**

Q.1) Choose the correct alternative for the following question. [05]

1) The type of integral equation $g(s) = f(s) + \lambda \int K(s,t) g(t) dt$ b a is -----
   a) Volterra integral equation of 1st kind
   b) Fredholm integral equation of 1st kind
   c) Homogeneous Fredholm integral equation of 2nd kind
   d) Non-homogeneous Fredholm integral equation of 2nd kind
2) The homogeneous Fredholm integral equation has trivial solution, if -----
   a) $D(\lambda) = 0$ b) $D(\lambda) \neq 0$ c) $D(\lambda)$ does not exist d) none of these
3) The eigen values of non-zero symmetric kernel are -----
   a) real b) zero c) only imaginary d) none of these
4) If $\{\phi k\}$ is orthonormal set, then $< \phi i , \phi i >$ = ----- for all i.
   a) 0 b) 1 c) -1 d) ∞
5) A symmetric kernel possesses ----- eigen value.
   a) only one b) at-least one c) at-most one d) none of these

Q.2) Attempt any three [15]

1) Convert the following initial value problem to an integral equation.
   $y'' + y = cosx, y(0) = 0, y'(0) = -1$
2) Prove that eigen functions $g(s)$ and $\psi(s)$ corresponding to distinct eigen values λ_1 and λ_2 respectively of the homogeneous integral equation $g(s) = \lambda \int K(s,t) g(t) dt$ and its transpose are orthogonal.
3) Convert the following boundary value problem to an integral equation. $y'' + xy = 1$, $y(0) = 0, y(1) = 1, 0 \leq x \leq 1$
4) Find the eigen values and eigen functions of the homogeneous integral equation
   $g(s) = \lambda \int_0^1 (6s - 2t) g(t) dt$

Q.3) Attempt any One [10]

1) Describe the procedure of finding eigen values and eigen functions for the homogeneous Fredholm integral equation of 2nd kind with separable kernel.
2) Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 (1 - 3st) g(t) dt$ by discussing all possible cases.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV
Internal Examination(2018-19)
Algebraic Number Theory

Time: 3:00PM–4:00PM
Date:10/04/2019

Total Marks: 30

Q.1) Choose the correct alternative for the following question. **[05]**

1)The element $a \neq 0 \in R$, the commutative ring is an integral domain if-

(a) $ab = 0, b \in R$ and $b = 0$ (b) $ab = 0$, $b \in R$ and $b \neq 0$

(c) $ab \neq 0$, $b \in R$ and $b = 0$ (d) $ab \neq 0$, $b \in R$ and $b \neq 0$

2)E is set of integers under ordinary addition and multiplication, then E is a ring, E is also a ......

(a) Commutative ring (b) Integral domain

(c) Both (a) and (b) (d) None of these

3)The homomorphism ϕ of rings R into R is an isomorphism iff the kernel $I(\phi)$ is ......

(a) $I(\phi) = \{0\}$ (b) $I(\phi) = R$ (c) $I(\phi) = R$ (d) None of these

4)Let p and q are non-zero polynomials in $R[t]$ such that R is an integral domain then

(a)$\partial pq = \partial p \partial q$ (b) $\partial pq = \partial p - \partial q$ (c) $\partial pq = \partial p + \partial q$ (d) $\partial pq = \partial p(\partial q)^2$

5)Let R be a ring And X and Y are ideals of R. Which of the following is true?

(a) $X + Y$ is ideal (b) $X - Y$ is ideal

(c) Both (a) and (b) is true (d) None of the above

Q.2) Attempt any three **[15]**

1) Let $M = M_1 \oplus M_2$ then prove that $\frac{M}{M_1} \cong M_2$ and $\frac{M}{M_2} \cong M_1$
2) With usual notations prove that The Field polynomial f_α is a power of minimal polynomial p_α
3) Show that the ring of integer β is a subring of the field of algebraic number $\mathbb{A}$.
4) Show that the coefficient of the field polynomial are rational numbers so that

$f_\alpha(t) \in Q(t)$.

