Date : 02/09/2019

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
M. Sc. I Sem. I and M.Sc. II Sem III
Internal Examination 2019-20

All the students of M.Sc. I and M.Sc. II are hereby informed that their Internal Examination of Mathematics will be conducted on **as given below timetable.** The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus and timetable for examination will be as mentioned in following table.

Timetable

Date and Time	Time	Class	Subject
16/09/2019	03:00 PM to 04: 00 PM	M.Sc. I	Algebra
	03:00 PM to 04: 00 PM	M.Sc. II	Functional Analysis
17/09/2019	03:00 PM to 04: 00 PM	M.Sc. I	Advanced Calculus
	03:00 PM to 04: 00 PM	M.Sc. II	Advanced Discrete Mathematics
18/09/2019	03:00 PM to 04: 00 PM	M.Sc. I	Complex analysis
	03:00 PM to 04: 00 PM	M.Sc. II	Lattice Theory
19/09/2019	03:00 PM to 04: 00 PM	M.Sc. I	Ordinary Differential Equation
	03:00 PM to 04: 00 PM	M.Sc. II	Number theory
20/09/2019	03:00 PM to 04: 00 PM	M.Sc. I	Classical Mechanics
	03:00 PM to 04: 00 PM	M.Sc. II	Operational Research -I

Syllabus for M. Sc. I Sem. I

Sr. No.	Name of Paper	Topics
1	CP-1170A : Algebra	Unit I
2	CP-1171A: Advanced Calculus	Unit I
3	CP-1172A: Complex analysis	Unit I
4	CP-1173A: Ordinary Differential Equation	Unit I
5	CP-1174A: Classical Mechanics	Unit I

Syllabus for M. Sc. II Sem. III

Sr. No.	Name of Paper	Topics
1	CP-1180C: Functional Analysis	Unit I
2	CP-1181C: Advanced Discrete Mathematics	Unit I
3	CP-1182C : Number theory	Unit I
4	CP-1184C : Operational Research -I	Unit I
5	CP-1185C :Lattice Theory	Unit I

Nature of question paper

Time:-1 Hours **Total Marks: 30**

Q.1) Choose the correct alternative for the following question. [05]

Five questions

Q.2) Attempt any three [15]

Four questions

Q.3) Attempt any One [10]

Two questions

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Vivekanand College, Kolhapur (Autonomous)

M.Sc. I Semester-I Internal Examination: 2019-20

MATHEMATICS

Sub: Algebra (CP-1170A) **Time: 03:00PM -04:00PM**

Date:16/09/2019 **Total Marks:30**

Q1) Select the correct alternatives **(5)**

1] i) Every permutation is one - one function. ii) Every one-one function is permutation.

a) (ii) is true. b) (i) is true. c) both statement are true. d) both statement are false.

2] *How many proper subgroups does the group* $\mathbb{Z}\oplus\mathbb{Z}$ *have*?

$a)$ 1 $b)$ 2 $c)$ 3 $d)$ *infinitely many.*

3] *In a group of order* 15 *the number of subgroups of order* 3 *is* ...

$a)$ 3 $b)$ 5 $c)$ 1 $d)$ 2

4] with usual notation which is true?

a) $\{\sigma_0\}$, $\{\sigma_0, \sigma_1, \sigma_2\}$ are not subgroup of S_3.

b) $\{\sigma_0, \mu_1\}$, $\{\sigma_0, \mu_2\}$,$\{\sigma_0, \mu_3\}$ ae only subgroups of S_3.

c) $\{\sigma_0, \sigma_1, \sigma_2\}$,$\{\sigma_0, \mu_1\}$ are subgroup of S_3.

d) S_3 has no subgroup of order 3

5] Order of A_5 is…

a) 60 b) 120 c) 5 d) 5!

Q2) Solve any THREE of the following. **(15)**

1] Prove that every permutation σ of a finite set A is a product of disjoint cycles.

2] Define commutator subgroup of group G. Show that G is abelian if and only if commutator subgroup is {e}.

3] Show that for $n \geq 3$, Subgroup generated by 3 – cycle of A_n is A_n.

4] Define index of subgroup . Find index of A_n in S_n . Show that A_n is normal in S_n.

Q3) Solve any ONE of the following. **(10)**

1] If H and K are subgroups of G and H* and K* normal subgroups of H and K respectively, Then show t that i) $H^*(H \cap K^*)$ is normal subgroup of $H^*(H \cap K)$

ii) $K^*(H^* \cap K)$ is normal subgroup of $K^*(H \cap K)$

iii) $H^*(H \cap K)/H^*(H \cap K^*) \cong K^*(H \cap K)/ K^*(H^* \cap K) \cong (H \cap K)/ [(H^* \cap K)(H \cap K^*)]$

2] State and prove Caley's theorem.

Vivekanand College, Kolhapur (Autonomous)

M.Sc. (Part-I) Semester-I Internal Examination: 2019-20

Subject : Advanced Calculus **Date and Time: 17/09/2019 03:00pm onwards**

Total Marks: 30

Q. 1 Select the correct alternative for each of the following: [5]

i. *he series* $\sum_{n=1}^{\infty} a_n sin(nx)$ *converges uniformly on* $\mathbb{R}$ *if*

A) $\sum_{n=1}^{\infty} a_n$ *converges* B) $\sum_{n=1}^{\infty} |a_n|$ *converges*

C) $\sum_{n=1}^{\infty} sin(nx)$ *converges* D) $\sum_{n=1}^{\infty} |sin(nx)|$ *converges.*

ii. *Radius of convergence for the seris* $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$ *is* ________

A) 2 B) 1/2

C) 1 D) *series always diverges.*

iii. *If* $\bar{f}$ *is linear then* $\bar{f}'(\bar{c};\bar{u}) =$ ________

A) 0 B) $\bar{f}(\bar{u})$ C) $\bar{f}(\bar{c})$ D) $\bar{f}'(\bar{u})$

iv. *Stokes theorem relates a surface integral to*

A) *Volume integral* B) *Line integral*

C) *Vector integral* D) *Real integral*

v. If α_1v1+α_2v2+ …+$\alpha_n$vn=0, where v1, v2, … ,vn are linearly independent vectors in a vector space V(F), then ________.

i) i=0 for all i=1,2, …, n ii) i≠0 for all i=1,2, …, n

iii) i=0 for at least one i iv) i≠0 for at least one i

Q.2. Attempt any three of the following: [15]

1) *State & prove Green's theorem for plane region bounded by piecewise smooth Jordon curve.*

2) *Evaluate* $\iint e^{\frac{y-x}{y+x}} dxdy$ *over triangle bounded by lines* $x + y = 2$ *& two cordinate axes x & y.*

3) Let $\bar{f}$ *be a vector field given by* $\bar{f}(x,y) = \sqrt{y}i + (x^3 + y)j$ *where* $(x,y) \in \mathbb{R}^2$. $y \geq 0$ *obtain the integral of* $\bar{f}$ *from* $(0,0)$ *to* $(1,1)$ *along the path* $\alpha(t) = ti + tj$

4) *Let f be a double sequence.&* $\lim_{p,q\to\infty} f(p,q) = a$. *Assume that* $\lim_{q\to\infty} f(p,q)$ *exist for each fixed integer p. Prove that the iterated limit,* $\lim_{p\to\infty}(\lim_{q\to\infty} f(p,q))$ *also exist &has same value a.*

Q.3. Attempt any one of the following: [10]

1) *If* $\sum_n a_n(z - z_0)^n$ coverges for each $z \in B(z_0; r)$. Prove that the function

$$f(z) = \sum_{n=0}^{\infty} a_n(z - z_0)^n \text{ has derivative } f'(z) = \sum_{n=1}^{\infty} na_n(z - z_0)^{n-1}$$

& series for $f'(z)$ *has same radius of convergence r.* **[10]**

2) *If* $\{M_n\}$ *be a sequence of non − negative real numbers such that* $0 \leq |f_n(x)| \leq M_n$ $\forall n \in \mathbb{N}$ & $\forall x \in S$. *Prove that* $\sum f_n$ *converges uniformely on S if* $\sum M_n$ *converges.*

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-I
Internal Examination : 2019-20
Department of mathematics

Subject : Complex Analysis **Date:18/09/2019**
Time: 03: 00 PM-04:00pm **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i) If $f(z) = x^2y^2 + i2xy$ and $g(z) = 2xy + i(x^2 - y^2)\ \forall z \in c$, then in the complex plane C.

