Date:15/10/2018

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
B. Sc. I Sem. I Mathematics
Internal Examination 2018-19

All the students of B.Sc. I Mathematics (A and C) are hereby informed that their theory Internal Examination of Mathematics will be conducted on **26th October, 2018 at 2.00 pm to 3.00 pm.** Syllabus for examination will be as mentioned in following table.

Name of Paper	DSC-1003A-Section-I: Differential Calculus-I	DSC-1003A-Section-II: Differential Calculus-II
Syllabus	Unit I	Unit I

***Nature of question paper:-**
Q.1) Attempt any two (20 marks)
Q.2) Attempt any two (10 marks)

Venue- Room No. 41

VIVEKANAND COLLEGE ESTD. JUNE 1964 KOLHAPUR-416003

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Vivekanand College, Kolhapur

Vivekanand College, Kolhapur (Autonomous)

Internal Examination- Oct 2018

Differential Calculus-I and II

Course code : DSC-1003 A

Marks-30

Q.1) Attempt any two of the following. [20]

i) State and Prove Leibnitz's theorem.

ii) State and prove Cauchy mean Value theorem theorem.

iii) Find the Maclaurin's series of cos x

Q.2) Attempt any two of the following [10]

i) If $y = \sin(ax + b)\ then\ find\ y_n$.

ii) For $f(x) = e^x\ and\ g(x) = e^{-x}, show\ that\ c\ of\ CMVT$ is A.M. between a and b.

iii) If $y = e^{ax} \sin bx$ then find y_n.

Date :06/10/2018

Vivekanand College, Kolhapur (Autonomous)

Department of Mathematics

B. Sc. II Sem. III

Internal Examination 2018-19

It is hereby informed that the student of B.Sc. II Sem-III their Internal Examination, 2018 of mathematics will be held according to the following schedule:

Sr. No.	Date of examination	Paper	Units	Time	Room No.
1	16/10/2018 (Tuesday)	Section-I (Differential Calculus)	Unit III-Extreme Values Unit IV-Vector Calculus	2.00 - 3.00	41
2		Section II (Differential Equation)	Unit II- Linear Differential equations of the second order Unit III- Ordinary simultaneous differential equations		

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Vivekanand College, Kolhapur

Nature of question Paper:

Time: 1 Hr. **Total Marks: 30**

Section-I

Q.1 Attempt any One **[10]**

1)

2)

Q.2 Attempt any One **[05]**

1)

2)

Section-II

Q.1 Attempt any One **[10]**

1)

2)

Q.2 Attempt any One **[05]**

1)

2)

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-II) Semester-III Internal Examination 2018-2019

MATHEMATICS

Subject Code: **Time: 1 Hr.**

Date: 16/10/2018 **Total Marks: 30**

Section-I

Differential calculus

Q.1 Attempt any One **[10]**

1) Prove that $\nabla\left[\nabla\frac{\bar{r}}{r}\right]=\frac{-2\bar{r}}{r^3}$.

2) Find extreme values of function $u=2x^3+5x^2+xy^2+y^2$

Q.2 Attempt any One **[05]**

1) Prove that stationary value of $u=xyz$ subject to condition $x+y+z=1$ is $\frac{1}{27}$

2) Find unit vector normal to the surface $xy^3z^2=c$ at point $(-1,-1,2)$

Section-II

Differential equations

Q.1 Attempt any One **[10]**

1) Solve $x^6\frac{d^2y}{dx^2}+3x^5\frac{dy}{dx}+a^2y=\frac{1}{x^2}$

2) Solve by using method of variation of parameter $x^2y''+xy'-y=x^3e^x$

Q.2 Attempt any One **[05]**

1) Solve $x\frac{d^2y}{dx^2}-(2x-1)\frac{dy}{dx}+(x-1)y=0$

2) Solve $\frac{adx}{yz(b-c)}=\frac{bdy}{zx(c-a)}=\frac{cdz}{xy(a-b)}$

Date:15/10/2018

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
B. Sc. III Sem. V
Internal Examination 2018-19

All the students of B.Sc. III are hereby informed that their Internal Examination of Mathematics will be conducted from 23th October 2018 to 26th October 2018 at time 3.00- 4.00. Syllabus, timetable and nature of question paper of examination is given below:

Sr. No.	Name of Paper	Units	Date
1	Real Analysis	UNIT II, III	23/10/2018
2	Modern Algebra	UNIT I, II, III	24/10/2018
3	Partial Differential Equations	UNIT I, II, III	25/10/2018
4	Numerical Methods I	UNIT I, II, III, IV	26/10/2018

*** Nature of question paper:**

Time: 1hour **Total Marks: 30**

Q.1 Attempt any four **(16 Marks)**

Five questions

Q.2 Attempt any two **(14 Marks)**

Three questions

Venue: Roome No. 39

VIVEKANAND COLLEGE ESTD. JUNE 1964 KOLHAPUR-416003

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Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-V Internal Examination 2018-19

MATHEMATICS

Real Analysis

Subject Code: **Date: 23/10/2018**

Total Marks: 30   **Time: 3.00 - 4.00**

Q.1 Attempt any four of the following **[16]**

i) Prove that Cauchy sequence of real number is bounded.

ii) Show that series $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges.

iii) Show that $f(x) = sinx$ is R – integrable over $[0, \pi/2]$.

iv) If P = {0, 1, 2, 4} is partition of the interval [0, 4] and f(x) = x^2. Find

a) norm P   b) U(P, f)   c)L(P, f)   d) U(P, f) – L(P, f)

v) Prove that $\lim_{n\to\infty}(\sqrt{n+1}-\sqrt{n}) = 0$

Q.2. Attempt any two of the following: **[14]**

i) Show that the sequence $\{\left(1+\frac{1}{n}\right)^n\}_{n=1}^{\infty}$ is convergent.

ii) State and prove Cauchy convergence criterion theorem

ii) State and prove Monotone convergence theorem and show that the sequence

$S_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots\ldots\ldots \frac{1}{2n}$ $\forall\, n \in N$ is convergent.

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-V Internal Examination 2018-19

MATHEMATICS

Modern Algebra

Subject Code: | **Date: 24/10/2018**

Total Marks: 30 | **Time: 3.00 - 4.00**

Q.1 Attempt any four of the following **[16]**

i) Define kernel of homomorphism θ where, $\theta : G \to G'$. Show that, θ is one to one if and only if $\ker\theta = \{e\}$.

ii) Define normaliser of element a in group G. Show that, Centre of group G is subgroup of G.

iii) Prove that, an infinite cyclic group has precisely two generators.

iv) Define cyclic group. Show that, Subgroup of cyclic group is cyclic.

v) Define even permutation, Odd permutation. Find order of $f \in S_8$, where $f = (1\ 2\ 5)(6\ 7)$

Q.2. Attempt any two of the following: **[14]**

i) State and prove Lagrange's Theorem.

ii) Prove that Infinite cyclic group have exactly two generators

iii)If $F : R \to R'$ is onto ring homomorphism then prove that $\frac{R}{A} \approx R'$ where A is kernel of F.