Q.3) Attempt any One **[10]**

1) Show that a complex number θ is an algebraic integer if and only if the additive group generated by all powers $1, \theta, \theta^2$, ... is finitely generated.

2) Let d be a square free rational integer then the integer if $Q(\sqrt{d})$ are

a) $Z(\sqrt{d})$ if $d \not\equiv 1(mod 4)$

b) $Z\left(\frac{1}{2} + \frac{1}{2}\sqrt{d}\right)$ if $d \equiv 1(mod 4)$.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV
Internal Examination(2018-19)
Operational Research-II

Time:03:00PM to 04:00 PM **Total Marks: 30**
Date : 11/04/2019

Q.1) Choose the correct alternative for the following question. [05]

i) In Retrogressive failure, probability of failure .... with the increase in the life of an item.

A) increases B) Remains constant C) Decreases D) None of these

ii) The problem of replacement is not concerned about the.....

A) item that deteriorate graphically

B) items that fail suddenly

C) determination of optimum replacement interval

D) maintenance of an item to work out profitably

iii) In dummy activity in a project network always has a .... duration.

A) one B) two C) zero D) Three

iv) In model I(b), minimum average inventory cost is...........

A) $\sqrt{\frac{2C_1C_3q}{T}}$ B) $\sqrt{\frac{2C_1C_3D}{T}}$ C) $\sqrt{2C_1C_3D}$ D) None of these

v) .... occurs when a waiting customer leaves the queue due to impatience.

A) Reneging B) Balking C) Jockeying D) None of these

Q.2) Attempt any three [15]

1) Explain the types of inventory model?

2) An aircraft uses rivets at an approximately constant rate of 5000 kg per year. The rivets cost Rs. 20 per kg and the company personnel estimate that it costs Rs. 200 to place an order, and the carrying cost of inventory is 10% per year

a) How frequently should be ordered for rivets be placed and what quantities should be ordered for?

b) If the actual costs are Rs. 500 to place an order and 15% for carrying cost, the optimum policy would change. How much is the company losing per year because of imperfect cost information?

3) In each of the following cases, stock is replenishment instantaneously and no

shortage allowed. Find the economic lot size, the associated total cost and length of time between two orders

a) $C_3 = Rs. 100, C_1 = Rs\ 0.05, R = 30\ units/$ year

b) $C_3 = Rs. 100, C_1 = Rs\ 0.01, R = 40\ units/$ year

c) $C_3 = Rs. 100, C_1 = Rs\ 0.04\ R = 20\ units/$ year

4) Define the following terms

a) Activity b) Dummy activity c) critical path

d) critical activity e) event

Q.3) Attempt any One **[10]**

i) Discuss the policy of replacement of items whose maintenance cost increases with time but the value of money remains constant.

ii) Show that the distribution of the number of births up to time T in a simple birth process follows the Poisson law

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV Internal Examination:2018-19
MATHEMATICS

Subject: Combinatorics **Time: 03: 00 PM-04:00 pm**

Date: 12/4/2019 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

I. *The weight of permutation* $(1,3,2,4) \in S_4$ *is*

$a)$ 2 $b)$ 4 $c)$ 6 $d)$ 8

II. *The number of derangements of* $(1,2,3)$ *is/are* _____

$a)$ 1 $b)$ 2 $c)$ 3 $d)$ *none of these*

III. *The number of circular permutation of* 6 *objects is*

$a)$ 120 $b)$ 24 $c)$ 6 $d)$ 5

IV. *The Ramsey Number* $R(3,3) =$ ______

$a)$ 6 $b)$ 5 $c)$ 4 $d)$ 0

V. *The weight of permutation* $(1,3,2,4) \in S_4$ *is*

$a)$ 2 $b)$ 4 $c)$ 6 $d)$ 8

Q.2. Attempt any three of the following: **[15]**

1) *Compute the number of square free integers not exceeding the given integer'n'.*
2) *Find a cycle index of dihedral group on symmetries of square*
3) *In how many ways'n'boys & 'n' girls be seated in row of* 2*n chairs if the two sexes must be alternate.*
4) *There are n married couples at a party each person shakes hands other than her or his spouse . Find the total number of handshakes.*