A) f is analytic and g is not analytic B) f is not analytic and g is analytic
C) f is analytic and g is analytic D) f is not analytic and g is not

ii) If C is the circle |z - a | = r then $\int_C \frac{dz}{(z-a)^n} = 2\pi i,$ when ……

A) n =1 B) n = 0 C) n≠ 1 D) None of these

iii In the Laurent series expansion f(z) = $\frac{1}{z(z-1)}$ valid for |z-1| >1, the coefficient of $\frac{1}{z-1}$ is….

A) 1 B) 0 C) -1 D) 6

iv) If f(z) is analytic for |z| < R and satisfies the condition |f(z)| $\leq M$ in |z|< R and f (0) = 0 then |f(z)| ≤……

A) $\frac{M}{R}|z|$ B) $\frac{M}{R}|z|^2$ C) $\frac{1}{R}|z|$ D) $\frac{M}{R}$

v) The radius of convergence of $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$ is…..

A) -e B) 1/e C) e D)1/e

Q.2. Attempt any three of the following: **[15]**

1) Find radius of convergence of f(z) = $\sum_{n=1}^{\infty} a_n z^n$
2) Prove that mobius transformation preserves the cross ratio.
3) If $u(x,y) = x^3 + ax^2y + bxy^2 + 2y^3$ is harmonic function and $v(x,y)$ its harmonic conjugate.
4) If γ is a contour with parameter interval [a, b] and $f(z) = u(x,y) + iv(x,y)$ is continuous function on the contour γ with $|f(z)| \leq M, \forall z \in \gamma$, then prove that $|\int_C f(z)dz| \leq ML$ where L is the length of contour given by $\int_a^b |\gamma'(t)|dt$

Q.3. Attempt any one of the following: **[10]**

1) If f : D → C is analytic then Prove that

a) If |f| is constant then f is constant

b) If Arg(f) is constant then f is constant

2) Prove that to each power series $\sum a_n z^n$ there exists a corresponding R with $0 \leq R < \infty$ called the radius of convergence with the following properties

i) $\sum a_n z^n$ converges absolutely for every z with |z| < R.

ii) If | z| > R, the terms of power series become unbounded and so the series diverges.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-I
Internal Examination(2019-20)
Ordinary Differential Equations

Subject: Ordinary Differential Equations **Total Marks: 30**
Date: 19/09/2019 **Time: 03:00PM to 04:00PM**

Q.1) Choose the correct alternative for the following question. [05]

i)If $\emptyset_1$ and $\emptyset_2$ are two solutions of $L(y) = 0$ then ..... is also solution of $L(y) = 0$ where c_1 and c_2 are any two constants.

A) $c_1\emptyset_1 + c_2\emptyset_2$ B) $c_1\emptyset_1 - c_2\emptyset_2$ C)Both A and B D) None of these

ii)The functions $\emptyset_1(x) = \cos x$, $\emptyset_2(x) = \sin x$ are .... on interval $-\infty \leq x \leq \infty$

A)Linearly Dependent B) Linearly Independent

C)Both A and B D)None of these

iii) In Legendary equation If α is a non-negative even integer, then $\emptyset_1$ is polynomial of degree 'n' containing only ..... powers of x.

A) Odd B)Even

C)Odd and zero D)Even and zero

iv) If $f(x,y) = xy^2$, $R = \{(x,y)||x| \leq 1, |y| < \infty\}$ and K is Lipschitz constant then ......

A) F satisfies Lipschitz Condition on R with $k = 2b$

B) F satisfies Lipschitz Condition on R with $k = 0$

C) F satisfies Lipschitz Condition on R with $k = 1$

D) F do not satisfy Lipschitz Condition on R

v) Which of the following is not solution of $y''' - 3r_1y'' + 3{r_1}^2y' - {r_1}^3y = 0$, where r_1 is constant

A)$\emptyset(x) = e^{r_1x}$ B) $\emptyset(x) = x^2e^{r_1x}$

C) $\emptyset(x) = xe^{r_1x}$ D) $\emptyset(x) = x^3e^{r_1x}$

Q.2) Attempt any three [15]

i) Suppose $\emptyset_1(x)$ and $\emptyset_2(x)$ are linearly independent solution of the constant coefficient equation $L(y) = y'' + a_1y' + a_2y = 0$, and if $W(\emptyset_1, \emptyset_2)$ is denoted by W than show that, W is constant iff $a_1 = 0$

ii) Compute the equation $y''' - 4y' = 0$

a) Compute the linearly independent solutions.

b)Compute the Wronskian of the solutions.

c)Find the solution $\emptyset$ satisfying $\emptyset(0) = 0$, $\emptyset'(0) = 2, \emptyset''(0) = 3$

iii) Classify the singular points in the finite plane $x^2(x^2 - 4)y'' + 2x^3y' + 3y = 0$

iv) Show that $\emptyset(x) = \frac{d^n}{dx^n}[(x^2 - 1)^n]$ satisfies the Legendre equation hence show that $\emptyset(1) = 2^n n!$

Q.3) Attempt any One **[10]**

i) If $b(x)$ be the continuous function on an interval I every solution φ of $L(y) = b(x)$ on I can be written as $\varphi = \varphi_p + c_1\emptyset_1 + c_2\emptyset_2$ where φ_p is particular solution and $\emptyset_1(x)$ and $\emptyset_2(x)$ are linearly independent solutions of $L(y) = 0$ with c_1 and c_2 are constants, then show that particular solution φ_p is given by

$$\varphi_p = \int_{x_0}^{x} \frac{\emptyset_1(t)\,\emptyset_2(x) - \emptyset_2(t)\emptyset_1(x)}{W(\emptyset_1, \emptyset_2)(x)} b(t)dt$$

Conversely, every such solution φ is a solution of $L(y) = b(x)$

ii) Find all the solutions of equation $y'' + 4y = \cos x$

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-I
Internal Examination(2019-20)
Classical Mechanics

Time: 3:00PM–4:00PM **Total Marks: 30**
Date:20/09/2019

Q.1) Choose the correct alternative for the following question. [05]

1) Lagrangian is defined as ....

A) L=T-V B) L=TV
C) L=T/V D) L=T+V

2) Equation of constraints that does not contain time as explicit variable are referred as ....

A) HOLONOMIC CONSTRAINTS B) NON HOLONOMIC CONSTRAINTS
C) RHEONOMIC CONSTRAINTS D) SCLERONOMIC CONSTRAINTS

3) Which of the following represents the spherical co-ordinates?

A) (x, y, z) B) (r, θ, φ)
C) (r, θ, z) D) NONE OF THESE

4) If the work done by forces on the particle around a closed path in the force field is zero then ....

A) THE FORCES ARE CONSERVATIVE B) THE FORCES ARE NON CONSERVATIVE
C) BOTH OPTION 1 AND 2 D) NONE OF THESE

5) Limitations imposed on the motion of a system are referred as ......

A) CONSTRAINTS B) FRAME OF REFERENCE
C) DEGREE OF FREEDOM D) NONE OF THESE

Q.2) Attempt any three [15]

1)Show that curve is Catenary for which the area of the surface of revolution is minimum when revolved about Y- axis.

2) Find the extremal of the functional $\int_0^{\frac{\pi}{2}}(y'^2 - y^2 + 2xy)dx$ subject to the conditions that $y(0) = 0, y\left(\frac{\pi}{2}\right) = 0$.

3) Use Hamilton's Principle to find the equation of motion of a simple pendulum.

4) Find the plain curve of fixed perimeter that encloses maximum area.

Q.3) Attempt any One [10]

1)If the cyclic generalized co-ordinate q_j is such that dq_j represents the translation of the system, then prove that the total linear momentum is conserved.

2)State and prove Basic lemma of variational calculus

Vivekanand College, Kolhapur (Autonomous)
M.Sc. II Semester-III Internal Examination2019-20
MATHEMATICS

Sub: Functional Analysis **Time: 03: 00PM -04:00 PM**
Total Marks:30 **Date: 16/09/2019**

Q.1 . Choose correct Alternative for the following. **(5)**

i) Consider following two statements;

I) Every normed linear space is a metric space.

II) Every metric space is normed linear space.

A) Only II is true. B) I is true and II is false

C) Only I is false D) II is true and I is false.

ii) Every projection on a Banach space B is___.

A) linear, bounded, idempotent B) linear, idempotent, continuous

C) linear, norm preserving, nilpotent D) Both A and B

iii) In Hilbert space every sequence is ____.