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-V Internal Examination 2018-19

MATHEMATICS

Partial Differential Equations

Subject Code: **Date: 25/10/2018**

Total Marks: 30 **Time: 3.00 - 4.00**

Q.1 Attempt any four of the following **[16]**

$i)$ *Form partial differential equation from* $z^2(1+a^3) = 8(x+ay+b)^3$

$ii)$*Find the general solution of* $z(xp - yq) = y^2 - x^2$

$iii)$ *Show that thesolution of lagrange's linear equation* $Pp + Qq =$ R *is given by* $F(u,v) = 0$ *where* $u(x,y,z) = a, v(x,y,z) =$ b *are solutions of* $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$iv)$ *Show that the equations*

$f = p^2 + q^2 - 1 = 0, g = (p^2+q^2)x - pz = 0$
are compatible & find one parameter family of solutions

$v)$ *Show that Pfaffian PDE'S*

$(1+yz)dx + x(z-x)dy - (1+xy)dz = 0$ *is integrable & hence find the corresponding solution*

Q.2. Attempt any two of the following: **[14]**

i) *Find the complete integral of*

$2z + p^2 + qy + 2y^2 = 0$ *by using charpits method*

ii) *Reduce the equation* $xu_{xx} - yu_{yy} = 0$ *into canonical form*

iii) *Show that* $2z = (ax+y)^2 + b$ *is complete integral of*

$px + qy - q^2 = 0$

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-V Internal Examination 2018-19

MATHEMATICS

Numerical Methods I

Subject Code: **Date: 26/10/2018**

Total Marks: 30 **Time: 3.00 - 4.00**

Q. 1 Attempt any four of the following: **[16]**

i) Find the approximate root of the equation $x^3 - 2x - 5 = 0$ by the method of Regula falsi method upto three places of decimal.

ii) Find the approximate root of the equation $cosx - xe^x = 0$ by the method of Secant method upto four places of decimal.

iii) Solve the given system of equation by using Gauss – elimination method

$2x - 3y - 4z = 11,\ \ 9x + 2y - 8z = 1.9,\qquad 15x - 8y + 6z = 14.7$

iv) Solve the given system of equation by using Gauss – Jordan method

$3x + 2y + 4z = 7,\quad 2x + y + z = 7,\quad x + 3y + 5z = 2$

v) Find the eigen value and corresponding eigen vector of A = $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix}$

Q.2. Attempt any two of the following: **[14]**

i) Explain Bisection method for finding an approximate root of non – linear equation and give it's graphical representation.

ii) Solve the given system of equation by using Jacobi's method

$3x - 6y + 2z = 23, -4x + y - z = -15, x - 3y + 7z = 16$

(Do 8 iterations).

iii) Find numerically largest eigen value and it's corresponding eigen vector for the given matrix

$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ starting with $[1 \quad 0 \quad 0]^T$

Date:15/11/2018

Vivekanand College, Kolhapur (Autonomous)

Department of Mathematics

B. Com. I Sem. I

Internal Examination 2018-19

All the students of B.Com. I Mathematics are hereby informed that their Internal Examination of Mathematics will be conducted on **26 November, 2018 from 10.00 am to 11.00 am.** Syllabus for examination will be as mentioned in following table.

Name of Paper	Topics
Business Mathematics-I GEC-1045A	Unit 1 Arithmetic Progration And Geometric Progration Unit 2 Compound Interest, Ratio, Percentage, Proportion and Partnership

Venue- Room No. 41

VIVEKANAND COLLEGE ESTD. JUNE 1964 KOLHAPUR-416003

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Vivekanand College, Kolhapur

Nature of question Paper

Time: 1 Hr. **Total Marks: 30**

Q.1) Attempt any two [20]

1)

2)

3)

Q.2) Attempt any two [10]

1)

2)

3)

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Vivekanand College, Kolhapur (Autonomous)
B.Com. (Part-I) Semester-I
Internal Examination
Business Mathematics Paper- I

Time: 1 Hr. **Total Marks: 30**

Q.1) Attempt any two **[20]**

1) Explain the following terms
   (a) Simple interest (b) Compound Interest (c) Direct Proportion
   (d) Inverse Proportion (e) Arithmetic and Geometric Mean

2) If 75 persons can perform a piece of work in 12 days of 10 hours, How many persons could perform a piece of work twice as large in half the number of days, working 8 hours daily?

3) Find the sum of $\frac{1}{\sqrt{3}}, 1, \sqrt{3} \ldots$

Q.2) Attempt any two **[10]**

1) Find the sum of $7 + 77 + 777 + 7777 + \ldots$to n terms

2) If for a geometric progression (G.P.) $r = 2,\ t_9 = 128$ find a and S_5

3) A sum of Rs. 7000 amounts to Rs. 11500 in a certain period. If the rate of simple interest is 6 % p.a. find the period.

***

Date-04/03/2019

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
B. Sc. I Sem. II Mathematics
Theory Internal Examination 2018-19

All the students of B.Sc. I Mathematics (A and C) are hereby informed that their theory Internal Examination of Mathematics will be conducted on 16th **March, 2019 at 2.00 pm to 3.00 pm.** Syllabus for examination will be as mentioned in following table.

Name of Paper	DSC-1003B-Section-I: Differential Equations-I	DSC-1003B-Section-II: Differential Equations-II
Syllabus	Unit I	Unit I

***Nature of question paper:-**

Q.1) Attempt any two (20 marks)

Q.2) Attempt any two (10 marks)

Venue:- Room No. 41

VIVEKANAND COLLEGE ESTD. JUNE 1964 KOLHAPUR-416003

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Vivekanand College, Kolhapur

Vivekanand College, Kolhapur (Autonomous)

Internal Examination- March 2019

Differential Equations-I and II

Course code : DSC-1003 B

Q.1) Attempt any two of the following. **[20]**

i) State and prove Necessary and sufficient condition for differential equation Mdx +Ndy=0 to be exact.

ii) Define Clairaut's equation and explain the method of solving it. Hence solve

$$sinpx\ cosy = cospx\ siny + p$$

iii) Explain method of variation of parameters.

Q.2) Attempt any two of the following **[10]**

i) Solve $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{\cos x}{1+x^2}$

ii) Solve $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$

iii) Solve the second order differential equation $y'' - 9y' + 20y = 0$.

iv) Solve $(D^2 - 5D + 6)y = e^{9x}$

Date:01/04/2019

Vivekanand College, Kolhapur (Autonomous)

Department of Mathematics

B. Sc. II Sem. IV

Internal Examination 2018-19

It is hereby informed that the student of B.Sc. II Sem-IV their Internal Examination, 2019 of mathematics will be held according to the following schedule:

Sr. No.	Date of examination	Paper	Units	Time	Room No.
1	08/04/2019 (Monday)	Section-I (Integral Calculus)	Unit II- Multiple integral Unit III- Fourier series	12.30 - 1.30	33
2		Section II (Discrete mathematics)	Unit-IV Graph theory		

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VIVEKANAND COLLEGE ESTD. JUNE 1964 KOLHAPUR-416003

Nature of question Paper:

Time: 1 Hr. **Total Marks: 30**

Section-I

Q.1 Attempt any One **[10]**

1)

2)

Q.2 Attempt any One **[05]**

1)

2)

Section-II

Q.1 Attempt any One **[10]**

1)

2)

Q.2 Attempt any One **[05]**

1)

2)

Vivekanand College, Kolhapur (Autonomous)
B.Sc. (Part-II) Semester-IV Internal Examination 2018-2019
MATHEMATICS

Subject Code: Time: 1 Hr.
Date: 08/04/2019 Total Marks: 30

Section-I
Integral Calculus

Q.1 Attempt any One [10]

1) Derive the Euler's formulae for the Fourier series of the function $f(x)$ in the interval $c \le x \le c+2\pi$ given by $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$.

