"Education for Knowledge, Science, and Culture"
- Shikshanmaharshi Dr. Bapuji Salunkhe

Shri Swami Vivekanand Shikshan Sanstha's

Vivekanand College, Kolhapur (Autonomous)

KOLHAPUR (AUTONOMOUS)

Date : 07/11/2022

M. Sc. I Sem. I and M.Sc. II Sem III
Internal Examination 2022-23

All the students of M.Sc. I and M.Sc. II are hereby informed that their Internal Examination of Mathematics will be conducted on **as given below timetable.** The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus and timetable for examination will be as mentioned in following table.

Syllabus for M. Sc. I Sem. I

Sr. No.	Name of Paper	Topics
1	CP-1170A: Algebra	Unit I
2	CP-1171A: Advanced Calculus	Unit I
3	CP-1172A: Complex analysis	Unit I
4	CP-1173A: Ordinary Differential Equation	Unit I
5	CP-1174A: Classical Mechanics	Unit I

Syllabus for M. Sc. II Sem. III

Sr. No.	Name of Paper	Topics
1	CC-1180C: Functional Analysis	Unit I
2	CC-1181C: Advanced Discrete Mathematics	Unit I
3	CBC-1182C: Lattice Theory	Unit I
4	CBC-1183C: Number theory	Unit I
5	CBC-1184C: Operational Research -I	Unit I

Timetable

Day and Date	Class	Time	Subject
Monday, 21/11/2022	M.Sc. I	03:00PM to 04:00PM	Algebra
	M.Sc. II	12:30 PM to 01:30 PM	Functional Analysis
Tuesday,22/11/2022	M.Sc. I	11:30AM to 12:30PM	Ordinary Differential Equation
	M.Sc. II	11:30AM to 12:30PM	Advanced Discrete Mathematics
Wednesday,23/11/2022	M.Sc. I	12:30PM to 01:30PM	Advanced calculus
	M.Sc. II	12:30PM to 01:30PM	Lattice Theory
Thursday,24/11/2022	M.Sc. I	11:30AM to 12:30PM	Complex analysis
	M.Sc. II	11:30AM to 12:30PM	Number theory
Friday,25/11/2022	M.Sc. I	02:00PM to 03:00PM	Classical Mechanics
	M.Sc. II	02:00PM to 03:00PM	Operational Research -I

Nature of question paper

Time:-1 Hours **Total Marks: 30**

Q.1) Choose the correct alternative for the following question. [05]

Five questions

Q.2) Attempt any three [15]

Four questions

Q.3) Attempt any One [10]

Two questions

VIVEKANAND COLLEGE ESTD. JUNE 1964 KOLHAPUR-416 003

(Prof. S. P. Patankar)
HEAD
Department of Mathematics
Vivekanand College, Kolhapur

Vivekanand College, Kolhapur (Autonomous)

M.Sc. I Semester-I Internal Examination: 2022-23

MATHEMATICS

Sub: Algebra (CP-1170A) **Time: 03:00 pm- 04:00 pm**

Date: 21/11/2022 **Total Marks:30**

Q1) Select the correct alternatives **(5)**

1] i) Every permutation is one - one function. ii) Every one-one function is permutation.

a) (ii) is true. b) (i) is true. c) both statement are true. d) both statement are false.

2] Subgroup of order 2 is always …

a) normal b) abelian c) Both a and b d) None

3] A_n contain every 3-cycle if ….

a) $n \geq 3$ b) $n \geq 5$ c) A_n is normal. d) A_n is simple.

4] with usual notation which is true?

a) $\{\sigma_0\}$, $\{\sigma_0, \sigma_1, \sigma_2\}$ are not subgroup of S_3.

b) $\{\sigma_0, \mu_1\}$, $\{\sigma_0, \mu_2\}$,$\{\sigma_0, \mu_3\}$ ae only subgroups of S_3.

c) $\{\sigma_0, \sigma_1, \sigma_2\}$,$\{\sigma_0, \mu_1\}$ are subgroup of S_3.

d) S_3 has no subgroup of order 3

5] Order of A_5 is…

a) 60 b) 120 c) 5 d) 5!

Q2) Solve any THREE of the following. **(15)**

1] Prove that every permutation σ of a finite set A is a product of disjoint cycles.

2] Define commutator subgroup of group G. Show that G is abelian if and only if commutator subgroup is {e}.

3] Show that for $n \geq 3$, Subgroup generated by $3-$cycle of A_n is A_n.

4] Define index of subgroup . Find index of A_n in S_n . Show that A_n is normal in S_n.

Q3) Solve any ONE of the following. **(10)**

1] Define symmetric group of G, |G| = k.

Show that if A is non-empty set and S_A is collection of all permutations of A. Then, S_A is a group under permutation multiplication.

2] State and prove Caley's theorem.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-I
Internal Examination(2022-23)
Ordinary Differential Equations

Subject :Ordinary Differential Equations **Total Marks: 30**
Date: 22/11/2022 **Time:11:30 AM to 12:30 PM**

Q.1) Choose the correct alternative for the following question. **[05]**

i) If $p_n(x)$ and $p_m(x)$ are n^{th} and m^{th} Legendary polynomials respectively, then $\int_{-1}^{1} p_n(x)p_m(x) = 0$ is possible when ……

A) $m = n$ B) $m \leq n$ C)$m \geq n$ D)$m \neq n$

ii) If $f(x,y) = y^{\frac{2}{3}}$, $R = \{(x,y) | |x| \leq 1, |y| \leq 1\}$ and K is Lipschitz constant then ……

A) F satisfies Lipschitz Condition on R with $k = \frac{1}{2}$

B) F satisfies Lipschitz Condition on R with $k = 0$

C) F satisfies Lipschitz Condition on R with $k = 1$

D) F do not satisfy Lipschitz Condition on R

iii)If $\emptyset_1$ and $\emptyset_2$ are two solutions of $L(y) = 0$ then ….. is also solution of $L(y) = 0$ where c_1 and c_2 are any two constants.

A) $c_1\emptyset_1 + c_2\emptyset_2$ B) $c_1\emptyset_1 - c_2\emptyset_2$ C)Both A and B D) None of these

iv)The functions $\emptyset_1(x) = \cos x$, $\emptyset_2(x) = \sin x$ are ……. on interval $-\infty \leq x \leq \infty$

A)Linearly Dependent B) Linearly Independent

C)Both A and B D)None of these

v) Which of the following is not solution of $y''' - 3r_1y'' + 3{r_1}^2y' - {r_1}^3y = 0$, where r_1 is constant

A)$\emptyset(x) = e^{r_1x}$ B) $\emptyset(x) = x^2e^{r_1x}$

C) $\emptyset(x) = xe^{r_1x}$ D) $\emptyset(x) = x^3e^{r_1x}$

Q.2) Attempt any three **[15]**

i) Classify the singular points in the finite plane $x^2(x^2 - 4)y'' + 2x^3y' + 3y = 0$

ii) Show that $\emptyset(x) = \frac{d^n}{dx^n}[(x^2 - 1)^n]$ satisfies the Legendre equation hence show that $\emptyset(1) = 2^n n!$

iii) Classify the singular points in the finite plane $x^2(x^2-4)y'' + 2x^3y' + 3y = 0$

iv) Show that $\emptyset(x) = \frac{d^n}{dx^n}[(x^2-1)^n]$ satisfies the Legendre equation hence show that $\emptyset(1) = 2^n n!$

Q.3) Attempt any One **[10]**

i) If $\emptyset_1$ is a solution of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I and $\emptyset_1(x) \neq 0$ on an interval I , then show that the second solution $\emptyset_2$ of $L(y) = 0$ on I given by, $\emptyset_2(x) = \emptyset_1(x)\int_{x_0}^{x} \frac{1}{[\emptyset_1(s)]^2}\, exp[-\int_{x_0}^{s} a_1(t)dt]ds$ and function $\emptyset_1, \emptyset_2$ form a basis for the solution of $L(y) = 0$ on I.

ii) Find all the solutions of $y''' + y'' + y' + y = 1$, $\varphi(0) = 0$, $\varphi'(0) = 1$, $\varphi''(0) = 0$

Vivekanand College, Kolhapur (Autonomous)

M.Sc. (Part-I) Semester-I Internal Examination: 2022-23

Subject : Advanced Calculus **Date:23/11/2022**

Time: 12:30pm -01:30 PM **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

1] $For\ a\ vector\ field\ \bar{f}, \bar{f}'(\bar{c};\bar{0}) =$ _______

A) 0 B) $\bar{c}$

C) $\bar{0}$ D) $\|\bar{c}\|$

2] $If\ T:\mathbb{R}^n \longrightarrow \mathbb{R}^m \&\ S:\mathbb{R}^m \longrightarrow \mathbb{R}^p\ are\ linear\ then\ order\ of\ matrix\ of\ (\bar{S}.\bar{T})$ = _______

A) $n \times p$ B) $m \times p$ C) $p \times n$ D) $m \times n$

3] $If\ \bar{f}\ is\ linear\ then\ \bar{f}'(\bar{c};\bar{u}) =$ _______

A) 0 B) $\bar{f}(\bar{u})$ C) $\bar{f}(\bar{c})$ D) $\bar{f}'(\bar{u})$

4] *Stokes theorem relates a surface integral to*

A) *Volume integral* B) *Line integral*

C) *Vector integral* D) *Real integral*

5] If α_1v1+α_2v2+ …+$\alpha_n$vn=0, where v1, v2, … ,vn are linearly independent vectors in a vector space V(F), then _______.