Q.3. Attempt any one of the following: **[10]**

1) *How many integers between* 1 & 1000 (*inclusive*) *are not divisible by* 2,3,5 *or* 7?
2) *Find a cycle index of dihedral group on symmetries of square*

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36593

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor:

Suppliment No. :

Roll No. : 1203

Class : MSc I

Subject : Numerical Analysis

Test / Tutorial No. : Internal Exam

Div. : –

$\frac{29}{30}$

Q1.

1) d) $\Delta p = \frac{b_{n-1} c_{n-2} - b_n c_{n-3}}{c^2_{n-2} + c_{n-3}(b_{n-1} - c_{n-1})}$

2) c) 2

3) a) $|\epsilon_{k+1}| \leq C.|\epsilon_k|^p$.

5

4) c) $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

5) d) Intermediate value theorem.

Q2.

1) Given equation is,

$$F(x) = \cos x - x \cdot e^{x} = 0$$

Initial approximations are $x_0 = 0$ and $x_1 = 1$

$f(x_0) = f(0) = 1$, $\quad f(x_1) = f(1) = -2.1780$

$\therefore f(x_0) \cdot f(x_1) < 0$

$\therefore$ Root lies betn $(0, 1)$

By secant method we have,

$$x_{k+1} = x_k - \frac{(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})} \cdot f(x_k)$$

$$= \frac{x_k f(x_k) - x_k \cdot f(x_{k-1}) - x_k \cdot f(x_k) + x_{k-1} \cdot f(x_k)}{f(x_k) - f(x_{k-1})}$$

$$x_{k+1} = \frac{x_{k-1} \cdot f(x_k) - x_k \cdot f(x_{k-1})}{f(x_k) - f(x_{k-1})}$$

i) For $k = 1$

$$x_2 = \frac{x_0 \cdot f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{(0) \cdot (-2.1780) - (1)(1)}{-2.1780 - 1}$$

$$= \frac{-1}{-3.1780}$$

$$= 0.3147.$$

$\therefore f(x_2) = 0.5198$

ii) For $k = 2$

~~$x_3 = x_2 \cdot f(x_1) - x_1 f(x$~~

$$x_3 = \frac{x_1 \cdot f(x_2) - x_2 \cdot f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{(1)(0.5198) - (0.3147)(-2.1780)}{0.5198 + 2.1780}$$

$$= \frac{1.2052}{2.6978} = 0.4467$$

$x_3 = 0.4467$.

$f(x_3) = 0.2036$

iii) For $k = 3$

$$x_4 = \frac{x_2 \cdot f(x_3) - x_3 \cdot f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{(0.3147).(0.2036) - (0.4467)(0.5198)}{0.2036 - 0.5198}$$

$$= \frac{-0.1681}{-0.3162}$$

$$= 0.5316$$

$f(x_4) = -0.0426$

iv) For $k = 4$

$$x_5 = \frac{x_3 \cdot f(x_4) - x_4 \, f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{(0.4467)(-0.0426) - (0.5316)(0.2036)}{-0.0426 - 0.2036}$$

$$= \frac{-0.0190 - 0.1082}{-0.2462}$$

$x_4 = 0.5167$.

$\therefore$ Approximate solⁿ upto four iterations is 0.5167.

2) Given equation is Polynomial is

$P_4(x) = x^4 + x^3 + 2x^2 + x + 1$

Initial approximations are

$p_0 = 0.5$ and $q_0 = 0.5$.

Now 1st iteration is.