2) Evaluate by changing the order of integration $\int_0^a \int_0^x \frac{\cos y\, dx\, dy}{\sqrt{(a-x)(a-y)}}$

Q.2 Attempt any One [05]

1) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2y^2}\, dy\, dx$

2) If $f(x) = x^2$ then find Fourier coefficient b_n in the interval $(0,2\pi)$

Section-II
Discrete mathematics

Q.1 Attempt any One [10]

1) a) Define (i) Adjacency matrix

(ii) Incidency matrix

b) Prove that the maximum number of edges in a graph consisting of n vertices is $\frac{n(n-1)}{2}$

2) a) State and prove Hand-Shaking lemma

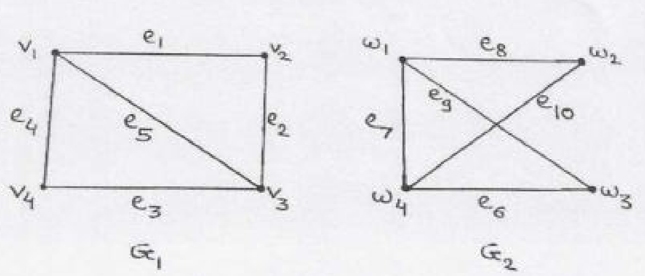
b) Draw the 4-regular graph.

Q.2 Attempt any One [05]

1) Draw the graph

(a) $K_{2,4}$ (b) $K_{3,2}$

2) Show that following pair of graphs are isomorphic.



Date:05/04/2019

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
B. Sc. III Sem. VI
Internal Examination 2018-19

All the students of B.Sc. III are hereby informed that their Internal Examination of Mathematics will be conducted from 15th April 2019. Syllabus and timetable and nature of question paper of examination is given below:

Sr. No.	Name of Paper	Units	Date	Time
1	Metric Space	UNIT II, III	15/04/2019	3.00 -4.00
2	Linear Algebra	UNIT I, II, III	16/04/2019	3.00 -4.00
3	Complex analysis	UNIT I, II, III	18/04/2019	3.00 -4.00
4	Numerical Methods II	UNIT I, II, III, IV	20/04/2019	3.00 -4.00

*** Nature of question paper:**

Time: 1hour **Total Marks: 30**

Q.1 Attempt any four **(16 Marks)**

Five questions

Q.2 Attempt any two **(14 Marks)**

Three questions

Venue: Roome No. 39

VIVEKANAND COLLEGE ESTD. JUNE 1964 KOLHAPUR-416003

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Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-VIInternal Examination 2018-19

MATHEMATICS

Metric Space

Subject Code: **Date: 15/04/2019**

Total Marks: 30 **Time: 3.00 - 4.00**

Q.1 Attempt any four of the following **[16]**

i) Show that the mapping d : $R^2 \to R^2$ defined by d (x, y) = max $(|x_1 - y_1|, |x_2 - y_2|)$ where $x = (x_1, x_2), y = (y_1, y_2)$ is metric on R^2

ii) Prove that continuous image of connected set is connected

iii) Define the following terms

a) Cluster point b) exterior point

c) Adherent point d) Interior point

iv) If d is a metric for non-empty set X and $d_1(x, y) = 2d(x, y)$ then show that d_1 is also metric defined on X

v) Prove that in a metric space, every closed sphere is closed.

Q.2 Attempt any two of the following **[14]**

i) Prove that if T is a contraction mapping on a complete metric space (X, d) then there exist a unique point x in X such that T(x) = x.

ii) Prove that any closed interval [a, b] is compact subset of R.

iii) Prove that A mapping f: X $\to Y$ is continuous on X iff $f^{-1}(G)$ is open in X for all open subsets of Y.

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-VI Internal Examination 2018-19

MATHEMATICS

Linear Algebra

Subject Code: **Date: 16/04/2019**

Total Marks: 30 **Time: 3.00 - 4.00**

Q.1 Attempt any four of the following **[16]**

i) A nonempty subset W of vector space V(F) is subspace of V iff $ax + by \in W$ for a, b $\in$ F , x, y $\in$ W

ii)S= {(1,1,2),(1,2,5)(5,3,4)}, Find whether S is linearly independent or linearly dependent.

iii) Define Inner product space, Euclidian Space, Unitary Space, Norm of Vector.

iv) Let V be an IPS then prove that $|| x+y|| \leq ||x|| + ||y||$

v) Define Characteristic values of matrix, Characteristic equation of matrix. Find Characteristic values of $A = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

Q.2 Attempt any two of the following **[14]**

i) Define Linear span of nonempty subset of Finite dimensional vector space V(F). Suppose S is finite subset of vector space V(F) such that L(S) = V. then their exist subset of S which is basis of V

ii) State and prove Rank - Nullity Theorem

iii) State and prove Cauchy-Schwartz Inequality.

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-VI Internal Examination 2018-19

MATHEMATICS

Complex Analysis

Subject Code: **Date: 18/04/2019**

Total Marks: 30 **Time: 3.00 - 4.00**

Q.1 Attempt any four of the following **[16]**

i) Evaluate $\int_0^{1+i}(x - y + ix^2)dz$ along the path from $z = 0$ to $z = 1 + i$.

ii) Prove that by using Contour integral $\int_{-\infty}^{+\infty} \frac{x^2 dx}{(x^2+1)^2} = \frac{\pi}{2}$.

iii) Find the analytic function whose imaginary part is $v(x, y) = \cos x \cosh y$.

iv) Find the residue of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ at $z = 1, -2$.

v) Obtain Laurent series for $f(z) = \frac{1}{(z+1)(z+3)}$ for region $1 < |z| < 3$

Q.2 Attempt any two of the following **[14]**

i) Define analytic function. If $f(z) = u + iv$ is an analytic function and $z = re^{i\theta}$ where u, v, r, θ are all real, show that the Cauchy-Riemann equations are

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

ii) If $f(z)$ is analytic in a domain D then $f(z)$ has at any point $z = a$ of D, derivatives of all orders are again analytic functions in D. Then prove that $f^n(a) = \frac{n!}{2\pi i}\int_C \frac{f(z)}{(z-a)^{n+1}} dz$ where C is any closed Contour in D surrounding the point $z = a$.

iii) state and prove Cauchy's Residue Theorem

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-VI Internal Examination 2018-19

MATHEMATICS

Numerical Methods II

Subject Code: **Date: 20/04/2019**

Total Marks: 30 **Time: 3.00 - 4.00**

Q.1 Attempt any four of the following **[16]**

i) Show that a) $\nabla = \Delta E^{-1}$, b) $(I + \Delta)(I - \nabla) = I$

ii) Find $f'(x)$ at x = 3.5 by using Newton's divided difference formula for the following table

x	0	1	2	3
y	0	1	8	27

iii) Using Euler's method, solve $\frac{dy}{dx} = \frac{x^2}{2y}$ for given boundary condition $y(0) = 1.2$ and find y at x = 0.8(take h = 0.4)

iv) Solve $\frac{dy}{dx} + y = 0$ using R-K method of second order under the boundary condition $y(0) = 1$ and find y at x = 0.1, 0.2. (take h = 0.1)

v) Evaluate the integration $\int_0^3 \frac{x^3}{1+x^2} dx$ using Simpson's 3/8 rule with 6 subdivisions.