i) i=0 for all i=1,2, …, n ii) i≠0 for all i=1,2, …, n

iii) i=0 for at least one i iv) i≠0 for at least one i

Q.2. Attempt any three of the following: **[15]**

1) Prove that the sequence $\{f_n\}_{n=1}^{\infty}$ *converges pointwise but not uniformly where* $f_n(x) = \frac{1}{nx+1}, 0 < x < 1$

2) *If* $\sum_n a_n$ *converges absolutely then prove that every subseris* $\sum_n b_n$ *also converges absolutely.*

3) Let $\bar{f}$ *be a vector field given by* $\bar{f}(x,y) = \sqrt{y}i + (x^3+y)j$ *where* $(x,y) \in \mathbb{R}^2,$ $y \geq 0$ *obtain the integral of* $\bar{f}$ *from* $(0,0)$ *to* $(1,1)$ *along the path* $\alpha(t) = ti + tj$

4) *Let* f *be a double sequence.&* $\lim_{p,q\to\infty} f(p,q) = a$. *Assume that* $\lim_{q\to\infty} f(p,q)$ *exist for each fixed integer* p. *Prove that the iterated limit,* $\lim_{p\to\infty}(\lim_{q\to\infty} f(p,q))$ *also exist &has same value* a.

Q.3. Attempt any one of the following: **[10]**

1) *If* $\bar{f}$ *be differentiable at* $\bar{c}$ *with total derivative* $\bar{T}_{\bar{c}}$. *Prove that* $\bar{f}'(\bar{c};\bar{u}) = \bar{T}_{\bar{c}}(\bar{u})$ *for every* $\bar{u} \in \mathbb{R}^n$

2) *If* $\{M_n\}$ *be a sequence of non − negative real numbers such that* $0 \leq |f_n(x)| \leq M_n$ $\forall n \in \mathbb{N}\ \&\ \forall x \in S$. *Prove that* $\sum f_n$ *converges uniformely on S if* $\sum M_n$ *converges.*

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-I
Internal Examination: 2022-23
MATHEMATICS

Subject : Complex Analysis
Date:24/11/2022

Time: 11:30am -12:30pm
Total Marks: 30

Q. 1 Select the correct alternative for each of the following: **[5]**

i) The radius of convergence of $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$ is.....

A) -e B) 1/e C) e D)1/e

ii) If f(z) is an analytic function within and on | z-2| = 3 and |f(z)| < 2 on |z-2| = 3, then $|f^4(2)| =$.......

A) 48/81 B) 24/27 C) 81/24 D) 27/48

iii) In the Laurent series expansion f(z) = $\frac{1}{z(z-1)}$ valid for |z-1| >1, the coefficient of $\frac{1}{z-1}$ is....

A) 1 B) 0 C) -1 D) 6

iv) The excess of the number of zeros over the number of poles of a meromorphic function is called ....

A) Maximum Modulus Principle B) Minimum Modulus Principle
C) Schwarz Lemma D) The Argument Principle

v) For the function f(z) =$\frac{z-sinz}{z^3}$, at the point z = 0 is.....

A) Pole of order 3 B) Pole of order 2

C) Essential singularity D) Removable singularity

Q.2. Attempt any three of the following: **[15]**

1) Find radius of convergence of f(z) = $\sum_{n=1}^{\infty} a_n z^n$.

2) Find radius of convergence of $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$.

3) If γ is a contour with parameter interval [a, b] and $f(z) = u(x,y) + iv(x,y)$ is continuous function on the contour γ with $|f(z)| \le M, \forall z \in \gamma$, then prove that $|\int_C f(z)dz| \le ML$ where L is the length of contour given by $\int_a^b |\gamma'(t)|dt$

4) If $u(x,y) = x^3 + ax^2y + bxy^2 + 2y^3$ is harmonic function and $v(x,y)$ its harmonic conjugate. If $v(0,0) = 1$, then $|a + b + v(1,1)|$ is equal to.......

Q.3. Attempt any one of the following: **[10]**

1) If $f(z) = u(x,y) + iv(x,y)$ and $f'(z)$ exists at $z_0 = x_0 + iy_0$. Then prove that the first order partial derivatives of u and v at (x_0, y_0) satisfy $u_x = v_y$ and $u_y = -v_x$ and prove that the function $f(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, f(0) = 0$, is continuous and satisfied C- R equations at the origin, yet $f'(z)$ does not exists.

2) Define harmonic conjugate and prove that the function $u = x^2 - y^2 + xy$ satisfies Laplace's equation and find the corresponding analytic function f(z)

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-I
Internal Examination(2022-23)
Classical Mechanics

Time: 2:00PM–3:00PM **Total Marks: 30**

Date:25/11/2022

Q.1) Choose the correct alternative for the following question. **[05]**

1) The system is said to be in equilibrium, if the generalized forces acting on the system …….

A) are equal to zero **B)** are non-zero **C)** are infinite **D)** none of these

2) If the force is conservative, the work done on the particle around a ………

A) closed path in the force field is zero **B)** open path in the force field is zero

C) closed path in the force field is non zero **D)** none of these

3) Lagranges equation of motion is…

A) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$ **B)** $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j$ **C)** $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = 0$ **D)** $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$

4) Lagranges equation of motion for conservative system is…

A) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$ **B)** $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j$ **C)** $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = 0$ **D)** $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$

19) Lagranges equation of motion for non-conservative system is…

A) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$ **B)** $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j$ **C)** $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = 0$**D)** $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$

5) Lagranges equation of motion for partially conservative and partially non-conservative system is…

A) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$ **B)** $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j$

C) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = \frac{-\partial R}{\partial q_j}$ **D)** $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = -\frac{\partial R}{\partial \dot{q}_j}$

Q.2) Attempt any three **[15]**

1)Find the differential equation of the geodesic on the surface of an inverted cone with semi-vertical angle θ.

2) Obtain the Lagrangian L from the Hamiltonian H and show that it satisfies Lagrange's equations of motion.

3)Show that curve is Catenary for which the area of the surface of revolution is minimum when revolved about Y- axis.

4) Find the extremal of the functional

$\int_0^{\frac{\pi}{2}}(y'^2 - y^2 + 2xy)dx$ subject to the conditions that

$y(0) = 0, y\left(\frac{\pi}{2}\right) = 0$.

Q.3) Attempt any One **[10]**

1) Describe the Routh's procedure to solve the problem involving cyclic and

2) Define orthogonal transformation. Show that in the case of an orthogonal transformation the inverse matrix is identified by the transpose of the matrix.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. II Semester-III Internal Examination :2022-23
MATHEMATICS

Sub: Functional Analysis **Date: 21/11/2022**
Time: 12:30 PM-01:30 PM **Total Marks:30**

Q.1 . Choose correct Alternative for the following. **(5)**

1) Consider following two statements
I) Every normed linear space is a metric space. II) Every metric space is normed linear space.
A) Only II is true. B) I is true and II is false C)Only I is false D) II is true and I is false.

2) Quotient space N/M = {x + M/ x in N} is norm linear space with respect to norm
A) $||x + M|| = \inf \{x + M / x \text{ in } N\}$ B) $||x + M|| = \inf \{x M / x \text{ in } N\}$
C) $||x + M|| = \sup \{x + M / x \text{ in } N\}$ D) $||x + M|| = \{x + M / x \text{ in } N\}$

3) For finite dimensional norm linear space N, dim(N) = 14 then dim(N*) = __
A) 7 B) 28 C) 14 D) 1

4)Every projection on a Banach space B is___
A)Linear, Bounded, Idempotent B) Linear , Idempotent, Continuous
C)Linear , Norm preserving , nilpotent D) Both A and B

5) Consider following two statements
I)Every Banach space is reflexive norm linear space
II)Every reflexive norm linear space is Banach Space
A) Only II is true. B) I is true and II is false C)Only I is false D) II is true and I is false.

Q2) Solve any THREE of the following. (15)

1)Define Banach space. Show that l_∞ (space of all bounded sequences of scalars) which is normed linear space with $|| \, ||_\infty$ given by $||x||_\infty = \sup|x_i|$ for all x in l_∞ is banach space.