$-p_0 = -0.5$	1	1	2	1	1
$-q_0 = -0.5$		-0.5	-0.25	-0.625	0.125
			-0.5	-0.25	-0.625
$-p_0 = -0.5$	1	0.5	1.25	0.125	$0.5 = b_4$
$-q_0 = -0.5$		-0.5	0	-0.375	
			-0.5	0	
	1	0	0.75	$-0.25 = c_3$	

Now,

$$\Delta P = \frac{-(b_n c_{n-3} - b_{n-1} c_{n-2})}{c_{n-2}^2 - c_{n-3}(c_{n-1} - b_{n-1})}$$

$$= \frac{-(b_4 c_1 - b_3 c_2)}{c_2^2 - c_1(c_3 - b_3)}$$

$$= -\frac{[(0.5)(0) - (0.125)(0.75)]}{(0.75)^2 - (0)(-0.25 - 0.125)}$$

$$= \frac{0.0938}{0.5625}$$

$$= 0.1668$$

$$\therefore P_1 = \Delta P + P_0$$

$$= 0.1668 + 0.5$$

$$P_1 = 0.6668.$$

$$\Delta q = \frac{-(b_{n-1}(c_{n-1} - b_{n-1}) - b_n \cdot c_{n-2})}{c_{n-2}^2 - c_{n-3}(c_{n-1} - b_{n-1})}$$

$$= \frac{-(b_3(c_3 - b_3) - b_4 c_2)}{c_2^2 - c_1(c_3 - b_3)}$$

$$= -\frac{(0.125(-0.25 - 0.125) - 0.5 \cdot (0.75)]}{(0.75)^2 - 0(c_3 - b_3)}$$

$$= \frac{0.0469 + 0.3750}{0.5625}$$

$$\Delta q = 0.75.$$

$$\therefore q_1 = \Delta q_0 + q_0$$

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SUPPLIMENT

Signature of Supervisor	

Suppliment No. : 1

Roll No. : 1203

Class : MSc - I

Subject : N A.

Test / Tutorial No. :

Div. :

$q_1 = 0.75 + 0.5$

$q_1 = 1.25.$

4\. Let,

$x = 17^{1/3}$

$x^3 = 17$

$\therefore F(x) = x^3 - 17$

$\therefore F'(x) = 3x^2$ $\qquad x_0 = 2.$

By Newton Raphson method

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$= x_k - \frac{x_k^3 - 17}{3x_k^2}$$

$$= \frac{3x_k^3 - x_k^3 + 17}{3x_k^2}$$

$$= \frac{2x_k^3 - 17}{3x_k^2}$$

For $k = 0$

$$x_1 = \frac{2x_0^3 + 17}{3x_0^2}$$

$$= \frac{2(2)^3 + 17}{3(2)^2}$$

$$= \frac{33}{12}$$

$$x_1 = -0.0833 \quad 2.75$$

For $k = 1$

$$x_2 = \frac{2x_1^3 + 17}{3(x_1)^2}$$

$$= \frac{2(2.75)^3 + 17}{2(2.75)^2}$$

$$= \frac{58.5938}{15.1250}$$

$$x_2 = 3.8740$$

For $k = 2$

$$x_3 = \frac{2x_2^3 + 17}{3(x_2)^2}$$

$$= \frac{2(3.8740)^3 + 17}{3(3.8740)^2}$$

$$= \frac{133.2810}{45.0236}$$

ok

$$= 2.9602$$

For $k = 3$

$$x_4 = \frac{2x_3^3 + 17}{3(x_3)^2}$$

$$= \frac{2(2.9602)^3 + 17}{3(2.9602)^2}$$

$$= \frac{68.8792}{26.2884}$$

$$x_4 = 2.6201$$

∴ Approximate solution is 2.6201.

Q3.

1) We assume that simple root of $f(x)=0$ is ξ.