Q.2 Attempt any two questions **[14]**

i) Derive the relation between operator E^{-1} and ∇ and find the values of y at x = 0.25 and x = 0.35 from the following data

X	0.1	0.2	0.3	0.4	0.5
Y	1.40	1.56	1.76	2.00	2.28

ii) Derive the formula for $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ by using Newton's backward interpolation. Find $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ at x = 3 from the following data

x	3	3.2	3.4	3.6	3.8	4.0
y	-14	-10.032	-5.296	-0.256	6.672	14

iii) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by a) Trapezoidal rule and b) Simpson's 1/3 rule take n = 6

Date-01/04/2019

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
B. Com. I Sem. II Mathematics
Internal Examination 2018-19

All the students of B.Com. I Mathematics are hereby informed that their theory Internal Examination of Mathematics will be conducted on **8th April, 2019 from 10.00 am to 11.00 am.** Syllabus for examination will be as mentioned in following table.

Name of Paper	Topics
Business Mathematics-II GEC-1045B	Unit 3: Differentiation Unit 4: Application of derivatives

Venue:- Room No. 41

VIVEKANAND COLLEGE ESTD. JUNE 1964 KOLHAPUR-416 003

Mr. S. P. Patankar
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Vivekanand College, Kolhapur

Nature of question Paper

Time: 1 Hr. **Total Marks: 30**

Q.1) Attempt any two [20]

1)

2)

3)

Q.2) Attempt any two [10]

1)

2)

3)

VIVEKANAND COLLEGE ESTD. JUNE 1964 KOLHAPUR-416 003

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Vivekanand College, Kolhapur (Autonomous)
B.Com. (Part-I) Semester-II
Internal Examination
Business Mathematics Paper- II

Time: 1 Hr. **Total Marks: 30**

Q.1) Attempt any two **[20]**

1) If $y = 2x^3 - 9x^2 + 12x + 5$, find x when $\frac{dy}{dx} = 0$. Also find the signs of the second order derivatives of these values of x.

2) If $y = x^{x^x} + log_x\, e$, find $\frac{dy}{dx}$

3) The expression $O = 40F + 3F^2 - \frac{F^3}{3}$ shows how total output O varies with input F.
   a) Give an algebraic expression for the average product and marginal product.
   b) Find the values of F and O for which total output is at maxima.
   c) Find the maximum value of the marginal product.
   d) When do diminishing returns set in?

Q.2) Attempt any two **[10]**

1) Find $\frac{dy}{dx}$ if $x = at^2, y = 2at$.

2) Find the maximum and minimum of the function $y = x^4 - 8x^3 + 16x^2 + 3$

3) Prove that if $y = x^n$ then $\frac{dy}{dx} = n \cdot x^{n-1}$

***

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Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

Signature of Supervisor	
Subject	: Maths.
Test / Tutorial No. :	
Div. :	A

SUPPLIMENT

Suppliment No. :

Roll No. : 7225

Class : BSC-I

15+8–23

Q.1 2) Leibnitz's theorem.

stament:- leibnitz's theorem shows that the n^{th} derivative of $(u \cdot v)$

$$y_n(u \cdot v) = {}^nC_0 u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + {}^nC_3 u_{n-3} v_3 + \dots$$
$$+ {}^nC_r u_{n-r} v_r + \dots + {}^nC_n u v_n \text{ ——— (A)}.$$

Proof :- We have to prove this theorem by mathematical Induction.

i) We have to prove theorem for $n=1$.

$$y_1 = u_1 v + u v_1 \text{ ——— (1)}.$$
$$= {}^1C_0 u_1 v + {}^1C_0 u v_1$$

∴ Theorem is true for $n=1$.

ii) We have to prove theorem for $n=2$.

Diff eq (1) ~~for~~ w.r.t. x.

$$y_2 = u_2 v + u_1 v_1 + u_1 v_1 + u v_2$$
$$= u_2 v + 2u_1 v_1 + u v_2$$
$$= {}^2C_0 u_2 v + {}^2C_1 2u_1 v_1 + {}^2C_2 u v_2$$

∴ Theorem is true for $n=2$.

Assume that theorem is true for The value n.

$$\therefore y_n(u.v) = {}^nc_0 u_n v + {}^nc_1 u_{n-1} v_1 + {}^nc_2 u_{n-2} v_2 + {}^nc_3 u_{n-3} v_3 + \dots$$
$$\dots {}^nc_r u_{n-r} v_r + \dots \; {}^nc_n u v_n \text{ ——— (A)}.$$

We have to prove this theorem for $y = n+1$.
Diff eq (A) w.r.t. x.

$$y_{n+1}(u.v) = {}^nc_0 u_{n+1} v + {}^nc_0 u_n v_1 + {}^nc_1 u_n v_1 + {}^nc_1 u_{n-1} v_2 +$$
$${}^nc_2 u_{n-1} v_2 + {}^nc_2 u_{n-2} v_3 + \dots + {}^nc_r u_{n-r+1} v_r + {}^nc_r u_r$$
$$+ \dots \; {}^nc_n u_1 v_n + {}^nc_n u v_{n+1}.$$

$$= {}^nc_0 u_{n+1} v + ({}^nc_0 + {}^nc_1) u_n v_1 + ({}^nc_1 + {}^nc_2) u_{n-1} v_2 +$$
$$({}^nc_2 + {}^nc_3) u_{n-2} v_3 + \dots + ({}^nc_{r-1} + {}^nc_r) u_{n-r+1} v_r + \dots$$
$$+ \cancel{{}^nc_n u_{n-r} v_{r+1}} + {}^nc_n u v_{n+1}$$

We know that.

$${}^nc_{r-1} + {}^nc_r = {}^{n+1}c_r$$

$$\therefore {}^nc_0 + {}^nc_1 = {}^{n+1}c_1$$
$${}^nc_1 + {}^nc_2 = {}^{n+1}c_2$$
$${}^nc_2 + {}^nc_3 = {}^{n+1}c_3$$
$$\vdots$$
$${}^nc_{r-1} + {}^nc_r = {}^{n+1}c_r$$

and

$${}^{n+1}c_0 = {}^nc_n = 1$$

$$y_{n+1} = {}^nc_0 u_{n+1} v + {}^{n+1}c_1 u_n v_1 + {}^{n+1}c_2 u_{n-1} v_2 + {}^{n+1}c_3 u_{n-2} v_3 +$$
$$\dots + {}^{n+1}c_r u_{n-r+1} v_r + \dots \; {}^{n+1}c_n u v_{n+1}.$$

$$= {}^{n+1}c_0 u_{n+1} v + {}^{n+1}c_1 u_n v_1 + {}^{n+1}c_2 u_{n-1} v_2 + {}^{n+1}c_3 u_{n-2} v_3 +$$
$$\dots + {}^{n+1}c_r u_{n-r+1} v_r + \dots \; {}^nc_n u v_{n+1} \dots$$