A) convergent B) not convergent C) oscillatory D) none of these

iv) If A and B are self-adjoint operators on H then their product AB is ___ if and only if___.

A) $A^2 = B^2$ B) AB = BA

C) A= B D) $AB \neq BA$

v) Consider following two statements

I)Every Banach space is reflexive norm linear space

II)Every reflexive norm linear space is Banach Space

A) Only II is true. B) I is true and II is false C)Only I is false D) II is true and I is false

Q2) Solve any THREE of the following. **(15)**

1)If N' is Banach space then prove B(N , N') is Banach space with respect to norm

$||T|| = \sup\{||T(x)||, x \text{ in } N\ ||x|| \leq 1\}$

2)If N is a normed linear space and x_0 is non zero vector in N then show that there exist a functional f_0 in N^* such that $f_0(x_0) = ||x_0||$ and $||f_0|| = 1$

3)Prove that, Two norms $|| \ ||_1$ and $|| \ ||_2$ on a linear space N are equivalent if and only if there are positive constants K_1 and K_2 such that, $K_1 ||x||_1 \leq ||x||_2 \leq K_2 ||x||_1$ for all x in N. [5]

4)If $\{T_n\}$ and $\{S_n\}$ are sequences in B(N) such that $T_n \to T$ and $S_n \to S$ as $n \to \infty$ then show that,

a) $T_n + S_n \to T + S$ b) $kT_n \to kT$ for k in F c) $T_nS_n \to TS$ as $n \to \infty$ [5].

Q3) Solve any ONE of the following. **(10)**

1)Define normed linear space. If N and N' are normed linear spaces ,T is linear transformation from N into N' then show that following conditions are equivalent

a)T is continuous on N

b)T is continuous at origin

c)there exist a real number $k \geq 0$ with property $||T(x)|| \leq k||x||$ for all x in N

d)If s = {x in N such that $||x|| \leq 1$ } is closed unit sphere in N then T(S) is bounded in N'

2)State and prove Hahn Banach theorem.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III
Internal Examination(2019-20)
Advanced Discrete Mathematics

Time:3:00PM–4:00PM **Total Marks: 30**

Date: 17/09/2019

Q.1) Choose the correct alternative for the following question. [05]

i) Complete bipartite graph $K_{n,n}$ is ----- regular graph.

A) n B) $n-1$ C) $n+1$ D) $n-2$

ii) If tree T has n vertices, then T has exactly ----- number of edges.

A) n B) $n-1$ C) $n+1$ D) $\frac{n}{2}$

iii) The homogeneous solution to recurrence relation $a_r + 2a_{r-1} - 8a_{r-2} = 0$ is -----

A) $A_1(2)^r + B_1(4)^r$ B) $A_1(2)^r + B_1(-4)^r$

C) $A_1(-2)^r + B_1(4)^r$ D) $A_1(-2)^r + B_1(-4)^r$

iv) For a Boolean algebra B, if $a + b = 0$, then -----

A) $a = 0, b = 0$ B) $a = 0, b \neq 0$ C) $a \neq 0, b = 0$ D) $a \neq 0, b \neq 0$

v) If G is connected graph, then G is tree iff every edge of G is -----

A) loop B) bridge C) not bridge D) none of these

Q.2) Attempt any three [15]

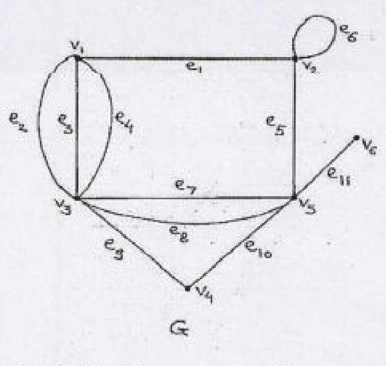
i) Prove that in group of 'n' people there are two persons having same number of friends.
ii) Solve the recurrence relation $a_r - 4a_{r-1} + 4a_{r-2} = 0$ with $a_0 = 1, a_1 = 1$
iii) State and prove De-Morgan's law in Boolean algebra.
iv) Define Boolean algebra. For a Boolean algebra B, show that

a) $a * (a + b) = a$

b) If $a + x = 1$ and $a * x = 0$, then $x = a'$

Q.3) Attempt any One [10]

i) Find the graphs G-U, G-F, G[U], G[F] where U = $\{v_1, v_2\}$ and F = $\{e_1, e_2, e_5, e_{11}\}$



ii) Among the integers 1 to 300, find how many integers are not divisible by 3 nor by 5. Also find how many are divisible by 3 but not by 7.

Vivekanand College, Kolhapur (Autonomous)

M.Sc. (Part-II) Semester-III Internal Examination: 2019-20

MATHEMATICS

Subject: Lattice Theory **Time: 03: 00 PM-04:00pm**

Date: 18/09/2019 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i. *Consider the following statements*

Statement – 1) M_3 *is modular lattice.*

Statement – 2) *Every chain need not be modular lattice.*

A) *Only* 1) *true* B) *Only* 2) *true* C) *Both* 1)&2) *true* D) *none of these*

ii. *Which of the following is incorrect regarding lattice?*

A) $[\{1,2,3,6,9,18\} \mid]$ *is bounded lattice* B) $[\mathbb{Z} \ \leq]$ *is not bounded lattice*

C) $[(0,1) \ \leq]$ *is bounded lattice* D) $[[0,1] \leq]$ *is bounded lattice*

iii. *Let* $P = \{2,3,6,12,24\}$ &

$x \leq y$ *iff* $x|y$ *then number of edges in Hasse diagram of* $(P, \leq)$ *is*______

A) 3 B) 4 C) 5 D) *none of these*

iv. *Select the incorrect statement.*

A) M_3 *has only one antichain.* B) M_3 *has more than one antichain.*

C) *Longest antichain of* M_3 *has three elements.* D) *none of these.*

v. *Consider the following statements*
Statement – 1) *Every ideal is heriditory subset.*
Statement – 2) *Every heriditory subset is ideal.*
A) *Only* 1) *true* B) *Only* 2) *true* C) *Both* 1)&2) *true* D) *Both* 1)&2) *false.*

Q.2. Attempt any three of the following: [15]

1) *A poset satisfies DCC then prove that it has minimal element.*

2) *If the algebra* $< L, \wedge, \vee >$ *be a lattice &* $a \leq b$ *iff* $a = a \wedge b$ *then prove that*

$< L, \leq >$ *is a poset & as a poset it is a lattice.*

3) *If* θ *is a congruence relation on lattice L then for every*

$a \subset L$ *prove that* $[a]_\theta$ *is convex sublattice of L.*

4) *Prove that the lattice L is distributive lattice iff* $\exists$*median* $\forall\, a, b, c \in L$.

Q.3. Attempt any one of the following: [10]

1) *State and prove Stones theorem.*

2) *If L is finite distributive lattice then prove that the map*

$f: L \to H(J(L))$ *is an isomorphism.*

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III
Internal Examination(2019-20)
Number Theory

Time: 03:00PM to 04:00PM **Total Marks: 30**
Date:19/09/2019

Q.1) Choose the correct alternative for the following question. [05]

1)The prime factorization of 7007 is ....

a) $7^3.11.13$ b) $7^2.11.13$
c) $11^3.7.13$ d) $13^3.7.13$

2) The least common multiple of $41 * 42$ and $42 * 41$ is......

$a)$ 41 $b)$ 42 $c)$ 84 $d)$ $41 * 42$

3) The sum of two positive integers is100. If one is divided by 7 the remainder is1, and if the other is divided by 9 the remainder is 7. Then the numbers are....

$a)$ 43, 57 $b)$ 57, 43 $c)$ 42, 58 $d)$ 58, 42

4) If a is odd then $gcd\ (3a, 3a + 2)\ = ...$

$a)$ 1 $b)$ 2 $c)$ 3 $d)$ 4

5) Applying Euclidian algorithm find the particular solution of $112x + 70y\ =\ 168.$

$a)$ 36, 24 $b)$ 24, −36 $c)$ − 24, −36 $d)$ 24, 36

6) Remainder obtained upon dividing $1! + 2! + \cdots + 100!$ by 12 is

$a)$ 3 $b)$ 5 $c)$ 7 $d)$ 9

Q.2) Attempt any three **[15]**

1) By using mathematical induction prove that $15/2^{4n-1}$.
2) State and Prove Euclid's theorem.
3) Prove that for any positive integer n and a, $\gcd(a, b)\ /n$ and hence prove that $\gcd(a, a + 1) = 1$
4) Solve the linear Diophantine equation $54x + 21y = 906$.