2)If N is a normed linear space and x_0 is non zero vector in N then show that there exist a functional f_0 in N* such that $f_0(x_0) = ||x_0||$ and $||f_0|| = 1$

3)If N and N' are norm linear space then show that the set B(N,N') of all continuous linear transformation of N into N' is norm linear space with respect to norm $||T|| = \sup\{||T(x)||, x \text{ is in } N \text{ and } ||x|| \leq 1\}$

4)If $\{T_n\}$ and $\{S_n\}$ are sequences in B(N) such that $T_n \to T$ and $S_n \to S$ as $n \to \infty$ then show that,

a) $T_n + S_n \to T + S$ b) $kT_n \to kT$ for k in F c) $T_nS_n \to TS$ as $n \to \infty$

Q3) Solve any ONE of the following. (10)

1) Define normed linear space. If N and N' are normed linear spaces ,T is linear transformation from N into N' then show that following conditions are equivalent

a)T is continuous on N

b)T is continuous at origin

c)there exist a real number $k \geq 0$ with property $||T(x)|| \leq k||x||$ for all x in N

d)If s = {x in N such that $||x|| \leq 1$ } is closed unit sphere in N then T(S) is bounded in N'

2) Define finite dimensional normed linear space. Prove that, If N is finite dimensional normed linear space then all norms on N are equivalent.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III
Internal Examination(2022-23)
Advanced Discrete Mathematics

Time: 11:30AM to 12:30 PM

Total Marks: 30
Date: 22/11/2022

Q.1) Choose the correct alternative for the following question. **[05]**

i) The order of recurrence relation $a_r - 4a_{r-2} + 3a_{r-3} = 5r + 2$ is -----

A) 0 B) 1 C) 2 D) 3

iii) For a bounded distributive lattice an element can have ----- complement if they exist.

A) only one B) exactly two C) more than two D) zero

iii) Complete bipartite graph $K_{n,n}$ is ----- regular graph.

A) n B) $n-1$ C) $n+1$ D) $n-2$

iv) If tree T has n vertices, then T has exactly ----- number of edges.

A) n B) $n-1$ C) $n+1$ D) $\frac{n}{2}$

v) The adjacency matrix of graph is ----- matrix.

A) diagonal B) scalar C) symmetric D) skew-symmetric

Q.2) Attempt any three **[15]**

i) If T is a tree with at-least two vertices and $P = u_0u_1u_2\ldots u_n$ be a longest path in T, then show that $d(u_0) = d(u_n) = 1$

ii) Prove that graph G is connected iff it has a spanning tree.

iii) Find particular solution of recurrence relation $a_r - 2a_{r-1} = 3(2^r)$

iv) Obtain generating function for the numeric function $a_r = 2r + 3, r \geq 0$

Q.3) Attempt any One **[10]**

i) If A, B, C are three finite sets, then show that
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

ii) Find total solution of recurrence relation $a_r - 5a_{r-1} + 6a_{r-2} = 7^r$

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III Internal Examination : 2022-23
MATHEMATICS

Subject: Lattice Theory **Time: 12:30PM -01:30PM**
Date: 23/11/2022 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i. *Consider the following statements*
Statement $-$ 1) *Every ideal is heriditory subset.*
Statement $-$ 2) *Every heriditory subset is ideal.*
A) *Only* 1) *true* B) *Only* 2) *true* C) *Both* 1)&2) *true* D) *Both* 1)&2) *false.*

ii. *A chain is*______
A) *Complemented lattice* B) *Not complemented lattice*
C) *may be complemented lattice* D) *None of these*

iii. *In the poset* $< \mathbb{Z}^+, | >$
where $\mathbb{Z}^+$ *is the set of positive integers* & | *is divides relation then* 3 & 18 *are*______
A) *Comparable* B) *Parellel* C) *both A)&B)* D) *neither a) nor b).*

iv. *L & M be two sublattices of lattice P then which of the following is also sublattice of P*
A) $L \cap M$ B) $L \cup M$
A) $L \times M$ D) *Both a) & c)*

v. *Consider the following statements*
Statement $-$ 1) $J(L)$ *is not ring of set.*
Statement $-$ 2) $H(J(L))$ *is ring of set.*
A) *Only* 1) *true* B) *Only* 2) *true* C) *Both* 1)&2) *true* D) *none of these*

Q.2. Attempt any three of the following: [15]

1) *Prove that every chain is lattice. what is your opinion about antichain? justify.*

2) *Prove that a poset* $< L, \leq >$ *is lattice iff* $supH$ & $infH$ *exists for every* $\emptyset \neq H \subseteq L$.

3) *If* θ *be a congruence relation on lattice L then show that* $\forall\ a \in L, [a]_\theta$ *is convex sublattice of L.*

4) *Show that set of all ideals of lattice L forms a lattice under set inclusion.*

Q.3. Attempt any one of the following: [10]

1) *In any lattice L prove the following conditions always holds.*

$i)\ (x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z)$

$ii)\ x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z) \quad \forall\, x, y, z \in L$

2) *Prove that a lattice is distributive iff it has no sublattice isomorphic to* M_3 *or* N_5.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III
Internal Examination(2022-23)
Number Theory

Time: 11:30AM to 12:30PM **Total Marks: 30**
Date:24/11/2022

Q.1) Choose the correct alternative for the following question. **[05]**

1) The product of positive divisors of $n > 1$ is equal to…
   A) n B) $n^{\frac{\tau(n)}{2}}$ C) $n^{\tau(n)}$ D) $\frac{\tau(n)}{2}$
2) The product of positive divisors of 15 is equal to…
   A) 15 B) $15^{\frac{\tau(15)}{2}}$ C) $15^{\tau(15)}$ D) $\frac{\tau(15)}{2}$
3) Product of all positive divisors of 16 are…
   A) 1024 B) 1025 C) 1026 D) 1027
4) $\mu(30)$ ...
   A) 0 B) 1 C) -1 D) 2
5) $\mu(2019)$ ...
   A) 0 B) 1 C) -1 D) 2

Q.2) Attempt any three **[15]**

1) By using mathematical induction prove that $21/4^{n+1} + 5^{2n-1}$.

2) Prove that for any positive integer n and a, $\gcd(a, b)\ /n$ and hence prove that $\gcd(a, a + 1) = 1$

3) State and Prove Euclid's Lemma.

4)Solve the linear Diophantine equation $172x + 20y = 1000$.

Q.3) Attempt any One **[10]**

1) Prove that the linear Diophantine equation $ax + by = c$ has a solution iff d/c where $d = \gcd(a, b)$. If $(x_0, y_0)$is any particular solution of this equation then all other solutions are given by $x = x_0 + \frac{b}{d}t$ and $y = y_0 - \frac{a}{d}t$.

2) State and Prove Division Algorithm.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-III
Internal Examination(2022-23)
Operational Research-I

Time: 02:00PM to 03:00 PM **Total Marks: 30**
Date: 25/11/2022

Q.1) Choose the correct alternative for the following question. **[05]**

i) In Big – M method, the coefficient of artificial variable in the objective function for maximization problem is….

A) +M B) -M C) Zero D) None of these

ii) The point at which $\nabla f(x) = 0$ are called …

A) boundary points B) interior points C) extreme points D) convex point

iii) A sufficient condition for a stationary point to be an extreme point is that the Hessian matrix H is evaluated at x_0 is … when x_0 is minimum point

A) Positive definite B) Negative definite
C) Positive semidefinite D) Negative semidefinite

iv)The solution of Dynamic Programming Problem is based upon…

A) Bellman's principle of calculus B) Principle of Optimality
C) Bellman's principle of optimality D) None of these

v) The general NLPP with inequality constraints….

A) Can be solved by using Kuhn –Tucker conditions
B) Can be solved by Lagrange's method
C) Can be solved only if the constraints are of ≤ type

Q.2) Attempt any three **[15]**

i) Explain the model of communication system.

ii) Show that the set S = $\{(x_1, x_2): x_1^2 + x_2^2 + x_3^2 \leq 1\}$ *is convex set*

iii) Solve the following non – linear programming problem

Min Z = $2x_1^2 + 2x_2^2 + 2x_3^2 - 24x_1 - 8x_2 - 12x_3 + 200,$

subject to $x_1 + x_2 + x_3 = 1,\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

iv) Find the extreme point of the function

$f(x) = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 64$

Q.3) Attempt any One **[10]**

i) Define quadratic programming problem. Solve the following quadratic programming problem by Beal's Method. Max $Z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$,

$subject\ to\ x_1 + 2x_2 + x_3 = 10,\ x_1 + x_2 + x_4 = 9,$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0$

ii) Solve the following LPP Max Z = $6x_1 + 4x_2$

$subject\ to\ 2x_1 + 3x_2 \leq 30,\ 3x_1 + 2x_2 \leq 24,\ x_1 + x_2 \geq 3, x_1 \geq 0, x_2 \geq 0$

"Education for Knowledge, Science, and Culture"
- Shikshanmaharshi Dr. Bapuji Salunkhe

Shri Swami Vivekanand Shikshan Sanstha's

Vivekanand College, Kolhapur (Autonomous)

KOLHAPUR (AUTONOMOUS)

Date : 17/04/2023

Department Of Mathematics
Notice(2022-2023)

All the students of M.Sc I(Mathematics)- semester II, M.Sc II(Mathematics) semester IV are hereby informed that internal examination will be conducted as follows.