This is same as eq Ⓐ by changing n to n+1

∴ Theorem is true for y_{n+1}

∴ This theorem is true for all values.

$$y = \frac{x}{1+3x+2x^2}, \quad y_n = ?$$

$$= \frac{x}{2x^2+3x+1}$$

$$= \frac{x}{2x^2+2x+x+1}$$

$$= \frac{x}{2x(x+1)+1(x+1)}$$

$$= \frac{x}{(2x+1)(x+1)}$$

$$= \frac{1/2}{(x \cdot 1/x+1)(x+1)} + \frac{1}{(2x+1)(1+1)}$$

$$= \frac{1/2}{2(x+1)} + \frac{1}{2(2x+1)}$$

$$= \frac{1}{2}\left[\frac{1}{(x+1)} + \frac{1}{(2x+1)}\right]$$

$$= \frac{1}{2}\left[\frac{(-1)^n n!(1)^n}{(x+1)^{n+1}} + \frac{(-1)^n n!(2)^n}{(2x+1)^{n+1}}\right]$$

4

36
6
216 1227

3) $f(x) = 2x^3 - 5x^2 + 3x + 2$. ; $x \in [0, 3/2]$.

i) As $f(x)$ is polynomial function
$\therefore f(x)$ continuos in $[0, 3/2]$

ii) $f(x)$ is Diff. in $(0, 3/2)$
$f'(x) = 2x^3 - 5x^2 + 3x + 2$
$f'(x) = 6x^2 - 10x + 3$

ii) $f(a) = f(0) = 2(0)^3 - 5(0)^2 + 3(0) + 2 = 2$ 415 − 37 = 19
$f(b) = f(3/2) = 2(3/2)^3 - 5(3/2)^2 + 3(3/2) + 2 = 2$

$\therefore$ funtion satisfy all the canditions of R.M.V.T.
$\therefore f'(c) = 0$. 1010 − 82 = 28
$f'(c) = 6(c)^2 - 10c + 3 = 0$
$6c^2 - 10c + 3 = 0$ 18

$= \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{10 \pm \sqrt{100 - 4 \times 6 \times 3}}{12}$ 1010 − 82 = 928 24 × 3 = 72

$= \dfrac{10 \pm \sqrt{100 - 72}}{12}$

$= \dfrac{10 \pm \sqrt{28}}{12}$

$= \dfrac{10 + \sqrt{28}}{12}$ $= \dfrac{10 - \sqrt{28}}{12}$

$= \dfrac{10 + \sqrt{4 \times 7}}{12}$ $= \dfrac{10 - \sqrt{4+7}}{12}$

$= \dfrac{10 + 2\sqrt{7}}{12}$ $= \dfrac{10 - 2\sqrt{7}}{12}$

$= \dfrac{10 + 2\sqrt{7}}{12} \in c$

$\therefore$ R.M.V.T is verify. $c = \dfrac{10 + 2\sqrt{7}}{12}$

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Class :		Test / Tutorial No. :	
		Div. :	

Q.1 2) caley - Hamilton's theorem states that Every squre matrixe have characteristic equation.

$a_0 I + a_1 A + a_2 A^2 + a_3 A^3 + \dots a_n A^n$.

Proof:- let A be the given squre matrixe.

$\therefore$ the characteristic equation of A is

$|A - xI| = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots a_n x^n$. ——— ①.

$adj[A - xI] = B_0 + B_1 x + B_2 x^2 + B_3 x^3 + \dots B_n x^n$.

We know that

$[A - xI] . adj[A - xI] = [A - xI] . I$.

$R.HS = a_0 I + a_1 x + a_2 x^2 + a_3 x^3 + \dots a_n x^n$.

$L.HS = [A - xI] . adj[A - xI]$

$= [A - xI](B_0 + B_1 x + B_2 x^2 + B_3 x^3 + \dots B_n x^n.)$

$= AB_0 + AB_1 x + AB_2 x^2 + AB_3 x^3 + \dots - IB_0 x - IB_1 x^2 + - IB_2 x^3 + IB_3 x^4 + \dots IB_n x^{n+1}$.

$= AB_0 + x[AB_1 - B_0] + x^2 I[AB_2 - B_1] + x^3 I[AB_2 - B_2]$ ——— ②.

comparing eq① wit eq②.

$a_0 = AB_0$, $a_1 = AB_1 - B_0$, $a_2 = AB_2 - B_1$,

$a_3 = AB_3 - B_2$

Multiplying these quantities by $I, A, A^2, A^3, \ldots$

$a_0 I = A B_0 I$
$a_1 A = B_1 A^2 - B_0 A$
$a_2 A^2 = B_2 A^3 - B_1 A^2$
$a_3 A^3 = B_3 A^4 - B_2 A^3$

adding these quantities.

$$a_0 I + a_1 A + a_2 A^2 + a_3 A^3 + \cdots = 1 + A B_0 I + B_1 A^2 - B_0 A + B_2 A \ldots$$
$$B_1 A^3 - B_1 A^2 + B_3 A^4 - B_2 A^3$$

$\therefore a_0 I + a_1 A + a_2 A^2 + a_3 A^3 + \cdots = 0.$

$\therefore$ Here caley - Hamilton's theorem is prove.

4

10° d.

3) $x^3 - 18x - 35 = 0$

$h = \frac{+(-a_1)}{n(a_0)} = \frac{-(-18)}{3} = \frac{18}{3} = 6$

f by sinthetic dividion.

$$\begin{array}{c|cc} 6 & 1 & -18 \end{array}$$

Here we have $t = x - 6$ $\therefore$ $x = t + 6$.

We have:

$t^3 - 18t - 35 = 0$ ——— ①.

$t = u^{1/3} + v^{1/3}$

$t^3 = (u+v) + 3 \cdot u^{1/3} \cdot v^{1/3} \cdot (u^{1/3} + v^{1/3})$

$t^3 = (u+v) + 3 \cdot u^{1/3} \cdot v^{1/3} \cdot t$

$t^3 - 3 \cdot u^{1/3} \cdot v^{1/3} \cdot t - (u+v) = 0$ ——— ②

comparing eq ① and ②.

3

$+3 \cdot u^{1/3} \cdot v^{1/3} = +18$ $\qquad$ $+(u+v) = +35$

$u^{1/3} \cdot v^{1/3} = +6$ $\qquad$ $u+v = 35$.

$u \cdot v = (+6)^3$

As we know that.

$t^2 - (u+v)t + (u \cdot v) = 0$

$t^2 - 35t + 6^3 = 0$

$$\begin{array}{r} 35 \\ -29 \\ \hline 16 \end{array} \qquad \begin{array}{r} 35 \\ \times 35 \\ \hline 175 \\ 1050 \\ \hline 1225 \end{array} \qquad \begin{array}{r} 19 \\ \times 19 \\ \hline 171 \\ 190 \\ \hline 361 \end{array}$$

by formula method.