Q.3) Attempt any One **[10]**

1) Prove that the linear Diophantine equation $ax + by = c$ has a solution iff d/c where $d = \gcd(a, b)$. If $(x_0, y_0)$is any particular solution of this equation then all other solutions are given by $x = x_0 + \frac{b}{d}t$ and $y = y_0 - \frac{a}{d}t$.

2) Prove that for given integers a and b not both zero, $lcm(a, b) \times \gcd(a, b) = ab$.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III
Internal Examination(2019-20)
Operational Research-I

Time: 03:00PM to 04:00 PM **Total Marks: 30**
Date: 20/09/2019

Q.1) Choose the correct alternative for the following question. [05]

i) Which of the following is correct?

A) An extreme point is boundary point of set

B) An extreme point cannot be between any other two point of set

C) Both A and B

D)None of these

ii) If an optimal solution is degenerate, then

A) it has an alternate optimal solution B) solution is infeasible

C) solution is unbounded D) None of these

iii) The solution of Dynamic Programming Problem is based upon…

A) Bellman's principle of calculus B) Principle of Optimality

C) Bellman's principle of optimality D) None of these

iv) The general NLPP with equality constraints….

A) Can be solved by using Kuhn –Tucker conditions

B) Can be solved by Lagrange's method

C) Can be solved only if the constraints are of $\leq$ type

D) Are usually solved by simplex method

v) The solution of Dynamic Programming Problem is based upon…

A) Bellman's principle of calculus B) Principle of Optimality

C) Bellman's principle of optimality D) None of these

Q.2) Attempt any three **[15]**

i) Rewrite the following LPP in standard form

1) Min Z = $2x_1 + x_2 + 4x_3$,

$subject\ to\ -2x_1 + 4x_2 \leq 4,\ x_1 + 2x_2 + x_3 \geq 5,\ 2x_1 + 3x_3 \leq 2,$

$x_1 \geq 0, x_2 \geq 0, x_3\ unrestricted\ in\ sign$

2) Min Z = $x_1 - 2x_2 + x_3$,

$subject\ to\ 2x_1 + 3x_2 + 4x_3 \geq -4,\ 3x_1 + 5x_2 + 2x_3 \geq 5,$

$x_1 \geq 0, x_2 \geq 0, x_3\ unrestricted\ in\ sign$

ii) Solve the following problem by dynamic programming Min $Z = x_1x_2x_3$

$subject\ to\ x_1 + x_2 + x_3 = 100,$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

iii) Prove that the set of all convex combinations of a finite number of points of set

S= $\{x: x = \sum_{i=1}^{m} x_i\lambda_i,\ \lambda_i \geq 0, \sum_{i=1}^{m} \lambda_i = 1\}$ is a convex set

iv) Define convex set. Show that the set S = $\{(x_1, x_2): 3x_1^2 + 2x_2^2 \leq 6\}\ is\ convex\ set$

Q.3) Attempt any One **[10]**

i) Prove that if X_0 is an optimum solution to the primal, then there exists a feasible solution W_0 to the dual such that $C^T X_0 = b^T W_0$.

Find the dual of the following primal problem Max Z = $2x_1 + 3x_2 - x_3$,

$subject\ to\ x_1 + x_2 - 3x_3 \leq 8,\ x_1 - x_2 + x_3 \leq 4,\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

ii) Explain the Branch and Bound method in detail.

Date : 24/02/2020

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
M. Sc. I Sem II and M.Sc. II Sem IV
Internal Examination 2019-20

All the students of M.Sc. I and M.Sc. II are hereby informed that their Internal Examination of Mathematics will be conducted on **as given below timetable.** The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus and timetable for examination will be as mentioned in following table.

Syllabus for M. Sc. I Sem. II

Sr. No.	Name of Paper	Topics
1	Linear Algebra (CP-1175B)	Unit I
2	Measure and Integration (CP-1176B)	Unit I
3	General Topology (CP-1177B)	Unit I
4	Partial Differential Equations (CP-1178B)	Unit I
5	Numerical Analysis (CP-1179B)	Unit I

Syllabus for M. Sc. II Sem. IV

Sr. No.	Name of Paper	Topics
1	Field Theory (CP-1190D)	Unit I
2	Integral Equation (CP-1191D)	Unit I
3	Algebraic Number Theory (CP-1192D)	Unit I
4	Operational Research II(CP-1194D)	Unit I
5	Combinatorics (CP-1197D)	Unit I

Timetable :

Date	Time	Class	Subject
09/03/2020	03:00 PM to 04: 00 PM	M.Sc. I	Linear Algebra (CP-1175B)
	03:00 PM to 04: 00 PM	M.Sc. II	Field Theory (CP-1190D)
10/03/2020	03:00 PM to 04: 00 PM	M.Sc. I	Measure and Integration (CP-1176B)
	03:00 PM to 04: 00 PM	M.Sc. II	Integral Equation (CP-1191D)
11/03/2020	03:00 PM to 04: 00 PM	M.Sc. I	General Topology (CP-1177B)
	03:00 PM to 04: 00 PM	M.Sc. II	Algebraic Number Theory (CP-1192D)
12/03/2020	03:00 PM to 04: 00 PM	M.Sc. I	Partial Differential Equations (CP-1178B)
	03:00 PM to 04: 00 PM	M.Sc. II	Operational Research II(CP-1194D)
13/03/2020	03:00 PM to 04: 00 PM	M.Sc. I	Numerical Analysis (CP-1179B)
	03:00 PM to 04: 00 PM	M.Sc. II	Combinatorics (CP-1197D)

Nature of question paper

Time:-1 Hours **Total Marks: 30**

Q.1) Choose the correct alternative for the following question. [05]

Five questions

Q.2) Attempt any three [15]

Four questions

Q.3) Attempt any One [10]

Two questions

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(Prof. S. P. Patankar)
HEAD
Department of Mathematics
Vivekanand College, Kolhapur

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II Internal Examination: 2019-20
MATHEMATICS

Subject: Linear Algebra **Time: 03: 00 PM-04:00 pm**
Date:09/03/2020 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i) Let V denote the vector space of n × n Skew symmetric matrices, over R. Then dim V as a vector space over R is

A) n^2 B) $(n^2 + n)/2$ C) $n^2 + n$ D) $(n^2 - n)/2$

ii) $W^{\perp\perp}$ = ………… (with usual notation)

A) $W^{\perp}$ B) W C) F D) V

iii) If T be a linear operator on R^3, given by $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x+2y+z \\ 2y+z \\ 2x+3y+z \end{bmatrix}$ Then the matrix representation of T with respect to standard basis of R^3 is…………

A) $\begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 3 & 1 \\ 0 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 2 \\ 2 & 2 & 3 \end{bmatrix}$

iv) If $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ then A is of nilpotence index ….

A) 4 B) 6 C) 3 D) 2

v] If dim V=n and S={v1,v2, …,vn} spans V then S is ________ of V.

i) a subspace ii) a basis
iii) a linearly dependent subset iv) the smallest subspace

Q.2. Attempt any three of the following: **[15]**

1.Prove that Sum , Scalar multiplication and Product of two linear transformation is again a linear transformation.

2.Show that, (au+bv , au+bv) = a$\bar{a}$(u,u) + a$\bar{b}$(u,v) + b$\bar{a}$ (v,u) + b$\bar{b}$ (v,v)

3.State and Prove Cauchy Swartz Inequality

4.Prove That, $W^{\perp}$ Is subspace of V

5,If B = { v_1 , v_2 , … v_n } is orthonormal set then vectors in B are Linearly Independent .
If w = $a_1v_1 + a_2v_2 ... + a_nv_n$ then a_i = <w, v_i>.