Class	Subject	Date and Time
M.Sc. I	Linear Algebra (CP-1175B)	25/04/2023 (3:00 PM- 4:00 PM)
M.Sc. II	Field Theory (CC-1190D)	25/04/2023 (3:00 PM- 4:00 PM)
M.Sc. I	Numerical Analysis (CP-1179B)	26/04/2023 (3:00 PM- 4:00 PM)
M.Sc. II	Measure and Integration (CC-1191D)	26/04/2023 (3:00 PM- 4:00 PM)
M.Sc. I	General Topology (CP-1177B)	27/04/2023 (3:00 PM- 4:00 PM)
M.Sc. II	Algebraic Number Theory (CBC-1192D)	27/04/2023 (3:00 PM- 4:00 PM)
M.Sc. I	Partial Differential Equations (CP-1178B)	28/04/2023 (3:00 PM- 4:00 PM)
M.Sc. II	Operational Research II(CBC-1194D)	28/04/2023 (3:00 PM- 4:00 PM)
M.Sc. I	Integral Equation (CP-1176B)	29/04/2023 (3:00 PM- 4:00 PM)
M.Sc. II	Combinatorics (CBC-1198D)	29/04/2023 (3:00 PM- 4:00 PM)

Syllabus for M. Sc. I Sem. II

Sr. No.	Name of Paper	Topics
1	Linear Algebra (CP-1175B)	Unit I
2	Numerical Analysis (CP-1179B)	Unit I
3	General Topology (CP-1177B)	Unit I
4	Partial Differential Equations (CP-1178B)	Unit I
5	Integral Equation (CP-1176B)	Unit I

Syllabus for M. Sc. II Sem. IV

Sr. No.	Name of Paper	Topics
1	Field Theory(CC-1190D)	Unit I
2	Measure and Integration(CC-1191D)	Unit I
3	Algebraic Number Theory(CBC-1192D)	Unit I
4	Operational Research II(CBC-1194D)	Unit I
5	Combinatorics (CBC-1198D)	Unit I

Nature of question paper

Time:-1 Hours **Total Marks: 30**

Q.1) Choose the correct alternative for the following question. [05]

Five questions

Q.2) Attempt any three [15]

Four questions

Q.3) Attempt any One [10]

Two questions

VIVEKANAND COLLEGE KOLHAPUR ESTD JUNE 1964 (Empowered Autonomous)

Sthorat

(Prof. S. P. Thorat)
HEAD
DEPARTMENT OF MATHEMATICS
VIVEKANAND COLLEGE, KOLHAPUR
(EMPOWERED AUTONOMOUS)

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II Internal Examination: 2022-23
MATHEMATICS

Subject: Linear Algebra
Date: 25/04/2023
Time: 03: 00 PM-04:00pm
Total Marks: 30

Q. 1 Select the correct alternative for each of the following: **[5]**

1] A zero vector is always ______.

i) linearly dependent ii) linearly independent
iii) member of any basis iv) none of these

2] If T: $\mathbb{R}^2 \to \mathbb{R}^2$ and S: $\mathbb{R}^2 \to \mathbb{R}^3$ defined by T(x,y)=(y,x) and S(x,y) = (x+y , x-y, y) then ST(x,y) = ______.

i) (y+x, y-x, x) ii)(x-y, x+y, x)
iii) (x-y, x+y, y) iv)(y+2x, y-x, x)

3] If dim V=n and S={v1,v2, …,vn} spans V then S is ______ of V.

i) a subspace ii) a basis
iii) a linearly dependent subset iv) the smallest subspace

4] If T is a linear operator on $\mathbb{R}^2$ defined by T(x_1 , x_2) = (0 , 0) then rank of T= ___ .

i) 3 ii) 0
iii) 2 iv) 1

5] If α_1v1+α_2v2+ …+$\alpha_n$vn=0, where v1, v2, … ,vn are linearly independent vectors in a vector space V(F), then ______.

i) i=0 for all i=1,2, …, n ii) i≠0 for all i=1,2, …, n
iii) i=0 for at least one i iv) i≠0 for at least one i

Q.2. Attempt any three of the following: **[15]**

1]Define intersection of two subspaces. Prove that intersection of two subspaces is again a subspace.

2] Define isomorphism. Show that identity transformation from vector space V to V is isomorphism.

3] Prove that, If W is subspace of vector space V(F) then L(W) = W and conversely.

4] Define basis of V(F). Show that {(1,0),(0,1)} forms basis of $\mathbb{R}^2(R)$.

5] If S: $\mathbb{R}^2 \to \mathbb{R}^3$ is linear transformation such that, S(a, b) = (a+b ,a-b ,b). Find range, rank, kernel and nullity.

Q.2. Attempt any one of the following: **[10]**

1] State and prove Rank-nullity theorem

2] Define direct sum. Prove that V is direct sum of U, W if and only if V = U + W and U ∩ W = {0}.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II
Internal Examination(2022-23)

Subject : Numerical Analysis **Total Marks: 30**

Date:26/04/2023 **Time: 03:00PM to 04:00PM**

Q.1) Choose the correct alternative for the following question. [05]

1) The error in Trapezoidal rule is -------

A) $-\frac{h^3}{12}f''(\xi)$ B) $-\frac{h^2}{12}f''(\xi)$ C) $-\frac{h}{12}f''(\xi)$ D) $\frac{h^3}{12}f''(\xi)$

2) Using Simpson's $\frac{1}{3}$rd rule the value of integral $\int_0^2 \frac{dx}{5+3x}$ = -----

A) 0.2909 B) 0.5273 C) 0.2636 D) 0.3626

3) If the method is explicit A is lower triangular matrix with diagonal entries -----

A) 1 B) -1 C) 0 D) 2

4) The order of a tree is the ------

A) Number of vertices in the tree B) Number of edges of tree

C) Number of roots of tree D) None Above

5) The stability region of fourth order Runge – Kutta method is -------

A) $\left\{z \in \mathbb{C} / \left|1 + z + \frac{z^2}{2!} + \frac{z^3}{3!}\right| < 1\right\}$ B) $\left\{z \in \mathbb{C} / \left|1 + z + \frac{z^2}{2!}\right| < 1\right\}$

C) $\left\{z \in \mathbb{C} / \left|1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!}\right| < 1\right\}$ D) $\left\{z \in \mathbb{C} / \left|1 + z + \frac{z^2}{2} + \frac{z^3}{3} + \frac{z^4}{4}\right| < 1\right\}$

Q.2) Attempt any three [15]

1) Determine the rate of convergence of Regula Falsi method.

2) Derive Gauss Legendre integration method for n=1

3) Estimate the eigen value of the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ by using Gerschgorin theorem and Brauer theorem.

4) Answer the following.

a)Statbility region of Runge-Kutta third order method b) Diagonally dominant matrix

c) Quadrature formula d) Stability region of Runge-Kutta second order method

e) The coefficient tableau

Q.3) Attempt any One [10]

1) State and prove Brauer theorem.

2) Evaluate the integral $\int_0^1 \frac{1}{1+x}dx$ using Gauss Legendre three point formulas.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II
Internal Examination(2022-23)

Subject: General Topology **Total Marks: 30**
Date: 27/04/2023 **Time: 03:00 PM to 04:00**

Q.1) Choose the correct alternative for the following question. **[05]**

1) Boundary point set of a set of integer Z is

a) N b) Z c) R d) Q

2) A property of a subspace of a topological space is .... if each of its subspace has the same property

a) connected b) compact c) hereditary d) countable

3) If (X, τ) is a topological space and $Y \subset X$ and (Y, τ_Y) is relative topology and $A \subseteq Y$. Then

a) $\mathrm{int}_X(A) \subseteq \mathrm{int}_Y(A)$ b) $\mathrm{int}_X(A) \supseteq \mathrm{int}_Y(A)$

c) $\mathrm{int}_X(A) = \mathrm{int}_Y(A)$ d) none of them

4) In Discrete topology , (X, D) is separable if and only if X is ....

a) uncountable b) countable c) infinite d) finite

5) Which of the following property is not hereditary property?

a) Discreteness b) indiscreetness c) separability d) T_1 space

Q.2) Attempt any three **[15]**

1) Let (Y, τ_Y) be a subspace of (X, τ) then show that a subset M of Y is a τ_Y neighbourhood of a point $y \in Y$ if and only if $M = N \cap Y$ for some τ neighbourhood of Y.
2) Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, X, \{b\}, \{a, b\}, \{a, b, c\}\}$ and $Y \subset X$ such that $Y = \{a, b, d\}$ Find a) τ *and* τ_Y closed set
   b) closure set of $A = \{a, d\}$ relative to Y and X
   c) derived set of $B = \{c, e\}$ relative to Y and X
3) Prove that regularity of topological space is hereditary
4) Prove that a topological space (X, τ) is T_1 if and only if every singleton subset $\{x\}$ of X is τ closed

Q.3) Attempt any One **[10]**

1) State and prove Lindelof theorem
2) Prove that every compact Hausdorff space is norma

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-I) Semester-II
Internal Examination(2022-23)
Partial Differential Equations

Time: 3:00 PM to 4:00PM **Total Marks: 30**
Date : 28/04/2023

Q.1) Choose the correct alternative for the following question. **[05]**

1) *The equation* $(x^2+z^2)p-xyq = z^3x$ *is*
   a) Linear b) semilinear c) Quasilinear d) Nonlinear
2) *The complete integral of* $z=px+qy+pq$ *is*
   a) $z=a+b+ab$ b) $z=ax+by+ab$ c) $z=c$ d) none of these
3) *The complete integral of* $z=px+qy+\sqrt{pq}$ *is*
   a) $z=a+b+ab$ b) $z=ax+by+\sqrt{pq}$ c) $z=c$ d) none of these
4) *The equation...represents the set of all right circular cones with x−axis as the axis of symmetry.*

   a) $(x^2+y^2) = (z-c)^2 tan^2(\alpha)$ b) $(x^2-y^2) = (z-c)^2 tan^2(\alpha)$

   c) $(z^2+y^2) = (x-c)^2 tan^2(\alpha)$ d) $(x^2+z^2) = (y-c)^2 tan^2(\alpha)$

5) *The equation* $Ruxx+Suxy+Tuyy+g=0$ *is parabolic if...*
   a) $S^2-4RT<0$ b) $S^2-4RT>0$ c) $S^2-4RT=0$ d) None of these

Q.2) Attempt any three **[15]**

1) *Find the general integral of* $(x^2+y^2)p+2xyq=(x+y)z$.