$\therefore f'(\xi) \neq 0$

Now Substituting $x_k = \varepsilon_k + \xi$ in secant method

$$x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} \cdot f(x_k)$$

$$\varepsilon_{k+1} + \xi = \varepsilon_k + \xi - \frac{(\varepsilon_k + \xi) - (\varepsilon_{k-1} + \xi)}{f(\varepsilon_k + \xi) - f(\varepsilon_{k-1} + \xi)} \cdot f(\varepsilon_k + \xi)$$

$$\varepsilon_{k+1} = \varepsilon_k - \frac{(\varepsilon_k - \varepsilon_{k-1}) \cdot f(\varepsilon_k + \xi)}{f(\varepsilon_k + \xi) - f(\varepsilon_{k-1} + \xi)} \quad \text{--- (1)}$$

$$f(\varepsilon_k + \xi) = f(\xi) + \varepsilon_k \cdot f'(\xi) + \frac{\varepsilon_k^2}{2!} f''(\xi) + \cdots$$

[Tailors series]

as $f(\xi) = 0$

$$f(\varepsilon_k + \xi) = \varepsilon_k \cdot f'(\xi) + \frac{\varepsilon_k^2}{2!} f''(\xi) + \cdots$$

Equation (1) becomes,

$$\varepsilon_{k+1} = \varepsilon_k - \frac{(\varepsilon_k - \varepsilon_{k-1})\left[\varepsilon_k \cdot f'(\xi) + \frac{\varepsilon_k^2}{2!} \cdot f''(\xi) + \cdots\right]}{\left[\varepsilon_k \cdot f'(\xi) + \frac{\varepsilon_k^2}{2!} f''(\xi) + \cdots\right] - \left[\varepsilon_{k-1} f'(\xi) + \frac{\varepsilon_{k-1}^2}{2!} f''(\xi) + \cdots\right]}$$

$$= \varepsilon_k - \frac{(\varepsilon_k - \varepsilon_{k-1})\left[\varepsilon_k \cdot f'(\xi) + \frac{\varepsilon_k^2}{2!} f''(\xi) + \cdots\right]}{f'(\xi)\left[\varepsilon_k - \varepsilon_{k-1}\right] + \left[\frac{\varepsilon_k^2}{2!} - \frac{\varepsilon_{k-1}^2}{2!}\right] f''(\xi)}$$

$$= \varepsilon_k - \frac{(\varepsilon_k - \varepsilon_{k-1})\left[\varepsilon_k f'(\xi) + \frac{\varepsilon_k^2}{2!} f''(\xi) + \cdots\right]}{(\varepsilon_k - \varepsilon_{k-1}) \cdot f'(\xi)\left[1 + \frac{\varepsilon_k + \varepsilon_{k-1}}{2} \frac{f''(\xi)}{f'(\xi)}\right]}$$

$$= \varepsilon_k - \frac{\left[\varepsilon_k + \frac{\varepsilon_k^2}{2!}\frac{f''(\xi)}{f'(\xi)} + \cdots\right]}{\left[1 + \frac{\varepsilon_k + \varepsilon_{k-1}}{2} \cdot \frac{f''(\xi)}{f'(\xi)} + \cdots\right]}$$

$$= \varepsilon_k - \left[\varepsilon_k + \frac{\varepsilon_k}{2}\frac{f''(\xi)}{f'(\xi)} + \cdots\right]\left[1 + \frac{\varepsilon_k + \varepsilon_{k-1}}{2} \cdot \frac{f''(\xi)}{f'(\xi)}\right]^{-1}$$

$$\doteq \varepsilon_k - \left[\varepsilon_k + \frac{\varepsilon_k}{2}\frac{f''(\xi)}{f'(\xi)} + \cdots\right]\left[1 - \frac{\varepsilon_k + \varepsilon_{k-1}}{2}\frac{f''(\xi)}{f'(\xi)} + \cdots\right]$$

$$= \varepsilon_k - \varepsilon_k + \varepsilon_k\left(\frac{\varepsilon_k + \varepsilon_{k-1}}{2}\right) \cdot \frac{f''(\xi)}{f'(\xi)} + \frac{\varepsilon_k}{2}\frac{f''(\xi)}{f'(\xi)} + O(\varepsilon_{k-1} \cdot \varepsilon_k + \varepsilon_k)$$