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+35 \pm \sqrt{(35)^2 - 4 \times 6^3}}{2}$$

$$\begin{array}{r} 216 \\ \times 4 \\ \hline 864 \end{array}$$

$$= \frac{35 + \sqrt{361}}{2} \qquad = \frac{35 - \sqrt{361}}{2}$$

$$\begin{array}{r} 1225 \\ -864 \\ \hline 361 \end{array}$$

t $\qquad = \frac{35+19}{2} \qquad = \frac{35-19}{2}$

$\cancel{x = t + 0}$

$\cancel{= \frac{35 + \sqrt{361}}{2}} \qquad = \frac{16}{2} \qquad = \frac{54}{2}$

$\cancel{x_2 = 35} \qquad = 8 \qquad = 27$

$t_1 = t_1 + t_2 = 8 + 27 = 35$

$t_2 = \omega \cdot t_1 + \omega^2 \cdot t_2 = 8\left(\frac{-1}{2} - \frac{i\sqrt{3}}{2}\right) + 27\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)$

$= (8+27)(-1)$

$= -35$

$t_3 = \omega^2 \cdot t_1 + \omega \cdot t_2 = 8\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right) + 27\left(\frac{-1}{2} - \frac{i\sqrt{3}}{2}\right)$

$= (8+27)(-1)$

$= -35.$

$$\boxed{\begin{aligned} \therefore t_1 &= 35 \\ t_2 &= -35 \\ t_3 &= -35 \end{aligned}}$$

$\therefore x_1 = 35$

$x_2 = -35$

$x_3 = -35$

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SUPPLIMENT

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Text

Suppliment No. : 1   14 + 12 = 26

Subject : Mathematics - I.

Roll No. : 7763.

Test / Tutorial No. : Internal Exam

Class : B.Sc - II

Div. : A.

1) – –

i) Division Algorithm

Statement = If any integer a and b with $b>0$ $\exists$ unique integers q and r satisfying $a=bq+r$ with $0\leq r\leq b$.

Proof -

let s be the set of integers.

$s=\{a-xb/x : a-xb \text{ is an integer}\}$.

claim: s is non-empty set

$x=-|a|$.

$\therefore a-xb = a+|a|b \geq 0$.

$= a+|a|\cdot 1$.

$= a+|a| \geq 0$.

Thus s is non-empty set.

Their exist integer ~~r <~~ $r\geq 0$

i.e. $r\in s$

claim : $r \geq b$.

consider contradictory $r<b$.

$r \in S$.

Then their exist integer q. so. $a-bq=r$.

$\therefore a-(q+1)b = a-qb-b$

$= r-b. \geq 0$.

i.e. $r-b \in S$. , $r-b<r$

i.e. It is contradiction to the minimality of r.

Thus. $r=a-bq$. where $0 \leq r < b$.

<u>Uniqueness</u>

let their exist integer q' and r' then $a=bq$ for $0 \leq r' < b$

$bq+r = bq'+r'$

$bq-bq' = r'-r$.

$b(q-q') = r'-r$.

$b|q-q'| = |r'-r|$

Now, $0 \leq r < b$. , $0 \geq -r > b$.

$b<-r \leq 0$. and $0 \geq r > b$.

6 i.e. $-b<-r'<b. \Rightarrow -b<r'-r<b. \leq 0$.

i.e. $r'-r<b \leq 0 \Rightarrow b|q-q'|<b$

$1|q-q'|<1$.

$\therefore q=q'$ and $r=r'$.

This proves uniqueness.

i) Euclid's theorem.

Statement : There is infinite prime numbers.

Proof :

$p_1 = 2,\ p_2 = 3,\ p_3 = 5,\ p_4 = 7 \ldots\ldots$

This is the prime numbers in natural way.

There is upto p_n prime numbers.

$p_1, p_2, p_3, p_4, \ldots\ldots p_n$

$N = p_1 p_2 p_3 \ldots\ldots p_n + 1$

By fundamental theorem of arithmetic there exist integer p_k $(0 \leq k < n)$.

4

$\Rightarrow p_k \mid N$

$\Rightarrow p_k \mid N - p_1 p_2 p_3 \ldots$

$\Rightarrow p_k \mid 1$

This is contradictory.

It proves there is infinite prime numbers.

ii) Diophantine equation.

$221x + 35y = 11$

$\cancel{221 = 35 \times 6 + 11}$

$\cancel{35} = \cancel{1 \times 35} + \cancel{0}$

$\cancel{gcd(221, 35) = 1}$

$221 = 35 \times 6 + 11$

$35 = 11 \times 3 + 2$

$11 = 2 \times 5 + \boxed{1}$

$2 = 1 \times 2 + 0$

$gcd(221, 35) = 1$.

3535

Linear recombination.

~~11 = 221 - 35 × 6~~

221 ~~= 221(1) + 35(-6)~~.

$1 = 11 - (2\times5)$

$= 11 - ((35 - 11\times3)\times5)$

$= 11\times16 - 35\times5.$

$= (221 - 35\times6)\times16 - 35\times5.$

$= 221\times16 - 35\times101.$

$= 221\times16 + 35(-101)$

~~$x = x_0 + \frac{b}{d}t$~~

~~= 1~~.

176 -1111

The diophantine eq^n is...

$11 = 221\times176 + 35(-1111).$

4

$x = x_0 + \frac{b}{d}t$ $\quad y = y_0 - \frac{a}{d}t$

$x = 176 + \frac{35}{1}t$ $\quad = -1111 - \frac{221}{1}t.$

$x = 176 + 35t.$ $\quad = -1111 - 221t.$

$(176 + 35t, -1111 - 221t)$ is the sol^n of given diophantine eq^n.

iii) Theorem.

Statement : If a and b be integers not both zero then $\gcd(a,b) * \mathrm{lcm}(a,b) = a * b$.

Proof

let $d = \gcd(a,b)$ and $m = \frac{ab}{d}$.

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Suppliment No. : 2

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Class :

Subject :

Test / Tutorial No. :

Div. :

We have to prove that $m = lcm(a,b)$

then ~~a=df~~ there exist integer integer a and b.

$dr = a$ and $ds = b$.

$\therefore m = \frac{ab}{d} = \frac{drb}{d} = rb \Rightarrow$ ~~m|b~~ b|m or ~~mt~~ r|m.

$m = \frac{ab}{d} = \frac{dsa}{d} = as \Rightarrow$ ~~m|a~~ a|m. or s|m

Here we prove first condition.

let there exist a common divisor c.

c|a and c|b.

there exist integer x and y $= ax = c$ and $by = c$.

$gcd(a,b) = d$ then there exist integer u and v. $d = au + bv$.

let $\frac{c}{m} = \frac{cd}{ab} = \frac{c(au+bv)}{ab}$

$$\frac{c}{m} = \frac{c(au+bv)}{ab}$$

$$\frac{c}{m} = \frac{c}{b}u + \frac{c}{a}v$$

$$\frac{c}{m} = yu + xv.$$

$m \leq c$. i.e. c/m is an integer.

Now.

4

$$m = \frac{ab}{d}$$

$$\Rightarrow md = ab$$

$$\Rightarrow \operatorname{lcm}(a,b) \cdot \gcd(a,b) = a \times b.$$

Hence proved.

Section II - Integral Calculus.

12/15

We know that,

$$\int_0^{\pi/2} \sin^p\theta \cdot \cos^q\theta = \frac{1}{2}\,\frac{\Gamma\frac{p+1}{2}\;\Gamma\frac{q+1}{2}}{\Gamma\frac{p+q+2}{2}}$$

Here put $q = p$.