Q.3. Attempt any one of the following: **[10]**

1.If T is homomorphism from U onto V with kernel W . Then V is isomorphic to U\W
Conversely, If U is vector space and W is subspace of U then there is homomorphism of U onto U\W

2.If V is internal direct sum of U 1 , U 2 ,…U n ,Then prove that V is isomorphic to external direct sum of U 1 , U 2 ,…U n

Vivekanand College, Kolhapur (Autonomous)

M.Sc. (Part-I) Semester-II Internal Examination: 2019-2020

Subject :Measure and Integration **Time: 03: 00 PM**

Date: 10/03/2020 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i. $If\ A_n = \left(\frac{-1}{n+1}, \frac{1}{n+1}\right)\ then\ \bigcap_{n=1}^{\infty} A_n\ is$ ________

$a)\ 1$ $b)\ 0$ $c)\ \infty$ $d)\ \frac{2}{n}$

ii. $If\ \mathbb{Q}\ is\ set\ of\ all\ rational\ numbers\ then\ m^*(\mathbb{R} - \mathbb{Q}^c) =$ ______

$a)\ 1$ $b)\ 0$ $c)\ 2^C$ $d)\ \infty$

iii. $A\ set\ F\ is\ F_\sigma\ set\ if\ it\ is$ ______

a) Countable union of open sets b) Countable intersection of open sets

c) Countable union of closed sets d) Countable intersection of closed sets

iv. $Let\ f(x) = |x|, x \in [-1, 1]\ then$ ________

$a)\ D^+f(0) = 1$ $b)\ D^-f(0) = 0$

$c)\ D^+f(0) = 0$ $d)\ D^-f(0) = 1$

v. $If\ A_n = \left(\frac{-1}{n+1}, \frac{1}{n+1}\right)\ then\ \bigcap_{n=1}^{\infty} A_n\ is$ ________

$a)\ 1$ $b)\ 0$ $c)\ \infty$ $d)\ \frac{2}{n}$

Q.2. Attempt any three of the following: **[15]**

1) Prove that $L^1(E)$ is normed linear space

2) If a function f is measurable then prove that the set $\{x | f(x) = c\}$ is measurable.

for all c in $\mathbb{R}$

3) Prove that f is measurable if and only if f^+ & f^- are measurable

4) If $A \subseteq B$ then prove that $m^*(A) \leq m^*(B)$

Q.3. Attempt any one of the following: **[10]**

1) If f is monotone on (a, b) then prove that f is differentiable a. e on (a, b).

2) State & prove Lebesgue convergence theorem.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II
Internal Examination(2019-20)

Subject: General Topology **Total Marks: 30**

Date: 11/03/2020 **Time:03:00 PM to 04:00PM**

Q.1) Choose the correct alternative for the following question. **[05]**

1) In a topology, every open set can be expressed as.................
   a) union of some member of subbases
   b) intersection of bases
   c) union of intersection of some member of subbases
   d) intersection of union of some member of subbase
2) Every closed and bounded interval in R is ....
   a) connected b) compact c) both a and b d) none of them
3) (R, U) space is.............
   a) T_0 but not T_1 b) T_0 but not T_2 c) T_0, T_1 and T_2 d) none of them
4) A topological space X is connected if and only if ....
   a) there exists a non empty proper subset of X which is both open and closed.
   b) there does not exist a non-empty proper subset of X which is both open andclosed.
   c) there exist proper subset of X which is both open and closed.
   d) none of these
5) Limit point of a subset of $\{1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots\ldots\}$ of R is.......
   a) 1 b) ∞ c) 0 d) 2

Q.2) Attempt any three **[15]**

1) Define upper ray topology and lower ray topology
2) Let R be the set of all real numbers and let S consist of subset of R defined as follows:
   i) $\emptyset \in S$
   ii) a non – empty subset G of R belongs to S if and only if for each p ∈ G there exists a left half open interval (a, b] where $a, b \in R$, a < b such that $p \in (a, b] \subset G$
   Show that S is a topology for R
3) Consider the topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b, c, d\}, \{c, d\}\}$ on X ={a, b, c, d, e} and find the derived set of each of the following sets.
   a) A ={a, b} b) B = {b, c, d} c) C ={a, b, c} d) D = {b, d}
4) Show that A ∪ D(A) is closed set

Q.3) Attempt any One **[10]**

1) Prove that let X be any non – empty set and B be family of some subset of τ. Then B is base for τ on X if and only if
   a) $X = \cup \{B_i : B_i \in B\}$
   b) $\forall B_1, B_2 \in B, \forall x \in B_1 \cap B_2 \; \exists B_3 \in B$ such that $x \in B_3 \subseteq B_1 \cap B_2$
2) Show that $f: (X, \tau) \rightarrow (Y, V)$ is continuous if and only if inverse image of each V close set is τ closed set.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II
Internal Examination(2019-20)
Partial Differential Equations

Time:3:00PMto4:00PM **Total Marks: 30**
Date :12/03/2020

Q.1) Choose the correct alternative for the following question. [05]

1) *The equation xp−yq=z is*
   a) Linear b) semilinear c) Quasilinear d) Nonlinear
2) *The equation Ruxx+Suxy+Tuyy+g=0 is parabolic if…*
   a) $S^2 - 4RT < 0$ b) $S^2 - 4RT > 0$ c) $S^2 - 4RT = 0$ d) None of these
3) *The complete integral of z=px+qy+√pq is*
   a) z=a+b+ab b) z=ax+by+√pq c) z=c d) none of these
4) *The equation Ruxx+Suxy+Tuyy+g=0 is elliptic if…*
   a) $S^2 - 4RT < 0$ b) $S^2 - 4RT > 0$ c) $S^2 - 4RT = 0$ d) None of these
5) *The equation…represents the set of all right circular cones with x−axis as the axis of symmetry.*

   a) $(x^2 + y^2) = (z - c)^2 tan^2(\alpha)$ b) $(x^2 - y^2) = (z - c)^2 tan^2(\alpha)$

   c) $(z^2 + y^2) = (x - c)^2 tan^2(\alpha)$ d) $(x^2 + z^2) = (y - c)^2 tan^2(\alpha)$

Q.2) Attempt any three [15]

1) Find the general solution of $z(xp - yq) = y^2 - x^2$.

2) Form partial differential equation from $z^2(1 + a^3) = 8(x + ay + b)^3$

3) Obtaine pde by eliminating a,b from $z = ax^2 + by^2 + c$

4) Form partial differential equation from the relation $F(x - y, x - \sqrt{z}) = 0$

Q.3) Attempt any One [10]

1) Reduce the equation $xuxx - yuyy = 0$ into canonical form

2) If $\bar{X} = (P, Q, R)$ is a vector such that $\bar{X}.\, curl\, \bar{X} = 0$ & μ is an arbitrary differentiable of x, y, z then prove that $\mu\bar{X}.\, curl\, \mu\bar{X} = 0$

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II
Internal Examination(2019-20)

Subject : Numerical Analysis **Total Marks: 30**

Date: 13/03/2020 **Time: 03:00PM to 04:00PM**

Q.1) Choose the correct alternative for the following question. **[05]**

1) The formula of Newton – Raphson method is -------

A) $x_{k+1} = x_0 - \frac{f(x_0)}{f(x_k)}$ B) $x_{k+1} = x_0 - \frac{f(x_k)}{f'(x_k)}$

C) $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$ D) $x_{k+1} = x_k + \frac{f(x_k)}{f'(x_k)}$

2) If f(x) is continuous function in the interval $[a, b]$ & f(a).f(b) < a , then the equation f(x) = 0 has at least one real root or an odd number of real roots in (a,b) is known as ------

A) Bisection Method B)Iterative Method

C) Direct Method D)Intermediate Value Theorem

3) If $\{x_k\}$ is convergent sequence i.e. $\lim_{k \to \infty} \{x_k\} = x^*$ is root of f(x) = 0 and x_k is called ------- of f(x).

A) Order B) Approximate root

C) Zero D)Convergence

\

4) Suppose x^* is a root of f(x) = 0 with multiplicity 2 .Then order of Convergence of Newton – Raphson method is -------

A) 1 B)2 C)3 D)0

5) In matrix factorization method , if matrix A = U (Upper triangular matrix) Then this method is called -----

A) Backward Substitution Method B) Forward Substitution Method

C)Triangulation method D) Decomposition Method

Q.2) Attempt any three **[15]** .

1) Determine the rate of convergence of Newton Raphson method.
2) Derive Gauss Legendre integration method for n=1
3) Derive Trapezoidal rule by using Newton cotes formula. Find error term also.
4) Answer the following.
   a) Doolittle method b) Rate of convergence c) Truncation error
   d) Crout Method e) Newton cotes method

Q.3) Attempt any One **[10]**

1) Derive Simpson's rule by using Newton Cotes formula.

2) Describe third order Runge- Kutta method

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV Internal Examination: 2019-20
MATHEMATICS

Time: 03: 00 PM- 04:00pm **Subject : Field Theory**
Date:09/03/2020 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i. e and π are .... elements over Q

A) Transcendental B) Algebraic C) Irreducible D) Reducible

ii. Polynomial of degree one is always ......