$$= -\frac{\varepsilon_k^2}{2} \cdot \frac{f''(\xi)}{f'(\xi)} + \frac{\varepsilon_k(\varepsilon_{k-1})}{2}\frac{f''(\xi)}{f'(\xi)} + \frac{\varepsilon_k^2}{2}\frac{f''(\xi)}{f'(\xi)} + O(\varepsilon_{k-1} \cdot \varepsilon_k + \varepsilon_k)$$

$$= \frac{\varepsilon_k \cdot \varepsilon_{k-1}}{2} \cdot \frac{f''(\xi)}{f'(\xi)} + O(\varepsilon_{k-1} \cdot \varepsilon_k + \varepsilon_k)$$

$$\varepsilon_{k+1} = \varepsilon_k \cdot \varepsilon_{k-1} \cdot \frac{1}{2}\frac{f''(\xi)}{f'(\xi)} + O(\varepsilon_{k-1} \cdot \varepsilon_k + \varepsilon_k)$$

$$\therefore \varepsilon_{k+1} = \varepsilon_k \cdot \varepsilon_{k-1} \cdot c \quad \text{--- (2)}$$

where, $c = \frac{1}{2}\frac{f''(\xi)}{f'(\xi)}$

eqn (2) is called erroror eqn.

To determine rate of convergence of Secant method

By defn of Rate of convergence

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VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor	
Subject :	
Test / Tutorial No. :	
Div. :	

Suppliment No. : 2

Roll No. : 1203

Class : MSc I (Maths)

$\varepsilon_{k+1} = A \cdot \varepsilon_k^{p}$ ——— ③

By eqn ③

$\varepsilon_k = A \cdot \varepsilon_{k-1}^{p}$

$\varepsilon_k^{1/p} = A^{1/p} \cdot \varepsilon_{k-1}$

$\varepsilon_{k-1} = \varepsilon_k^{1/p} \cdot A^{-1/p}$

Eqn ② becomes

$\varepsilon_{k+1} = C \cdot \varepsilon_k \cdot \varepsilon_{k-1}$

$A \cdot \varepsilon_k^{p} = C \cdot \varepsilon_k \cdot A^{-1/p} \cdot \varepsilon_k^{1/p}$

$A \cdot \varepsilon_k^{p} = A^{-1/p} \cdot C \;\; \varepsilon_k^{\frac{p+1}{p}}$

10

Comparing powers of ε_k

$p = \frac{p+1}{p}$

$p^2 - p - 1 = 0$

$p = \frac{1 \pm \sqrt{1+4}}{2}$

$= \frac{1 \pm \sqrt{5}}{2}$

The largest value of p is

$p = \frac{1 + \sqrt{5}}{2}$ ——— ④

$\simeq 1.6108.$

$c \cdot A^{-1/p} = 1$

$\therefore \quad A = c^{p/1+p}$

eqn (4) is called Rate of convergence of secant method.

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SUPPLIMENT

Signature of Supervisor

Suppliment No. : 1

Roll No. : 2219

Class : M.Sc II

26/30

Subject : Lattice theory

Test / Tutorial No. :

Div. :

Q.1.

1) False
In antichain no element is comparable
∴ antichain is not lattice.

2) ~~False~~ True
Every lattice is poset but converse need not be t
every semi lattice is lattice. ∴ Poset is semi lattice

3) ~~False~~ True
~~4a is lattice~~ 4a is lattice, so it is also sublattice

4) True
$a \cap (a \vee (a \cap \underbrace{\{c \vee b\}}_{b})) = a \cap (a) = a$

5) True
length of chain is element in chain minus 1.

03

Q.2.

1) Let A be any set.

Let $P(A)$ be the power set of A.

Clearly, $P(A)$ is non empty set.

Let, $B, C \in P(A)$ be any element in $P(A)$

Claim - $P(A)$ is lattice

Case 1] - If $B \leq C$, then

$\sup\{B,C\} = B \cup C = C$ exist

$\inf\{B,C\} = B \cap C = B$ exist

In this case $\sup\{B,C\}$ & $\inf\{B,C\}$ exist

$\therefore$ $P(A)$ is lattice.