We get,

$$\frac{1}{2}\,\frac{\left(\Gamma\frac{p+1}{2}\right)^2}{\Gamma\frac{2p+2}{2}} = \int_0^{\pi/2} \sin^p\theta \cdot d\theta$$

$$\frac{1}{2}\,\frac{\left(\Gamma\frac{p+1}{2}\right)^2}{\Gamma p+1} = \frac{1}{2^p}\int_0^{\pi/2} (2^p \sin^p\theta)\, d\theta$$

$$= \frac{1}{2^p}\int_0^{\pi/2} (2\sin\theta)^p\, d\theta$$

$$= \cancel{\frac{1}{2^p}}\int_0^{\pi/2} \cancel{2\sin^p\theta \cdot \cos\theta \cdot d\theta}$$

$$= \cancel{\frac{1}{2^p}}\int_0^{\pi/2} \cancel{2^p \sin^p\theta \cos^p\theta}$$

$$= \frac{1}{2^p}\int_0^{\pi/2} (\sin 2\theta)^p\, d\theta$$

Put $2\theta = t$.

$2d\theta = dt \qquad d\theta = \frac{dt}{2}$

$$= \frac{1}{2^p}\int_0^{\pi/2} (\sin t)^p \frac{dt}{2}$$

$$= \frac{1}{2^p}\,\frac{1}{\cancel{2}}\,\cancel{2}\int_0^{\pi} \sin^p t \; \cos^0 t \; dt.$$

$$= \frac{1}{2^p}\,\frac{1}{2}\,\frac{\Gamma\frac{p+1}{2}\,\Gamma\frac{0+1}{2}}{\Gamma\frac{p+0+2}{2}}$$

$$= \frac{1}{2^p}\,\frac{1}{2}\,\frac{\Gamma\frac{p+1}{2}\,\Gamma\frac{1}{2}}{\Gamma\frac{p+2}{2}}$$

$$\frac{1}{2}\,\frac{\left(\Gamma\frac{p+1}{2}\right)^2}{\Gamma p+1} = \frac{1}{2^p}\,\frac{1}{2}\,\frac{\Gamma\frac{p+1}{2}\;\sqrt{\pi}}{\Gamma\frac{p+2}{2}}$$

$$\cancel{\frac{1}{2}}\,\frac{\Gamma\frac{p+1}{2}}{\Gamma p+1} = \frac{1}{2^p}\,\frac{\sqrt{\pi}}{\Gamma\frac{p+2}{2}}$$

Put $\frac{p+1}{2} = m$

$$p+1 = 2m.$$

$$p = 2m-1.$$

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Roll No. : 7763.

Class :

Subject :

Test / Tutorial No. :

Div. :

$$\therefore \frac{\Gamma m}{\Gamma 2m} = \frac{1}{2^{2m-1}} \frac{\sqrt{\pi}}{\Gamma m+\frac{1}{2}}$$

$$\therefore 2^{2m-1} \Gamma m \, \Gamma m+\tfrac{1}{2} = \sqrt{\pi} \, \Gamma 2m.$$

Now put $m=n$.

we get.

$$2^{2n-1} \Gamma n \, \Gamma n+\tfrac{1}{2} = \sqrt{\pi} \, \Gamma 2n.$$

2. ii) $\int_0^{\pi/6} \sin^2 6\theta \cdot \cos^6 3\theta = \frac{2}{3} B\left(\frac{3}{2}, \frac{9}{2}\right)$.

Here, put $3\theta = t$.

$$d\theta = \frac{1}{3} dt.$$

limit $\theta \quad 0 \quad \pi/6$

$\qquad t \quad 0 \quad 1$

$$I = \int_0^1 \sin^2 2t \cdot \cos^6 t \, \frac{1}{3} dt$$

$$= \frac{\cancel{2}1}{3} \int_0^1 \sin^2 2t \cdot \cos^6 t \, dt$$

$$= \frac{1}{3} \int_0^1 (\sin 2t)^2 \cos^6 t \, dt.$$

04

$$= \frac{1}{3} \int_0^1 2\sin^2 t \cdot \cos^2 t \cdot \cos^6 t \, dt.$$

$$= \frac{2}{3} \int_0^1 \sin^2 t \cdot \cos^8 t \cdot dt.$$

$$= \frac{2}{3} \int_0^1 \sin^2 t \cdot \cos^8 t \, dt$$

$$= \frac{2}{3} B\left(\frac{2+1}{2}, \frac{8+1}{2}\right).$$

$$= \frac{2}{3} B\left(\frac{3}{2}, \frac{9}{2}\right).$$ Hence proved.

iii) $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta.$

$$\rightarrow I = \int_0^{\pi/2} \sqrt{\frac{\cos\theta}{\sin\theta}}\, d\theta.$$

$$= \int_0^{\pi/2} \cos^{1/2}\theta \,.\, \sin^{-1/2}\theta \, d\theta.$$

02

$$= B\left(\frac{\frac{1}{2}+1}{2}, \frac{-\frac{1}{2}+1}{2}\right)$$

$$= B\left(\frac{3}{4}, \frac{1}{4}\right).$$

i) $\int_0^{2a} \sqrt{2ax - x^2}\, dx.$

$\rightarrow$ Put $x = 2at$

$dx = 2a\,dt$

limit x 0 $2a$

t 0 1

02

$$I = \int_0^1 \sqrt{2a2at - (2at)^2}\; 2a\,dt$$

$$= \int_0^1 \left[4a^2t - 4a^2t^2\right]^{1/2} 2a\,dt$$

$$= \int_0^1 (4a^2)^{1/2} \left[t - t^2\right]^{1/2} 2a\,dt.$$

$$= 2a^{1+1} \int_0^1 t\,(1-t)^{1/2}\, dt.$$

$$= 2a^2 \int_0^1 t^{2-1} (1-t)^{1/2+1-1} \, dt$$

$$= 2a^2 \int_0^1 t^{2-1} (1-t)^{3/2-1} \, dt.$$

$$= 2a^2 B\left(2, \frac{3}{2}\right).$$

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VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Suppliment No. :

Roll No. : 8302

Class : BSc – TY

Subject : Mathematics

Test / Tutorial No. : Internal examination

Div. :

29/30

Q1. (i) (c) 1

(ii) (a) $\frac{1}{\sqrt{3}}$

(iii) ~~(b) $\lim_{n\to\infty} 1/a_n = 0$~~ (a) $\sup\{a_n \mid n \in \mathbb{N}\} = 3$ and $\inf\{a_n \mid n \in \mathbb{N}\} = 1$

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(iv) (b) $\lim_{n\to\infty} \frac{1}{a_n} = 0$

(v) (c) $\{-1, 0, 1\}$

Q2. Monotone convergence theorem

(i) A Statement: A real sequence x_n, which is monoto[ne]
is convergent such that,

(i) if $x = x_n$ is bounded and increasing it converg[es]
to its supremum i.e $\lim x_n = \sup\{x_n : n \in \mathbb{N}\}$

(ii) if $x = x_n$ is bounded and decreasing it converge[s]
to its infimum i.e $\lim x_n = \inf\{x_n : n \in \mathbb{N}\}$