A) Inseparable B) Separable C) Monic D) Simple

iii. If $f(x)$ is of degree 3 , Then $f(x)$ has .... Root.

A) Complex B) Unique C) Distinct D) Real

iv. Any subgroup and any quotient group of a .... group is solvable.

A)Solvable B) Normal C) Separable D) None

v. If F ⊆ K ⊆ L are fields. If a ∈ L be algebraic over K and K is an algebraic extension of F. Then, a is ....

A) Algebraic over K B) Algebraic Over F C) Algebraic D) Separable

Q.2. Attempt any three of the following: **[15]**

1. If F ⊆ K be fields , $p(x)$ a monic irreducible polynomial over F and a and b ∈ K be roots of p(x). Then prove that , there exists an isomorphism j : F(a) → F(b) such that j(a) = b and j (s) = s for all s ∈ F.
2. Prove that an algebraic extension E of F is finitely generated over F if and only iff E is a finite extension of F.
3. Find the smallest extension of Q having a root of $x^4 - 2 \in Q[x]$
4. If F ⊆ E be fields such that [E : F] is prime number. Prove that there are no fields property between F and E.

Q.3. Attempt any one of the following: **[10]**

1. If F ⊆ K ⊆ L are field and [L : F] is finite , Then prove that [L : K] and [K : F] are divisors of [L : F]
2. A. Let F ⊆ K be field and a ∈ K be algebraic over F. Then prove that there exist a unique monic irreducible polynomial p(x) over F such that p(a) = 0. Also ,for any f(x) ∈ F[x] , f(a) = 0 if and only if p(x) divides f(x) in F[x]

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV
Internal Examination: 2019-20

Sub: Integral Equations **Date : 10/03/2020**

Total Marks: 30 **Time : 03:00pm-04:00pm**

Q.1) Choose the correct alternative for the following question. [05]

1) The type of integral equation $g(s) = \lambda \int K(s,t)g(t)dt\ b\ a$ is -----
   a) Volterra integral equation of 1st kind
   b) Fredholm integral equation of 1st kind
   c) Homogeneous Fredholm integral equation of 2nd kind
   d) Non-homogeneous Fredholm integral equation of 2nd kind
2) The homogeneous Fredholm integral equation has trivial solution, if -----
   a) $D(\lambda) = 0$ b) $D(\lambda) \neq 0$ c) $D(\lambda)$ does not exist d) none of these
3) The eigen values of non-zero symmetric kernel are -----
   a) real b) zero c) only imaginary d) none of these
4)) Spectrum of symmetric kernel is always -----
   a) empty b) non-empty c) does not exist d) none of these
5) If $\{\phi k\}$ is orthogonal set, then $< \phi i , \phi j > =$ ----- for all i,j.
   a) 0 b) 1 c) -1 d) ∞

Q.2) Attempt any three [15]

1) Convert the following boundary value problem to an integral equation.
   $y'' + xy = 1, y(0) = 0, y(1) = 1, 0 \leq x \leq 1$
2) Prove that eigen functions $g(s)$ and $\psi(s)$ corresponding to distinct eigen values λ_1 and λ_2 respectively of the homogeneous integral equation $g(s) = \lambda \int K(s,t)g(t)dt$ and its transpose are orthogonal.
3) Find the eigen values and eigen functions of the homogeneous integral equation
   $g(s) = \lambda \int st\ g(t)dt\ 1\ 0$
4)) Find the eigen values and eigen functions of the homogeneous integral equation
   $g(s) = \lambda \int \sin(s + t)g(t)dt\ 2\pi\ 0$

Q.3) Attempt any One [10]

1) Describe the procedure of solving non-homogeneous Fredholm integral equation of 2nd kind with separable kernel.
2) Solve the integral equation $g(s) = f(s) + \lambda \int_0^{2\pi} \cos(s + t)\, g(t)dt$ by discussing all possible cases.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV
Internal Examination(2019-20)
Algebraic Number Theory

Time:3:00PM–4:00PM **Total Marks: 30**
Date:11/03/2020

Q.1) Choose the correct alternative for the following question. [05]

1) Let $g_1, g_2, \dots, g_n$ are linearly independent in a abelian group G (over Z). $g_1m_1 + g_2m_2 + \dots + g_nm_n = 0$ ($m_i \in$ Z) if and only if

(a) $m_i = 0$ for all i (b) $m_i = 0$ for some i (c) $m_i = 0$ for exactly i (d) None of the above

2)If $\{g_1,\dots,g_n\}$ is a basis for G.
P : every $g \in G$ has a unique representation:$g = m_1g_1 + \dots + m_ng_n$ $(m_i \in \mathbf{Z}\}$

Q: $\{g_1,g_2,\dots,g_n\}$ is linearly independent set.

(a)P is true and Q is false (b) P is false and Q is true
(c) P and Q are false (d) P and Q are true

3) Let G be a finitely generated Z–Module.
P : Every basis has same number of elements.
Q : A linearly dependent set which generates G is called a basis.

(a)P is true and Q is false (b) P is false and Q is true
(c) P and Q are false (d) P and Q are true

4) P: A complex number α will be called algebraic if it is algebraic over **Q**, that is, it satisfies a non-zero polynomial equation with coefficients in **R**.

Q : A complex number α will be called algebraic if it is algebraic over **Q**, that is, it satisfies a non-zero polynomial equation with coefficients in **Q**.

(a)P is true and Q is false (b) P is false and Q is true
(c) P and Q are false (d) P and Q are true

5) Which of the following is not algebraic over Q

(a) $\sqrt{2}$ (b) i
(c) $-i$ (d) None of the above

Q.2) Attempt any three [15]

1) Show that a R-module M is cyclic if and only if $M \cong \frac{R}{I}$ for some ideal I of R.
2) Show that the set $\mathbb{A}$ of algebraic number is subfield of the complex field $\mathbb{C}$
3) Show that the ring of integer β is a subring of the field of algebraic number $\mathbb{A}$.
4) Let O be the ring of integers in number field K and let $x, y \in O$ then
a] x is unit iff $N(x) \pm 1$

b] x, y are associates then $N(x) = \pm N(y)$
c] If N(x) is rational prime then x is irreducible in O

Q.3) Attempt any One [10]

1) Show that a complex number θ is an algebraic integer if and only if the additive group generated by all powers $1, \theta, \theta^2$, ... is finitely generated.

2) Show that the ring of integers O of $Q(\zeta)$ is $Z(\zeta)$

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV
Internal Examination(2019-20)
Operational Research-II

Time:03:00PM to 04:00 PM **Total Marks: 30**
Date : 12/03/2020

Q.1) Choose the correct alternative for the following question. [05]

i) In progressive failure, probability of failure .... with the increase in the life of an item

A) increases B) Remains constant C) Decreases D) None of these

ii) The sequence of critical activity in network diagram is called..........

A) Dummy activity B) Total float C) Critical path D) None of these

iii) Queue can form only when......

A) arrivals exceed service facility

B) arrivals equals service facility

C) service facility is capable to serve all the arrivals at a time

D) there are more than one service facility

iv) .... occurs when a waiting customer leaves the queue due to impatience.

A) Reneging B) Balking C) Jockeying D) None of these

v) In model I(b), minimum average inventory cost is...........

A) $\sqrt{\frac{2C_1C_3q}{T}}$ B) $\sqrt{\frac{2C_1C_3D}{T}}$ C) $\sqrt{2C_1C_3D}$ D) None of these

Q.2) Attempt any three [15]

i) Customers arrive at a box office window, being managed by single individual, according to Poisson input process with mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 90 second. Find the average waiting time of a customer. Also determine the average number of customers in the system and average queue length

ii) Explain the model of communication system.

iii) A self – service store employs one cashier at its counter. Nine customers arrive on an average every 5 minutes while the cashier can serve 10 customers in 5 minutes.

Assuming Poisson distribution for arrival and exponential distribution for service time, find

a) Average number of customers. in the system

b) Average number of customers in the queue

c) Average time a customer spends in the system

d) Average time a customer waits before served.

iv) Let the value of money be assumed to be 10% per year and suppose that machine A is replaced after every 3 years whereas machine B is replaced after every six years. The yearly costs of both the machine are given as under:

Year	1	2	3	4	5	6
Machine A	1000	200	400	1000	200	400
Machine B	1700	100	200	300	400	500

Determine which machine should be purchased.