Case 2] - If $C \leq B$ then

$\sup\{B,C\} = B$ exists

$\inf\{B,C\} = C$ exist

In this case $P(A)$ is lattice

Case 3] . If $B \nleq C$, $C \nleq B$ then

i) ~~Su~~ Let $B \cap C \neq \phi$, $\exists\ a \in B \cap C$

$\therefore$ $\sup\{B,C\} = B \cup C$

$\inf\{B,C\} = a$

ii) If $B \cap C = \phi$, then

$\sup\{B,C\} = B \cup C$

$\inf\{B,C\} = \phi$

$\therefore$ Power set of any set is lattice under set inclusion.

2) Let $< P, \leq >$ be a poset satisfying ACC.
let $x_0 \in P$ be any element
If x_0 is maximal element the we are done
If x_0 is not maximal element then $\exists\ x_1 \in P$
s.t. $x_0 \leq x_1$
If x_1 is maximal element then we are done
if not then $\exists\ x_2 \in P$, s.t. $x_0 \leq x_1 \leq x_2$
Continuing this process we get increasing chain satisfying ACC
i.e. $x_0 \leq x_1 \leq x_2 \cdots \leq x_n \leq x_{n+1} \leq \cdots$
This chain satisfying ACC then it must be terminates
$\exists\ i \in \mathbb{N}$ s.t. $x_i = x_{i+1} = \cdots$
As no element in this chain is greater than x_i
i.e. x_i covers all the element. then x_i is maximal element then we are done.
If not then $\exists\ y_0 \in P$, $y_0 \neq x_i$, s.t.
$x_i \leq y_0$
If y_0 is maximal element then we are done
If not then $\exists\ y_1$, s.t $x_i \leq y_0 \leq y_1$
Continuing this way we get increasing chain satisfying ACC
i.e. $x_i \leq y_0 \leq y_1 \leq \cdots y_n \leq y_{n+1} \leq \cdots$
This chain satisfying ACC then it must be terminates, $\exists\ j \in \mathbb{N}$, s.t.
$y_j = y_{j+1} = \cdots$
If y_j is cover all the element then it is upper bound.
Do same process for all possible chain.
every chain is satisfying ACC has upper bound
by Zorn's lemma, It has maximal element.

3) Let I be non empty subset of L

$\phi \neq I \subseteq L$

~~claim~~ - Let $a, b \in L$, & $a \vee b \in I$

We know,

$a \leq a \vee b$

$\Rightarrow a = a \wedge (a \vee b) \in I$

$\Rightarrow a \in I$ — ①

As, $b \leq a \vee b$

$b = b \wedge (a \vee b) \in I$

$\Rightarrow b \in I$ — ②

$\therefore a, b \in I$ from ① & ②

Conversly, suppose $a, b \in L$, $a \vee b \in I \Rightarrow a, b \in I$

We know,

$a = a \vee (a \wedge b) \in I$

$\Rightarrow a \wedge b \in I$

$\therefore I$ is sublattice of L

Let $l \in L$, $x \in I$ be any element

~~$l \wedge x \in I$~~ $x \leq a \wedge x$

$\Rightarrow$ ~~$x \notin$~~ $\Rightarrow a \wedge x \in I$

$\Rightarrow I$ is lattice of L

05

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Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor	

Suppliment No. : 2
Roll No. : 2219
Class : M.Sc II

Subject : LT
Test / Tutorial No. :
Div. :

Q.3.