2) *Form partial differential equation from* $z^2(1+a^3) = 8(x+ay+b)^3$

3) *Find the general solution of* $p+q=2\sqrt{z}$.

4) *Obtaine pde by eliminating a,b from* $z = ax^2+by^2+c$

Q.3) Attempt any One **[10]**

1) *Obtaine the d'Alemberts solution of the one dimensional wave equation which describes the vibrations of an semi infinite string.*

2) *If* $\overline{X} = (P,Q,R)$ *is a vector such that* $\overline{X}.\,curl\,\overline{X} = 0$ *&* μ *is an arbitrary differentiable of* x, y, z *then prove that* $\mu\overline{X}.\,curl\,\mu\overline{X} = 0$

Vivekanand College, Kolhapur (Autonomous)

M.Sc. (Part-I) Semester-II

Internal Examination: 2022-23

Sub: Integral Equations **Date : 29/04/2023**

Total Marks: 30 **Time : 03:00pm-04:00pm**

Q.1) Choose the correct alternative for the following question. [05]

1) The type of integral equation $g(s) = f(s) + \lambda \int K(s,t)g(t)dt\ b\ a$ is -----
   a) Volterra integral equation of 1st kind
   b) Fredholm integral equation of 1st kind
   c) Homogeneous Fredholm integral equation of 2nd kind
   d) Non-homogeneous Fredholm integral equation of 2nd kind
2) The homogeneous Fredholm integral equation has trivial solution, if -----
   a) $D(\lambda) = 0$ b) $D(\lambda) \neq 0$ c) $D(\lambda)$ does not exist d) none of these
3) The eigen values of non-zero symmetric kernel are -----
   a) real b) zero c) only imaginary d) none of these
4)) Spectrum of symmetric kernel is always -----
   a) empty b) non-empty c) does not exist d) none of these
5) A symmetric kernel possesses ----- eigen value.
   a) only one b) at-least one c) at-most one d) none of these

Q.2) Attempt any three [15]

1) Convert the following initial value problem to an integral equation.
   $y'' + y = cosx, y(0) = 0, y'(0) = -1$
2) Prove that eigen functions $g(s)$ and $\psi(s)$ corresponding to distinct eigen values λ_1 and λ_2 respectively of the homogeneous integral equation $g(s) = \lambda \int K(s,t)g(t)dt$ and its transpose are orthogonal.
3) Convert the following boundary value problem to an integral equation. $y'' + xy = 1$, $y(0) = 0, y(1) = 1, 0 \leq x \leq 1$
4) Find the eigen values and eigen functions of the homogeneous integral equation

   $g(s) = \lambda \int_0^1 (6s - 2t)g(t)dt$

Q.3) Attempt any One [10]

1) Describe the procedure of solving non-homogeneous Fredholm integral equation of 2nd kind with separable kernel.

2) Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 (1 - 3st)g(t)dt$ by discussing all possible cases.

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV Internal Examination : 2022-23
MATHEMATICS

Subject : Field Theory **Time: 03: 00-04:00pm**
Date:25/04/2023 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i. e and π are ……… elements over Q

A) Transcendental B) Algebraic C) Irreducible D) Reducible

ii. Polynomial of degree one is always ……

A) Inseparable B) Separable C) Monic D) Simple

iii. If $f(x)$ is of degree 3 , Then $f(x)$ has …. Root.

A) Complex B) Unique C) Distinct D) Real

iv. Any subgroup and any quotient group of a ……… group is solvable.

A)Solvable B) Normal C) Separable D) None

v. If $F \subseteq K \subseteq L$ are fields. If $a \in L$ be algebraic over K and K is an algebraic extension of F. Then, a is …….

A) Algebraic over K B) Algebraic Over F C) Algebraic D) Separable

Q.2. Attempt any three of the following: **[15]**

i. If $F \subseteq K \subseteq L$ are field and $[L : F]$ is prime number , Then $K = L$ or $K = F$

ii. If K be smallest subfield of R containing $Q \cup \{\sqrt{2}, \sqrt{3}\}$. Then, find a basis of K over Q.

iii. If F be any field and $p(x) \in F[x]$ be irreducible over F . Then prove that, there exist a field extension E of F such that E contains a root of p(x)

iv. If F be a field and f(x) be nonconstant polynomial over F. Then prove that, F has field extension E containing a root of f(x)

Q.3. Attempt any one of the following: **[10]**

i. A. If F be any field, K be a field extension of F and L be a field extension of K . Then prove that $[L : F]$ is finite if and only if both $[L : K]$ and $[K : F]$ are finite and in case, $[L : F] = [L : K][K : F]$

ii. If $F \subseteq K \subseteq L$ are field and $[L : F]$ is finite , Then prove that $[L : K]$ and $[K : F]$ are divisors of $[L : F]$

Vivekanand College, Kolhapur (Autonomous)

M.Sc. (Part-II) Semester-IV Internal Examination: 2022-23

Subject :Measure and Integration **Time: 03: 00 PM**

Date:26/04/2023 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

i. *If X is a set, which one of the following is the smallest* $\sigma -$ *algebra of subsets of* X?

$A)\ \{\emptyset, X\}$ $B)\ \{\emptyset\}$ $C)\ P(X)$ $D)\ \{X\}$

ii. *If* $\mathbb{Q}$ *is set of all rational numbers then* $m^*(\mathbb{Q} - \mathbb{Q}^c) =$ ______

$A)\ 0$ $B)\ 1$ $C)\ 2^C$ $D)\ \infty$

iii. *A set F is* G_δ *set if it is* ______

A) *Countable union of open sets* B) *Countable intersection of open sets*

C) *Countable union of closed sets* D) *Countable intersection of closed sets*

iv. *For* $1 < p < \infty$, q *the conjugate of* p, & *any two positive numbers* a & b

$A)\ ab \geq \frac{a^p}{p} + \frac{b^q}{q}$ $B)\ ab = \frac{a^p}{p} + \frac{b^q}{q}$

$C)\ ab > \frac{a^p}{p} + \frac{b^q}{q}$ $D)\ ab \leq \frac{a^p}{p} + \frac{b^q}{q}$

i. *If* $A_n = \left(\frac{-1}{n+1}, \frac{1}{n+1}\right)$ *then* $\bigcap_{n=1}^{\infty} A_n$ *is* ________

$a)\ 1$ $b)\ 0$ $c)\ \infty$ $d)\ \frac{2}{n}$

Q.2. Attempt any three of the following: **[15]**

1) *If* E_1 *is measurable set* & $m^*(E_1 \Delta E_2) = 0$ *then show that* E_2 *is measurable.*

2) *If a function* f *is measurable then prove that the set* $\{x | f(x) = c\}$ *is measurable. for all* c *in* $\mathbb{R}$.

3) *Prove that* f *is measurable if and only if* f^+ & f^- *are measurable.*

4) *If* $\phi = 2\chi_A + 3\chi_B$ *then find* $\int \phi$, *where* $A = [2,3]$ & $B = [4,7]$.

Q.3. Attempt any one of the following: **[10]**

1) *State & prove Holders inequality.*

2) *State & Prove Fatous lemma.*

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV
Internal Examination(2022-23)
Algebraic Number Theory

Time: 3:00 PM – 4:00 PM **Total Marks: 30**
Date:27/04/2023

Q.1) Choose the correct alternative for the following question. [05]

1)A commutative division ring is ....

(a)Finite integral domain (b) Integral domain

(c) Zero ring (d) None of these

2) Let $x, y \in D$. x and y are associates then which of the following conditions satisfies

(a)$x = yu$ for $u \in D$ (b) $xy = u$ for u is a unit in D

(c) $x \mid y$ and $y \mid x$ (d) All of the above

3)If U is an ideal of R and $1 \in U$, then ....

(a)U is a proper subset of R (b) U is equal R

(c) U is a super set of R (d) $U = \phi$

4)Every integral domain is not a ....

(a)Field (b) Commutative ring

(c) Ring (d) Abelian group with respect to addition

5)If the ring R is such that $(ab)^2 = a^2b^2$, $a, b \in R$, then ....