$\boxed{\text{Part a}}$

Proof: let a real sequence x_n be a bounded sequen[ce]
which is increasing monotonically i.e $x_1 < x_2 < x_3 \ldots$
so, for such sequence $\exists\ M > 0$ such that $M \geq x_n\ \forall n$
As the sequence is bounded it contains its
supremum say x^* such that $x^* \geq x_n\ \forall n \in \mathbb{N}$.
$\therefore$ supremum of sequence x_n is x^*.
Now we have to show that, this increasing bounded
sequence x_n converges to its supremum i.e x^*.
We know that, For $\varepsilon > 0$,

$x^* - \varepsilon$ is not the supremum of the sequ[ence]
x_n

$$\therefore \quad x^* - \varepsilon < x^*$$

Also we have $x^* > x_n\ \forall n \in \mathbb{N}$ & $x^* < x^* + \varepsilon$
$\therefore$ we can write,

$$x^* - \varepsilon < x^*$$
$$x^* - \varepsilon < x_n < x^* < x^* + \varepsilon$$
$$\Rightarrow \quad x^* - \varepsilon < x_n < x^* + \varepsilon \qquad \Rightarrow x_n \in (x^* - \varepsilon, x^* + [\varepsilon])$$

For $\varepsilon > 0$, $\exists\ K \in \mathbb{N}$ such that

$$|x_n - x^*| < \varepsilon \qquad \forall n \geq K$$

$\therefore$ we can say that sequence x_n converges to x^* i[ts]
supremum.

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Part b: For a bounded decreasing real sequence say y_n we can write that $(-y_n)$ is a bounded increasing sequence.

consider,

$$x_n = (-y_n) \quad \rightarrow ①$$

using the proof of part a we have, for bounded increasing sequence,

$$\lim x_n = \sup\{x_n : n \in \mathbb{N}\}$$

$\therefore$ using ① we can write

$$-\lim y_n = \sup\{-y_n : n \in \mathbb{N}\} \qquad [\because x_n = -y_n]$$

$$-\lim y_n = -\inf\{y_n : n \in \mathbb{N}\}$$

$$-\lim y_n = -\inf\{y_n : n \in \mathbb{N}\}$$

$$\Rightarrow \lim y_n = \inf\{y_n : n \in \mathbb{N}\} \qquad \lim y_n = \inf\{y_n : n \in \mathbb{N}\}$$

$\Rightarrow$ A decreasing bounded real sequence converges to infimum of its sequence.

Hence, For increasing bounded sequence $\lim x_n = \sup\{x_n : n$

For decreasing bounded sequence $\lim x_n = \inf\{y_n : n$

Hence the proof

Q3. Given that, $x_n = \frac{1}{1+n^2}$

<u>To prove</u> : $\lim x_n = 0$

For a real number n & $\varepsilon > 0$ using Archemedian property we can write that, $\exists$ natural no k.

$\frac{1}{k} < \varepsilon \rightarrow ①$ and $\frac{1}{n} < \frac{1}{k} \rightarrow ②$

we know ~~$n^2 > 1+n^2$~~ $1+n^2 > n^2$

$\Rightarrow$ ~~$\frac{1}{n^2} < \frac{1}{1+n}$~~ $\frac{1}{1+n^2} < \frac{1}{n^2}$

$\therefore$ From ②,

$\frac{1}{n^2} < \frac{1}{k}$

$\Rightarrow \frac{1}{1+n^2} < \frac{1}{k} < \varepsilon$ (From ①)

$\Rightarrow \frac{1}{1+n^2} < \varepsilon \rightarrow ③$

For $\varepsilon > 0$ $\exists$ $k \in N$ such that, $\left|\frac{1}{1+n^2} - 0\right| = \frac{1}{1+n^2} < \varepsilon$

From ③

$\therefore \left|\frac{1}{1+n^2} - 0\right| < \varepsilon$

$\Rightarrow$ 0 is the limit of x_n

$\Rightarrow \lim \frac{1}{1+n^2} = 0$

Hence proved.

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Q3.

(iii) Statement : A sequence in R can utmost have one limit

Proof :

Consider a real sequence x_n which converges to l i.e l is limit of x_n.

On contrary assume that x_n has 2 limits i.e l & l'

Then For limit l of x_n,

For $\varepsilon > 0$, $\exists$ $k_1 \in \mathbb{N}$ such that,

$$|x_n - l| < \varepsilon/2, \ \forall n \geq k_1$$

Also, For limit l' of x_n,

For $\varepsilon > 0$, $\exists$ $k_2 \in \mathbb{N}$, such that,

$$|x_n - l'| < \varepsilon/2, \ \forall n \geq k_2$$

~~Fk~~ let $k = \max\{k_1, k_2\}$

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$\therefore$ we can write,

$$|x_n - l| < \varepsilon/2, \ \forall n \geq k$$
$$|x_n - l'| < \varepsilon/2, \ \forall n \geq k$$

consider,

$$|l - l'| = |l - x_n + x_n - l'|$$
$$= |l - x_n| + |x_n - l'|$$
$$= |x_n - l| + |x_n - l'|$$
$$< \varepsilon/2 + \varepsilon/2$$

$$\Rightarrow |l - l'| < \varepsilon$$

For $\varepsilon = 0$, $[1 - l' = 0 \Rightarrow l = l']$

which is contradiction

$\therefore$ we have $1 = 1'$

$\Rightarrow$ A real sequence at atmost one limit.

(iv) consider a constant function 'K' on interval $[a, b]$

A fuction is integrable when, $\int_a^{-b} f(x)\,dx = \int_{-a}^{b} f(x)\,dx$

on the integral $[a, b]$ which is bounded.

length of interval is $b-a$. Here $M_r = m_r = K$

using Rieman integration we have,

$$\int_a^{-b} f(x)\,dx = \inf \{U(P, f)\} \quad \text{and}$$

$$\int_{-a}^{b} f(x)\,dx = \sup \{L(P, f)\}$$

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$$U(P, f) = \sum_{i=1}^{n} M_r \Delta x_r$$

$$= \cancel{k} \Delta x_r \sum_{i=1}^{n} M_r$$

$$= \cancel{k}. \Delta x_r \sum_{i=1}^{n} M_r$$

$$= (b-a) \sum_{i=}^{n} M_r \qquad \because \Delta x_r = b-a$$

$U(P, f) = K(b-a) \longrightarrow (1)$ $\because$ function is constant & $\sup\{x_n\} = M_r = K$

$$\therefore \int_a^{-b} f(x)\,dx = \inf \{K(b-a)\}$$

$$= K(b-a) \longrightarrow (2)$$

Also,

$$\sup \{L(P,f)\} = \int_{-a}^{b} f(x)\,dx)$$

$$L(P,f) = \sum_{i=1}^{n} m_r \Delta x_r$$

$$= \Delta x_r \sum_{i=1}^{n} m_r$$

$$= (b-a) \cdot K$$ $\because$ Function is constant & here $m_r = K$.

$\therefore L(P,f) = K.(b-a)$

$$\therefore \int_{-a}^{b} f(x)\,dx = \sup\{K(b-a)\}$$

$$\therefore \int_{-a}^{b} f(x)\,dx = K(b-a) \longrightarrow ③$$

From ② & ③ $\int_{-a}^{b} f(x)\,dx = \int_{a}^{-b} f(x)\,dx$

$\therefore$ constant function is integrable.