Q.3) Attempt any One **[10]**

i) Develop economic lot size formula with uniform demand without shortage and production rate is infinite.

ii) A project consist of a series or tasks labelled A, B, ...H, I with the following relationship (W < X, Y means X and Y cannot start until W is completed) with this notation construct the network diagram having the following constraints

A < D, E; B, D < F; C < G; G, B < H; F, G < I.

Find also optimum time of completion of the project. When the time of completion of each task is as follows: **[12]**

Task	A	B	C	D	E	F	G	H	I
Time	23	8	20	16	24	18	19	4	10

Vivekanand College, Kolhapur (Autonomous)

M.Sc. (Part-II) Semester-IV Internal Examination: 2019-20

MATHEMATICS

Time: 03: 00 PM-04:00pm **Subject: Combinatorics** **Date: 13/03/2020**

Total Marks: 30

Q. 1 Select the correct alternative for each of the following: [5]

I. *The weight of permutation* $(1,3,2,4) \in S_4$ *i*

$a)$ 2 $b)$ 4 $c)$ 6 $d)$ 8

II. The total number of 6 *digit number in which all the odd digits & only odd digits appear is*

$a)$ 6! $b)$ $\frac{5}{2}(6!)$ $c)$ $\frac{1}{2}(6!)$ $d)$ *none of these*

III. The coefficient of x^2 *in the expansion of* $(1-x)^{-2}$ *is*

$a)$ 1 $b)$ 2 $c)$ 3 $d)$ 4

IV. The weight of permutation $(1,3,2,4) \in S_4$ *is*

$a)$ 2 $b)$ 4 $c)$ 6 $d)$ 8

V. *The number of derangements of* $(1,2,3)$ *is/are* _____

$a)$ 1 $b)$ 2 $c)$ 3 $d)$ *none of these*

Q.2. Attempt any three of the following: [15]

1) *Define Ramsey number & with usual notations prove that* $R(2,p) = p$.

2) *With usual notations prove that* $D_n = (n-1)\{D_{n-1} + D_{n-2}\}$. *for* $n \geq 3$ (06+06+06)

3) *Find the coefficient of* x^{21} *in* $(x^2 + x^3 + \cdots)^6$.

4) *Solve* $a_r = 10a_{r-1} - 9a_{r-2}$ *with initial conditions* $a_0 = 3$ & $a_1 = 11$

Q.3. Attempt any one of the following: [10]

1) *State & prove principle of inclusion and exclusion for 'n' finite sets.*

2) *Find a cycle index of dihedral group on symmetries of square*

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Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Suppliment No. : 1.
Roll No. : 1207
Class : M.Sc-I

Subject : Modern Algebra.
Test / Tutorial No. : 29/30
Div. :

i) → c] Both I and II are true.

ii) → b] Normal Subgroup

iii) → d) Both (a) & (b)

04

iv) → c) 15

v) → a) Isomorphic

Q2.

1) → ~~Let~~ We have,

S_n = { All permutation from set containing element ~~to~~ to itself }

$S_n = n!$

i) Closure Property :-

Let f & g ∈ S_n be any two element.

→ We know that Composition of permutation is again ~~an~~ Permutation.

∴ $f \circ g \in S_n \quad \forall f, g \in S_n$

∴ S_n Satisfies Closure property. —(1)

ii) Associativity Property:-

We know that mapping Composition of Permu Satisfies Associativity trivaly.

$\therefore f, g, h \in S_n$

$\therefore (f \circ g) \circ h = f \circ (g \circ h) \cdots \forall f, g, h \in S_n$ — (2)

iii) Existance of ~~Inverse~~ Identity.

$\exists \; I = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ 1 & 2 & 3 & \cdots & n \end{pmatrix}$ ~~is ide~~

$\therefore I \circ f = f = f \circ I \cdots \forall f \in S_n$

$\therefore I$ is identity element. — (3)

iv) Existance of Inverse.

If $f \in S_n$

05 $\therefore f$ is one-one & onto Function.

$\therefore f^{-1}$ is exist.

$\therefore f^{-1}$ is one-one & onto i.e permutation.

$\therefore f^{-1} \in S_n$

$\therefore f^{-1} \circ f = I = f \circ f^{-1} \cdots \forall f \in S_n$. — (4)

$\therefore$ From (1), (2), (3) & (4).

$\therefore$ ~~Sn~~ i.e. $\langle S_n, \circ \rangle$ is a group.

2) Let, $\langle Z, + \rangle$ is a group.

Suppose $\langle Z, + \rangle$ has composition series.

$\therefore \{0\} = H_0 \subset H_1 \subset H_2 \subset H_3 \subset \cdots \subset H_n = Z$

$\therefore Z$ is abelian group & all subgroup of Z is normal Subgroup is of form mZ.

Let $H_1 = mZ$

$\therefore \frac{H_1}{H_0} = \frac{mZ}{\{0\}} \cong mZ$ is simple group.

05 ~~But mZ is not~~ i.e. Which is Contradiction to that mZ has infinitely many non-trival proper normal Subgr

$\therefore$ ~~mZ is not simple group.~~

$\therefore$ ~~Z~~ Our assumption is wrong

$\therefore$ mz is not simple group.

$\therefore$ <z, +> has no composition series.

4] → Let G be a group.

Let suppose G is abelian group.

$\therefore ab = ba \quad \forall a, b \in G$.

Claime :- $G' = \{e\}$

Let $a, b \in G$ be any elements.

$\therefore aba^{-1}b^{-1} = baa^{-1}b^{-1}$

$= beb^{-1}$

$= bb^{-1}$

$\therefore G' = \{e\}$

Now suppose that $G' = \{e\}$

~~we know that~~

Claim! G is abelian group.

we know that G/G' is abelian group.

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$$\therefore \frac{G}{G'} = \frac{G}{\{e\}} \cong G$$

$\therefore G' \leq G$

$\therefore G$ is abelian group.

Q3.

1] → There is no harm to consider Commutator elem generate G'

i) To prove $G' \trianglelefteq G$

Let $g \in G$ be any element.

$x \in G'$ be any element.

Consider.

$g^{-1} x g = g^{-1} (aba^{-1}b^{-1}) g$ ∴ [x is Commutator

$= (g^{-1}aba^{-1}) e (b^{-1}g)$ -- [e is identity]

$= (g^{-1}aba^{-1})(gb^{-1}bg^{-1})(b^{-1}g)$

$= (g^{-1}aba^{-1}gb^{-1})(bg^{-1}b^{-1}g)$ -- [Associativity]

$= [(g^{-1}a)b(g^{-1}a)^{-1}b^{-1}][bg^{-1}b^{-1}(g^{-1})^{-1}] \in G$

∴ $g^{-1}xg \in G'$

∴ $G' \trianglelefteq G$

∴ G' is ~~subgroup of G~~ normal subgroup of

ii) ~~To prove~~ Now G is group & G' is normal subgroup of G Now we talk about G/G'

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Let $G'a, G'b \in G/G'$ be any element.

∴ $G'a \cdot G'b = G'(ab)$

$=$ ~~G'(ab)~~ $G'ab(a^{-1}b^{-1}ba)$

$= G'ba$

∴ $G'a \cdot G'b = G'b \cdot G'a$

∴ $G'b \cdot G'a \in G/G'$

∴ It satisfies Commutative property.

∴ G/G' is abelian group.

iii) Let G/N is abelian.

∴ $a^{-1}N\, b^{-1}N = b^{-1}N\, a^{-1}N$

⇒ $a^{-1}b^{-1}N = b^{-1}a^{-1}N$

⇒ $a^{-1}b^{-1}N = (ab)^{-1}N$

⇒ $a^{-1}b^{-1}abN = N$

∴ $a^{-1}b^{-1}ab \in N$

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Signature of Supervisor:

Suppliment No. : 2.

Roll No. : 1201

Class : MSc1

Subject : Modern Algebra.

Test / Tutorial No. :

Div. :

$\therefore a^{-1}b^{-1}(a^{-1})^{-1}(b^{-1})^{-1} \in N$

$\therefore G' \leq N$

$\therefore G'$ is subgroup of N.

~~Let~~ Now suppose G' is subgroup of N

Claime: G/N is abelian group.

let $aN, bN \in G/N$

$\therefore aN \cdot bN = (ab)N$

$= ~~(ab)~~ N(ab)(a^{-1}b^{-1}ba)$

$aN \cdot bN = baN$

$aN\, bN = bN\, aN$

$\forall\, bNaN \in G/N$

$\therefore G/N$ is abelian group.

Q2.

3) There is no harm to consider.