(a) R is commutative (b) R is non-commutative

(c) R is Zero ring (d) None of these

Q.2) Attempt any three [15]

1) Let $M = M_1 \oplus M_2$ then prove that $\frac{M}{M_1} \cong M_2$ and $\frac{M}{M_2} \cong M_1$
2) Find the number θ such that $Q(\theta) = Q(\sqrt{2}, \sqrt[3]{3})$.
3) Show that a prime element in D is always irreducible.
4) Prove that every Euclidean Domain is principle ideal domain

Q.3) Attempt any One [10]

1) In a domain D, in which factorization into irreducible is possible, prove that the factorization is unique iff every irreducible is prime.
2) Show that factorization of elements of O into irreducible is unique if and only if every ideal of O is principal

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV
Internal Examination(2022-23)
Operational Research-II

Time: 03:00PM to 04:00 PM **Total Marks: 30**
Date :28/04/2023

Q.1) Choose the correct alternative for the following question. [05]

i) The problem of replacement is not concerned about the.....
A) item that deteriorate graphically
B) items that fail suddenly
C) determination of optimum replacement interval
D) maintenance of an item to work out profitably

ii) In dummy activity in a project network always has a .... duration.
A) one B) two C) zero D) Three

iii) Queue can form only when......
A) arrivals exceed service facility
B) arrivals equals service facility
C) service facility is capable to serve all the arrivals at a time
D) there are more than one service facility

iv) .... occurs when a waiting customer leaves the queue due to impatience.
A) Reneging B) Balking C) Jockeying D) None of these

v) The present worth factor of one rupee spent in n years with r interest rate is given by.......
A) $\frac{1}{1+r}$ B) $\frac{1}{(1+r)^n}$ C) $\frac{1}{(1+r)^{-n}}$ D) None of these

Q.2) Attempt any three [15]

i) A TV repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they come in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 – hour day, what is the repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?

ii) Draw a network diagram for the following project and number the events according to Fulkerson's rule

Activity no.	A	B	C	D	E	F	G	H	I	J	K	L

Preceding activity	-	-	A	B	C	E	F	F	H	G, I	D, G	K

iii) Explain the model of communication system

iv) Workers come to tool store room to receive special tools for accomplishing a particular project assigned to them. The average time between two arrivals is 60 second and arrival assumed to be in Poisson distribution. The average service time(of the tool room attendant) is 40 seconds. Determine **[05]**

a) Average queue length

b) average length of non-empty queue

c) average numbers of workers in the system including worker being attendant

d) mean waiting time of arrival

e) average waiting time of an arrival(worker) who waits.

Q.3) Attempt any One **[10]**

i) Obtain the steady state solution of $(M/M/1)$: $(\infty/FCFS)$ system and solve this Equation

ii) Derive the replacement policy of items whose maintenance costs increases with time. You may assume that money value also changes with time

Vivekanand College, Kolhapur (Autonomous)
M.Sc. (Part-II) Semester-IV
Internal Examination: 2022-23
MATHEMATICS

Subject: Combinatorics **Time: 3:00PM-4:00PM**

Date: 29/04/2023 **Total Marks: 30**

Q. 1 Select the correct alternative for each of the following: **[5]**

I. *The number of circular permutation of* 5 *objects is*

$a)$ 120 $b)$ 24 $c)$ 6 $d)$ 5

II. *The Ramsey Number* $R(3,3) =$ ______

$a)$ 6 $b)$ 5 $c)$ 4 $d)$ 0

III. *The number of three character string that can be formed using* 26 *letters of alphabate*

$a)$ 26 $b)$ 26^2 $c)$ 26^3 $d)$ *None of these*

IV. *The type of permutation* $(1,3,2,4) \in S_4$ *is*

$a)$ [1111] $b)$ [1000] $c)$ [0000] $d)$ [0001]

V. *The total number of* 6 *digit number in which all the odd digits & only odd digits appear is*

$a)$ 6! $b)$ $\frac{5}{2}(6!)$ $c)$ $\frac{1}{2}(6!)$ $d)$ *none of these*

Q.2. Attempt any three of the following: **[15]**

$i)$ *Give a combinatorial proof of* $C(m+n,2) - C(m,2) - C(n,2) = mn$

$ii)$ *By using product rule find the total number of divisors of* 88

$iii)$ *Find the coefficient of* $p^2q^3r^3s^4$ *in the expansion* $(2p - 3q + 2r - s)^{12}$

$iv)$ *Show that in agroup of* 6 *people there will always be a subgroup of* 3 *people who are pairwise friends or a subgroup of* 3 *people who are pairwise strangers.*

Q.3. Attempt any one of the following: **[10]**

$i)$*Let* X *be a finite set & Let* G *be a group of permutations of* X *then*

$a)$ *Prove that distinct orbits with respect to* G *forms a partition of* X.

$b)$ *With usual notations show that* $\sum_{x\in X}|G_x| = \sum_{g\in G}|F(g)|$ *where* G_x *is stabilizer &*$F(g)$*is permutation character.*

$ii)$ *Find a cycle index of dihedral group on symmetries of square*

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
- शिक्षणमहर्षी डॉ. बापूजी साळुंखे

36593

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Suppliment No. :

Roll No. : 1203

Class : MSc I

Subject : Numerical Analysis

Test / Tutorial No. : Internal Exam

Div. : —

29/30

Q1.

1) d) $\Delta p = \frac{b_{n-1} c_{n-2} - b_n c_{n-3}}{c^2_{n-2} + c_{n-3}(b_{n-1} - c_{n-1})}$

2) c) 2

3) a) $|\varepsilon_{k+1}| \le C \cdot |\varepsilon_k|^p$.

5

4) c) $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

5) d) Intermediate value theorem.

Q2.

1) Given equation is,

$$F(x) = \cos x - x \cdot e^{x} = 0$$

Initial approximations are $x_0 = 0$ and $x_1 = 1$

$f(x_0) = f(0) = 1, \quad f(x_1) = f(1) = -2.1780$

$\therefore f(x_0) \cdot f(x_1) < 0$

$\therefore$ Root lies betn $(0, 1)$

By secant method we have,

$$x_{k+1} = x_k - \frac{(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})} \cdot f(x_k)$$

$$= \frac{x_k f(x_k) - x_k \cdot f(x_{k-1}) - x_k \cdot f(x_k) + x_{k-1} \cdot f(x_k)}{f(x_k) - f(x_{k-1})}$$

$$x_{k+1} = \frac{x_{k-1} \cdot f(x_k) - x_k \cdot f(x_{k-1})}{f(x_k) - f(x_{k-1})}$$

i) For $k = 1$

$$x_2 = \frac{x_0 \cdot f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{(0) \cdot (-2.1780) - (1)(1)}{-2.1780 - 1}$$

$$= \frac{-1}{-3.1780}$$

$$= 0.3147.$$

$\therefore f(x_2) = 0.5198$

ii) For $k = 2$

~~$x_3 = x_2 \cdot f(x_1) - x_1 f(x$~~

$$x_3 = \frac{x_1 \cdot f(x_2) - x_2 \cdot f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{(1)(0.5198) - (0.3147)(-2.1780)}{0.5198 + 2.1780}$$

$$= \frac{1.2052}{2.6978} = 0.4467$$

$x_3 = 0.4467$.

$f(x_3) = 0.2036$

iii) For $k = 3$

$$x_4 = \frac{x_2 \cdot f(x_3) - x_3 \cdot f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{(0.3147) \cdot (0.2036) - (0.4467)(0.5198)}{0.2036 - 0.5198}$$

$$= \frac{-0.1681}{-0.3162}$$

$$= 0.5316$$

$f(x_4) = -0.0426$

0.5 iv) For $k = 4$

$$x_5 = \frac{x_3 \cdot f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{(0.4467)(-0.0426) - (0.5316)(0.2036)}{-0.0426 - 0.2036}$$

$$= \frac{-0.0190 - 0.1082}{-0.2462}$$

$x_4 = 0.5167$.

∴ Approximate sol^n upto four iterations is 0.5167.

2) Given equation is Polynomial is

$P_4(x) = x^4 + x^{3}_{+} 2x^2 + x + 1$

Initial approximations are

$p_0 = 0.5$ and $q_0 = 0.5$.

~~Now~~ 1^{st} iteration is.

$-p_0 = -0.5$	1	1	2	1	1
$-q_0 = -0.5$		-0.5	-0.25	-0.625	0.125
			-0.5	-0.25	-0.625
$-p_0 = -0.5$	1	0.5	1.25	0.125	$0.5 = b_4$
$-q_0 = -0.5$		-0.5	0	-0.375	
			-0.5	0	
	1	0	0.75	$-0.25 = c_3$	

Now,

$$\Delta P = \frac{-(b_n c_{n-3} - b_{n-1} c_{n-2})}{c_{n-2}^2 - c_{n-3}(c_{n-1} - b_{n-1})}$$

$$= \frac{-(b_4 c_1 - b_3 c_2)}{c_2^2 - c_1(c_3 - b_3)}$$

$$= \frac{-[(0.5)(0) - (0.125)(0.75)]}{(0.75)^2 - (0)(-0.25 - 0.125)}$$

$$= \frac{0.0938}{0.5625}$$

$$= 0.1668$$

$$\therefore P_1 = \Delta P + P_0$$

$$= 0.1668 + 0.5$$

$$P_1 = 0.6668.$$

$$\Delta q = \frac{-(b_{n-1}(c_{n-1} - b_{n-1}) - b_n \cdot c_{n-2})}{c_{n-2}^2 - c_{n-3}(c_{n-1} - b_{n-1})}$$

$$= \frac{-(b_3(c_3 - b_3) - b_4 c_2)}{c_2^2 - c_1(c_3 - b_3)}$$

$$= \frac{-[0.125(-0.25 - 0.125) - 0.5 \cdot (0.75)]}{(0.75)^2 - 0(c_3 - b_3)}$$

$$= \frac{0.0469 + 0.3750}{0.5625}$$

$$\Delta q = 0.75.$$

$$\therefore q_1 = \Delta q_0 + q_0$$

|| ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ||
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

36612

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Suppliment No. : 1

Roll No. : 1203

Class : MSc - I

Subject : N A.