1) Let $\langle L, \leq \rangle$ be lattice.
$\Rightarrow$ by def^n of lattice sup ~~∃~~ & inf ~~∃~~ exist
Let $\phi \neq H \subseteq L$
~~by~~ We use method of induction on no. of elemen in H
If $H = \{a\}$ then
Sup H = inf $H = a$ exist
If $H = \{a, b\}$ then
Sup$\{a,b\}$ & inf $\{a,b\}$ exists
If $H = \{a,b,c\}$ then
Sup $\{a,b\} = k$ (say)
& Sup $\{k, c\} = t$ (say)

08 claim - Sup $H = t$
As $a \leq t$, $b \leq t$, $c \leq t$
then t is upper ~~of~~ bound of H
08 Let t' be any upper bound of H
$\Rightarrow a \leq t'$, $b \leq t'$, $c \leq t'$
As ~~t~~ k is upper bound of a, b ~~xxx~~ but
t' is upper bound of H

$\Rightarrow k \leq t'$

As t' is upper bound of H but t is upper bound of k, c

$\Rightarrow t \leq t'$

$\Rightarrow \text{Sup } H = t$

Now, suppose Sup H exist

let $H = \{a_1, a_2, ---, a_k\}$

let $H = \{a_1, a_2, ---, a_k, a_{k+1}\}$

by hypothesis $\sup\{a_k, a_{k+1}\}$ exist

$\text{Sup } H = \{a_1, a_2, ---, a_{k-1}, \text{Sup}\{a_k, a_{k+1}\}\}$ exist

by hypothesis sup H exists

by duality principle inf H also exist.

$\therefore$ Sup H & inf H exist for any non empty subset H of L

conversly, Suppose Sup H & inf H exist for any non empty subset H of L

In particular

$H = \{a, b\}$

$\text{Sup}\{a, b\}$ & $\inf\{a, b\}$ exist.

As a, b are any arbitrary element in L

$\therefore$ $\langle L, \leq \rangle$ is lattice

Q.2.

4) Let L be lattice & θ be congruence relation on L

$\frac{L}{\theta} = \{ [a]\theta \mid a \in L \}$

We define join & meet by,

i) $[a]\theta \wedge [b]\theta = [a \wedge b]\theta$

ii) $[a]\theta \vee [b]\theta = [a \vee b]\theta$

Claim :- L/θ is lattice

1) Idempotent property

Let $[b]\theta \in L$

$[b]\theta \wedge [b]\theta = [b \wedge b]\theta$

$= [b]\theta$

$[b]\theta \vee [b]\theta = [b \vee b]\theta$

$= [b]\theta$

It satisfies idempotent property

ii) Associativity property

Let $[a]\theta, [b]\theta \in L$ be any element

$[a]\theta \wedge [b]\theta = [a \wedge b]\theta$

$= [b \wedge a]\theta$

$= [b]\theta \wedge [a]\theta$

$[a]\theta \vee [b]\theta = [a \vee b]\theta$

$= [b \vee a]\theta$

$= [b]\theta \vee [a]\theta$

It satisfies associativity

05

iii) Commutative property

let $[a]\theta, [b]\theta, [c]\theta \in L$ be any element

$[a]_\theta \wedge \{[b]_\theta \wedge [c]_\theta\} = [a]_\theta \wedge \{[b \wedge c]_\theta\}$

$= [a \wedge (b \wedge c)]_\theta$

$= [(a \wedge b) \wedge c]_\theta$

$= [a \wedge b]_\theta \wedge [c]_\theta$

$= \{[a]_\theta \wedge [b]_\theta\} \wedge [c]_\theta$

11^{ry} we prove.

$[a]_\theta \vee \{[b]_\theta \vee [c]_\theta\} = \{[a]_\theta \vee [b]_\theta\} \vee [c]_\theta$

It satisfies commutivity

iv) ~~Ap~~ Absorption property

let $[a]_\theta, [b]_\theta \in L$ be any element

$[a]_\theta \wedge \{[a]_\theta \vee [b]_\theta\} = [a]_\theta \wedge \{[a \vee b]_\theta\}$

$= \{a \wedge [a \vee b]\}_\theta$

$= \{(a \wedge a) \vee b\}_\theta$

$= [a]_\theta$

11^{ry} we prove

$[a]_\theta \vee \{[a]_\theta \wedge [b]_\theta\} = [a]_\theta$

by i, ii, iii, iv

$\therefore L_\theta$ is lattice