$A = \{1, 2, 3 \dots n\}$

Consider element

$1, 1\sigma, 1\sigma^2 \dots$

A is finite set. so these ~~can~~ element can not be distinct.

Let $1\sigma^r = 1$

$\therefore 1\sigma^r = 1\sigma^s$

~~$\therefore (1\sigma^r)^{-1} = (1\sigma)^{-1}$~~

~~$\therefore 1\sigma^{r-s} =$~~

$\therefore 1\sigma^{r-s} = 1$

which is contraction to assumption $1\sigma^r = 1$

$\therefore \tau_1 = (1, 1\sigma, 1\sigma^2 \dots 1\sigma^{r-1})$ — ①

If i is 1st element of above sequence

$\therefore$ above sequence become.

$\tau_2 = (i, i\sigma, i\sigma^2 \dots i\sigma^{r-1})$ — ②

05

From ① & ②

τ_1 & τ_2 are disjoint cycle of Finite set A

If any small $j \in A$ is ~~com~~ identical betn τ_1 & τ_2 then τ_1 & τ_2 become identical which is not true

If $j \in A$ is not in τ_1 & τ_2 then we have to construct τ_3

Here A is finite so these ~~st~~ procedure after finite steps.

$\therefore \sigma = \tau_1 . \tau_2 . \tau_3 \dots \tau_m$

$\therefore$ Every permutation σ of finite set A is product of disjoint cycle.

SUPPLIMENT

Signature of Supervisor: [signature]

Suppliment No. : 1

Roll No. : 1210

Class : MSC-II (mathematics)

Subject : Advance discrete mathematics

Test / Tutorial No. : Internal exam

Div. :

05 + 15 + 10 = 30/30

Q.1)

(1) (B) $W(G) = 1$

(2) (D) $\frac{n(n-1)}{2}$

(3) (C) $W_n = 2$

05

(4) (A) / Bipartite (B) complete.

(5) (A) k

(Q.7)

(1) → Let G be a graph.

The graph G has e no. of edges and n vertices.

We know that,

$$\sum_{v \in V}^{n} d_G(v) = 2e \quad , v \in G. \quad \text{—— (1)} \quad \text{— by handshaking lemma.}$$

Given that :— the degree of t element have k then (n-t) el vertices have degree (k+1)

∴ From (1)

$$\therefore \underbrace{k + k + k + \dots + k}_{\text{'t' times}} + \underbrace{(k+1) + (k+1) + (k+1) \dots + k+1}_{(n-t) \text{ times}} = 2$$

$$\therefore tk + (n-t)(k+1) = 2e$$

$$\therefore tk + n(k+1) - 1(k+1) = 2e$$

$$\therefore tk + n(k+1) - tk - t = 2e$$

$$\therefore -t - n(k+1) = 2e$$

$$\Rightarrow \boxed{t = (k+1)n - 2e}$$

∴ Ans $t = (k+1)n - 2e$

(3) Let G be a graph.
V be the set of vertex.
and 'e' is number of edges in G.

∴ by handshaking lemma,

$$\sum d_G(v) = 2e = \text{even} \quad \text{--- (1)}$$

let 'U' be the set of even vertex
and
'W' be the set of odd vertex.

let, $u \in U$, then $\sum_{i=1}^{n} d_G(u) = \text{even}$.

as all the vertex in U has even vertex.
so there sum is even. ——— (2)

0.5

∴ As $U \cup W = V$

$$\therefore \sum_{v \in V} d_G(v) = \sum_{u \in U} d_G(u) + \sum_{w \in W} d_G(v)$$

∴ from (1) & (2) ⇒ even = even + $\sum_{v \in V} d_G(w)$

$$\Rightarrow \sum_{v \in W} d_G(w) = \text{even}$$

The set V has all the element of odd vertex still the sum is positive.

⇒ The terms in 'W' are even in numbers.

⇒ In any graph G, there is even number of odd vertices.

(4) Let G be graph.
let u, v, w any vertices of graph G.

Case I :- let u and v are not connected.
⇒ $d(u,v) = \infty$
and $d(u,w) + d(w,v) = \infty$
⇒ there is no path containting between ~~(u,w) and (w,v) walk~~ u and v.
⇒ $\boxed{d(u,w) + d(w,v) = \infty}$ ——— (1)

05

Case II :- let u and v are connected.
⟶ The path (u-v) are (u-w) and (w-v) walk.
∴ i.e. (u-v) is is the path.
Now,
length of the path (u,v) $= d(u,w) + d(w,v)$
∴ by the deletion procedure the unique path contained in u-v path is.
the length of path u-v is given as
⇒ length of (u,v) ≤ walk of (u,v)
$= d(u,w) + d(w,v)$
and
also, $d(u,v) \le d(u,w) + d(w,v)$
($\because d(u,v) \le d(u,v)$)
∴ Hence the proof.

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SUPPLIMENT

Signature of Supervisor

Suppliment No. : 2

Roll No. : 1210

Class : MSC II

Subject : Advance discrete mathe

Test / Tutorial No. :

Div. :

(Q.2)

(a) Given that :-

$U = \{v_1, v_2\}$

$F = \{e_1, e_2, e_5, e_{11}\}$

(i) Here $V(G-U) = \{v_3, v_4, v_5, v_6\}$

$E(G-U) = \{e_7, e_8, e_9, e_{10}, e_{11}\}$

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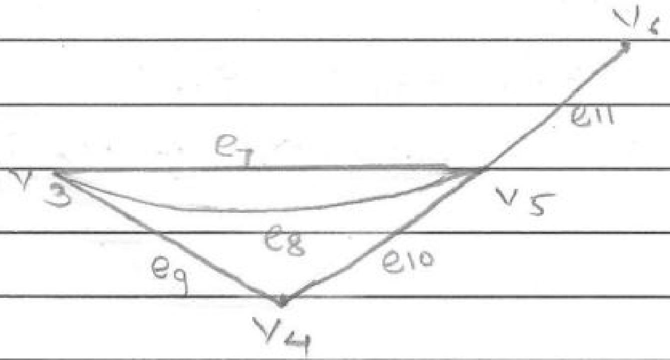
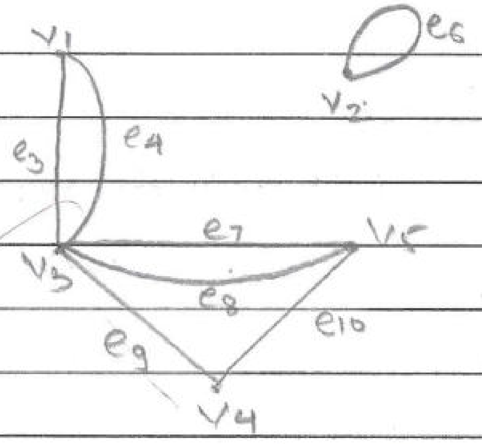


[G-U].

(ii) Now, $E(G-F) = \{e_3, e_4, e_6, e_7, e_8, e_9, e_{10}\}$

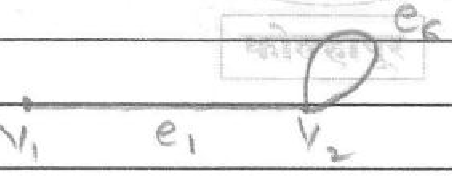
$V(G-F) = \{v_1, v_2, v_3, v_4, v_5\}$



(G-F)

(iii) $V[G(U)] = \{v_1, v_2\}$

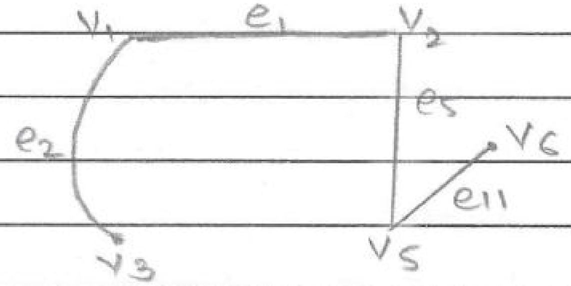
$E[G(U)] = \{e_1, e_6\}$



G(U)

(iv) $E[G(F)] = \{e_1, e_2, e_5, e_{11}\}$

$V[G(F)] = \{v_1, v_2, v_3, v_5, v_6\}$



G(F)

• Regular graph

(b) Definition :-

Let G be the ~~greg~~ graph and v be the vertex set of ~~and~~ G and E is edge set of G then graph is called regular graph if every $v \in V$ has same degree of vertex.