Test / Tutorial No. :

Div. :

$q_1 = 0.75 + 0.5$

$q_1 = 1.25.$

4. Let,

$x = 17^{1/3}$

$x^3 = 17$

$\therefore F(x) = x^3 - 17$

$\therefore F'(x) = 3x^2$ $\qquad x_0 = 2.$

By Newton Raphson method

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$= x_k - \frac{x_k^3 - 17}{3x_k^2}$$

$$= \frac{3x_k^3 - x_k^3 + 17}{3x_k^2}$$

$$= \frac{2x_k^3 - 17}{3x_k^2}$$

For $k = 0$

$$x_1 = \frac{2x_0^3 + 17}{3x_0^2}$$

$$= \frac{2(2)^3 + 17}{3(2)^2}$$

$$= \frac{33}{12}$$

$$x_1 = \underline{-0.0833} \quad 2.75$$

For $k = 1$

$$x_2 = \frac{2x_1^3 + 17}{3(x_1)^2}$$

$$= \frac{2(2.75)^3 + 17}{2(2.75)^2}$$

$$= \frac{58.5938}{15.1250}$$

$$x_2 = 3.8740$$

For $k = 2$

$$x_3 = \frac{2x_2^3 + 17}{3(x_2)^2}$$

$$= \frac{2(3.8740)^3 + 17}{3(3.8740)^2}$$

04

$$= \frac{133.2810}{45.0236}$$

$$= 2.9602$$

For $k = 3$

$$x_4 = \frac{2x_3^3 + 17}{3(x_3)^2}$$

$$= \frac{2(2.9602)^3 + 17}{3(2.9602)^2}$$

$$= \frac{68.8792}{26.2884}$$

$$x_4 = 2.6201$$

∴ Approximate solution is 2.6201.

Q3.

1) We assume that simple root of $f(x)=0$ is ξ.

$\therefore f'(\xi) \neq 0$

Now substituting $x_k = \varepsilon_k + \xi$ in secant method

$$x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} \cdot f(x_k)$$

$$\varepsilon_{k+1} + \xi = \varepsilon_k + \xi - \frac{(\varepsilon_k + \xi) - (\varepsilon_{k-1} + \xi) \cdot f(\varepsilon_k + \xi)}{f(\varepsilon_k + \xi) - f(\varepsilon_{k-1} + \xi)}$$

$$\varepsilon_{k+1} = \varepsilon_k - \frac{(\varepsilon_k - \varepsilon_{k-1}) \cdot f(\varepsilon_k + \xi)}{f(\varepsilon_k + \xi) - f(\varepsilon_{k-1} + \xi)} \quad \text{--- (1)}$$

$$f(\varepsilon_k + \xi) = f(\xi) + \varepsilon_k \cdot f'(\xi) + \frac{\varepsilon_k^2}{2!} f''(\xi) + \cdots$$

[Tailors series]

as $f(\xi) = 0$

$$f(\varepsilon_k + \xi) = \varepsilon_k \cdot f'(\xi) + \frac{\varepsilon_k^2}{2!} f''(\xi) + \cdots$$

Equation (1) becomes,

$$\varepsilon_{k+1} = \varepsilon_k - \frac{(\varepsilon_k - \varepsilon_{k-1}) \left[\varepsilon_k \cdot f'(\xi) + \frac{\varepsilon_k^2}{2!} \cdot f''(\xi) + \cdots\right]}{\left[\varepsilon_k \cdot f'(\xi) + \frac{\varepsilon_k^2}{2!} f''(\xi) + \cdots\right] - \left[\varepsilon_{k-1} f'(\xi) + \frac{\varepsilon_{k-1}^2}{2!} f''(\xi) + \cdots\right]}$$

$$= \varepsilon_k - \frac{(\varepsilon_k - \varepsilon_{k-1}) \left[\varepsilon_k \cdot f'(\xi) + \frac{\varepsilon_k^2}{2!} f''(\xi) + \cdots\right]}{f'(\xi)\left[\varepsilon_k - \varepsilon_{k-1}\right] + \left[\frac{\varepsilon_k^2}{2!} - \frac{\varepsilon_{k-1}^2}{2!}\right] f''(\xi)}$$

$$= \varepsilon_k - \frac{(\varepsilon_k - \varepsilon_{k-1}) \left[\varepsilon_k f'(\xi) + \frac{\varepsilon_k^2}{2!} f''(\xi) + \cdots\right]}{(\varepsilon_k - \varepsilon_{k-1}) \cdot f'(\xi) \left[1 + \frac{\varepsilon_k + \varepsilon_{k-1}}{2} \frac{f''(\xi)}{f'(\xi)}\right]}$$

$$= \varepsilon_k - \frac{\left[\varepsilon_k + \frac{\varepsilon_k^2}{2!}\frac{f''(\xi)}{f'(\xi)} + \cdots\right]}{\left[1 + \frac{\varepsilon_k + \varepsilon_{k-1}}{2} \cdot \frac{f''(\xi)}{f'(\xi)} + \cdots\right]}$$

$$= \varepsilon_k - \left[\varepsilon_k + \frac{\varepsilon_k}{2}\frac{f''(\xi)}{f'(\xi)} + \cdots\right]\left[1 + \frac{\varepsilon_k + \varepsilon_{k-1}}{2} \cdot \frac{f''(\xi)}{f'(\xi)} + \cdots\right]^{-1}$$

$$\doteq \varepsilon_k - \left[\varepsilon_k + \frac{\varepsilon_k}{2}\frac{f''(\xi)}{f'(\xi)} + \cdots\right]\left[1 - \frac{\varepsilon_k + \varepsilon_{k-1}}{2}\frac{f''(\xi)}{f'(\xi)} + \cdots\right]$$

$$= \varepsilon_k - \varepsilon_k + \varepsilon_k\left(\frac{\varepsilon_k + \varepsilon_{k-1}}{2}\right) \cdot \frac{f''(\xi)}{f'(\xi)} + \frac{\varepsilon_k}{2}\frac{f''(\xi)}{f'(\xi)} + O(\varepsilon_{k-1} \cdot \varepsilon_k + \varepsilon_k)$$

$$= -\frac{\varepsilon_k^2}{2} \cdot \frac{f''(\xi)}{f'(\xi)} + \frac{\varepsilon_k(\varepsilon_{k-1}) f''(\xi)}{2\, f'(\xi)} + \frac{\varepsilon_k^2}{2}\frac{f''(\xi)}{f'(\xi)} + O(\varepsilon_{k-1} \cdot \varepsilon_k + \varepsilon_k)$$

$$= \frac{\varepsilon_k \cdot \varepsilon_{k-1}}{2} \cdot \frac{f''(\xi)}{f'(\xi)} + O(\varepsilon_{k-1} \cdot \varepsilon_k + \varepsilon_k)$$

$$\varepsilon_{k+1} = \varepsilon_k \cdot \varepsilon_{k-1} \cdot \frac{1}{2}\frac{f''(\xi)}{f'(\xi)} + O(\varepsilon_{k-1} \cdot \varepsilon_k + \varepsilon_k)$$

$$\therefore \varepsilon_{k+1} = \varepsilon_k \cdot \varepsilon_{k-1} \cdot c \qquad \text{--- (2)}$$

where, $c = \frac{1}{2}\frac{f''(\xi)}{f'(\xi)}$

eqn (2) is called errorор eqn.

To determine rate of convergence of Secant method

By defn of Rate of convergence

|| ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ||
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

36505

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor:

Suppliment No. : 2

Roll No. : 1203

Class : MSc I (Maths)

Subject :

Test / Tutorial No. :

Div. :

$\varepsilon_{k+1} = A \cdot \varepsilon_k^p$ ——— ③

By eqn ③

$\varepsilon_k = A \cdot \varepsilon_{k-1}^p$

$\varepsilon_k^{1/p} = A^{1/p} \cdot \varepsilon_{k-1}$

$\varepsilon_{k-1} = \varepsilon_k^{1/p} \cdot A^{-1/p}$

Eqn ② becomes

$\varepsilon_{k+1} = C \cdot \varepsilon_k \cdot \varepsilon_{k-1}$

$A \cdot \varepsilon_k^p = C \cdot \varepsilon_k \cdot A^{-1/p} \cdot \varepsilon_k^{1/p}$

10

$A \cdot \varepsilon_k^p = A^{-1/p} \cdot C \; \varepsilon_k^{\frac{p+1}{p}}$

Comparing powers of ε_k

$p = \frac{p+1}{p}$

$p^2 - p - 1 = 0$

$p = \frac{1 \pm \sqrt{1+4}}{2}$

$= \frac{1 \pm \sqrt{5}}{2}$

The largest value of p is

$p = \frac{1 + \sqrt{5}}{2}$ ——— ④

$\simeq 1.6108.$

$c \cdot A^{-1-1/p} = 1$

$\therefore \; A = c^{p/1+p}$

eqn (4) is called
Rate of convergence of secant method.

|| ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ||

– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

36430

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor:

Suppliment No. : 1

Roll No. : 2219

Class : M.Sc II

26/30

Subject : Lattice theory

Test / Tutorial No. :

Div. :

Q.1.

1) False

In antichain no element is comparable

∴ antichain is not lattice.

2) ~~False~~ True

Every lattice is poset but converse need not be t

every semi lattice is lattice. ∵ Poset is semi lattice

3) ~~False~~ True

~~4Q is lattice~~ 4Q is lattice, so it is also sublattice

4) True

$a \cap (a \vee (a \cap \{c \vee b\})) = a \cap (a) = a$

5) True

length of chain is element in chain minus 1.

03

Q.2.

1) Let A be any set.

Let P(A) be the power set of A.

clearly, P(A) is non empty set.

Let, $B, C \in P(A)$ be any element in P(A)

Claim - P(A) is lattice

case 1] - If $B \subseteq C$, then

$Sup\{B, C\} = B \cup C = C$ exist

$inf\{B, C\} = B \cap C = B$ exist

In this case $sup\{B, C\}$ & $inf\{B, C\}$ exist

$\therefore$ P(A) is lattice.

case 2] - If $C \subseteq B$ then

$Sup\{B, C\} = B$ exists

$inf\{B, C\} = C$ exist

In this case P(A) is lattice

case 3] - If $B \nsubseteq C$, $C \nsubseteq B$ then

i) ~~Su~~ Let $B \cap C \neq \phi$, $\exists\, a \in B \cap C$

$\therefore$ $Sup\{B, C\} = B \cup C$

$inf\{B, C\} = a$

ii) If $B \cap C = \phi$, then

$Sup\{B, C\} = B \cup C$

$inf\{B, C\} = \phi$

05

$\therefore$ Power set of any set is lattice under set inclusion.

2) Let $<P, \leq>$ be a poset satisfying ACC.
let $x_0 \in P$ be any element
If x_0 is maximal element the we are done
If x_0 is not maximal element then $\exists\ x_1 \in P$
s.t. $x_0 \leq x_1$
If x_1 is maximal element then we are done
if not then $\exists\ x_2 \in P$, s.t. $x_0 \leq x_1 \leq x_2$
Continuing this process we get increasing chain satisfying ACC
i.e. $x_0 \leq x_1 \leq x_2 \cdots \leq x_n \leq x_{n+1} \leq \cdots$
This chain satisfying ACC then it must be terminates
$\exists\ i \in \mathbb{N}$ s.t. $x_i = x_{i+1} = \cdots$
As no element in this chain is greater than x_i
i.e. x_i covers all the element. then x_i is maximal element then we are done.
If not then $\exists\ y_0 \in P$, $y_0 \neq x_i$, s.t.
$x_i \leq y_0$
If y_0 is maximal element then we are done
If not then $\exists\ y_1$, s.t $x_i \leq y_0 \leq y_1$
Continuing this way we get increasing chain satisfying ACC
i.e. $x_i \leq y_0 \leq y_1 \leq \cdots y_n \leq y_{n+1} \leq \cdots$
This chain satisfying ACC then it must be terminates, $\exists\ j \in \mathbb{N}$, s.t.
$y_j = y_{j+1} = \cdots$
If y_j is cover all the element then it is upper bound.
Do same process for all possible chain.
every chain is satisfying ACC has upper bound
by Zorn's lemma., It has maximal element.

3) Let I be non empty subset of L

$\phi \neq I \subseteq L$

~~claim~~ - Let $a, b \in L$, & $a \vee b \in I$

We know,

$a \leq a \vee b$

$\Rightarrow a = a \wedge (a \vee b) \in I$

$\Rightarrow a \in I$ — (1)

As, $b \leq a \vee b$

$b = b \wedge (a \vee b) \in I$

$\Rightarrow b \in I$ — (2)

$\therefore a, b \in I$ from (1) & (2)

Conversly, suppose $a, b \in L$, $a \vee b \in I \Rightarrow a, b \in I$

We know,

$a = a \vee (a \wedge b) \in I$

$\Rightarrow a \wedge b \in I$

$\therefore I$ is sublattice of L

Let $l \in L$, $x \in I$ be any element

~~$l \wedge x \in I$~~ $x \leq a \wedge x$

$\Rightarrow$ ~~$x \in$~~ $\Rightarrow a \wedge x \in I$

$\Rightarrow I$ is lattice of L

05

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

36453

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Suppliment No. : 2
Roll No. : 2219
Class : M.Sc II

Subject : LT
Test / Tutorial No. :
Div. :

Q.3.

1) Let $\langle L, \leq \rangle$ be lattice.
$\Rightarrow$ by def^n of lattice sup ~~∃~~ & inf ~~∃~~ exist
Let $\phi \neq H \subseteq L$
~~by~~ We use method of induction on no. of elemen in H
If $H = \{a\}$ then
Sup H = inf H = a exist
If $H = \{a, b\}$ then
Sup$\{a,b\}$ & inf $\{a,b\}$ exists
If $H = \{a, b, c\}$ then
Sup$\{a,b\}$ = k (say)
& Sup$\{k, c\}$ = t (say)

claim – Sup H = t
As $a \leq t$, $b \leq t$, $c \leq t$
then t is upper ~~of~~ bound of H
Let t' be any upper bound of H
$\Rightarrow a \leq t'$, $b \leq t'$, $c \leq t'$
As ~~t~~ k is upper bound of a, b ~~xxx~~ but
t' is upper bound of H

08

~~05~~

$\Rightarrow k \leq t'$

As t' is upper bound of H but t is upper bound of k, c

$\Rightarrow t \leq t'$

$\Rightarrow$ Sup H = t

Now, suppose Sup H exist

let $H = \{a_1, a_2, ---, a_k\}$

let $H = \{a_1, a_2, ---, a_k, a_{k+1}\}$

by hypothesis sup $\{a_k, a_{k+1}\}$ exist

Sup $H = \{a_1, a_2, ---, a_{k-1}, \text{Sup}\{a_k, a_{k+1}\}\}$ exist

by hypothesis sup H exists

by duality principle inf H also exist.

$\therefore$ Sup H & inf H exist for any non empty subset H of L

conversly, Suppose Sup H & inf H exist for any non empty subset H of L

In particular

$H = \{a, b\}$

Sup $\{a, b\}$ & inf $\{a, b\}$ exist.

As a, b are any arbitrary element in L

$\therefore$ $< L, \leq >$ is lattice

Q.2.

4) Let L be lattice & θ be congruence relation on L

$\frac{L}{\theta} = \{[a]\theta \mid a \in L\}$

We define join & meet by,

i) $[a]\theta \wedge [b]\theta = [a \wedge b]\theta$

ii) $[a]\theta \vee [b]\theta = [a \vee b]\theta$

Claim :- L/θ is lattice

1) Idempotent property

Let $[b]\theta \in L$

$[b]\theta \wedge [b]\theta = [b \wedge b]\theta$

$= [b]\theta$

$[b]\theta \vee [b]\theta = [b \vee b]\theta$

$= [b]\theta$

It satisfies idempotent property

ii) Associativity property

Let $[a]\theta, [b]\theta \in L$ be any element

$[a]\theta \wedge [b]\theta = [a \wedge b]\theta$

$= [b \wedge a]\theta$

$= [b]\theta \wedge [a]\theta$

$[a]\theta \vee [b]\theta = [a \vee b]\theta$

$= [b \vee a]\theta$

$= [b]\theta \vee [a]\theta$

It satisfies associativity

05

iii) Commutative property

let $[a]\theta, [b]\theta, [c]\theta \in L$ be any element

$[a]_\theta \wedge \{[b]_\theta \wedge [c]_\theta\} = [a]_\theta \wedge \{[b \wedge c]_\theta\}$

$= [a \wedge (b \wedge c)]_\theta$

$= [(a \wedge b) \wedge c]_\theta$

$= [a \wedge b]_\theta \wedge [c]_\theta$

$= \{[a]_\theta \wedge [b]_\theta\} \wedge [c]_\theta$

$||^{ry}$ we prove.

$[a]_\theta \vee \{[b]_\theta \vee [c]_\theta\} = \{[a]_\theta \vee [b]_\theta\} \vee [c]_\theta$

It satisfies commutivity

iv) ~~Ap~~ Absorption property

let $[a]_\theta, [b]_\theta \in L$ be any element

$[a]_\theta \wedge \{[a]_\theta \vee [b]_\theta\} = [a]_\theta \wedge \{[a \vee b]_\theta\}$

$= \{a \wedge [a \vee b]\}_\theta$

$= \{(a \wedge a) \vee b\}_\theta$

$= [a]_\theta$

$||^{ry}$ we prove

$[a]_\theta \vee \{[a]_\theta \wedge [b]_\theta\} = [a]_\theta$

by i, ii, iii, iv

$\therefore L/\theta$ is lattice