Date:12/10/2019

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
B. Sc. I Sem. I Mathematics
Internal Examination 2019-20

All the students of B.Sc. I Mathematics (A and C) are hereby informed that their theory Internal Examination of Mathematics will be conducted on **18th October, 2019 at 3.00 pm to 4.00 pm.** The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus for examination will be as mentioned in following table.

Sr. No.	Name of Paper	Syllabus
1	DSC-1003A: Section I- Differential Calculus-I	Successive Differentiation nth order derivative of standard functions, Examples on nth order derivatives, Leibnitz's theorem, Partial differentiation, Chain rule (without proof) and its examples. Euler's theorem on homogenous functions. Maxima and Minima for functions of two variables. Lagrange's method of undetermined multipliers
2	DSC-1003A-section-II: Differential Calculus-II	Rolle's Theorem, Geometrical interpretation of Rolle's Theorem. Examples on Rolle's Theorem, Lagrange's Mean Value Theorem (L.M.V.T.) Geometrical interpretation of L.M.V.T., Examples on L.M.V.T., Cauchy's Mean Value Theorem (C.M.V.T.) Examples on C.M.V.T., Taylor's Theorem with Lagrange's and Cauchy's form of remainder, Maclaurin's Theorem with Lagrange's and Cauchy's form of remainder, Maclaurin's series, Examples on Maclaurin's series, Examples on maxima and minima of function, Indeterminate Forms

***Nature of question paper:-**

Q.1) Attempt any two (20 marks)

Q.2) Attempt any two (10 marks)

ESTD. JUNE 1964

Mr. S. P. Patankar

HEAD

Department of Mathematics

Vivekanand College, Kolhapur

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

B.SC. Part- I (Mathematics) (Sem-I)
Internal Examination- Nov 2019
Differential Calculus-I and II
Course code : DSC-1003 A

Marks-30

Q.1) Attempt any two of the following. [20]

1) State and Prove Leibnitz's theorem.
2) State and prove Roll's theorem.
3) Find the Maclaurin's series of cos x

Q.2) Attempt any two of the following [10]

1) If $y = \sin(ax + b)\ then\ find\ y_n$.
2) For $f(x) = e^x\ and\ g(x) = e^{-x}, show\ that\ c\ of\ CMVT$ is A.M. between a and b.
3) If $y = e^{ax} \sin bx$ then find y_n.

Date: 05/10/2019

Vivekanand College, Kolhapur (Autonomous)

Department of Mathematics

B. Sc. II Sem. III

Internal Examination 2019-20

All the students of B. Sc. II are hereby informed that their Internal Examination of Mathematics will be conducted on **14th** October 2019 at time 12.30 p.m. To 1:30 p.m. in Room No. 14. Syllabus and timetable of examination is given below.

Sr. No.	Paper	Units
1	Section I- Differential Calculus	Unit I- Jacobian Unit II- Curvature Unit III- Asymptotes
2	Section II- Integral Calculus	Unit I- Beta and Gamma function

ESTD.
JUNE
1964

(Prof. S. P. Patankar)
HEAD
Department of Mathematics
Vivekanand College, Kolhapur

Nature of Question Paper

Time:-1 Hours Total Marks:30

Section -I

Q.1) Attempt any One [07]

i)

ii)

Q.2) Attempt any two [08]

i)

ii)

iii)

Section-II

Q.1) Attempt any One [07]

i)

ii)

Q.2) Attempt any two [08]

i)

ii)

iii)

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-II) Semester-III Internal Examination 2019-2020

MATHEMATICS

Subject Code: **Time: 1 Hr.**

Date: 14/10/2019 **Total Marks: 30**

Section-I

Differential Calculus

Q.1) Attempt any one. **[07]**

1) If J is the Jacobian of u, v w. r. t. x, y and J' is the Jacobian of x, y w. r. t. u, v prove that JJ'=1.
2) Define Radius of curvature. Find the formula for radius of curvature for cartesian equation $y = f(x)$.

Q.2) Attempt any three **[08]**

1) Find the Jacobian of the following transformations $u = x^2 - y^2, v = xy$
2) Find the asymptote of the curve $y^2 = x$
3) Show that the curve $y = x^4$ is concave upwards at the origin.
4) Find the radius of curvature for the curve $y^2 = 8x$ at point $\left(\frac{9}{8}, 3\right)$.

Section-II

Integral Calculus

Q.1 Select the correct alternatives for each of the following: **[07]**

1) Prove that $\int_0^{\frac{\pi}{2}} sin^p\theta \ cos^q\theta \ d\theta = \frac{1}{2} B\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$
2) $\Gamma n = 2\int_0^\infty e^{-t^2} t^{2n-1}\, dt, n > 0$ is known as ….. form of Gamma function.

Q.2) Attempt any three **[08]**

1) Evaluate $\int_0^1 \sqrt[3]{\left(log\frac{1}{x}\right) dx}$
2) Prove that $B(m,n) = B(m, n+1) + B(m+1, n)$
3) Evaluate $\int_0^\infty e^{-h^2x^2} dx$.
4) Show that $\int_0^\infty e^{-x^3} dx = \frac{1}{3}\Gamma\frac{1}{3}$

Date: 1/10/2019

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
B. Sc. III Sem. V
Internal Examination 2019-20

All the students of B.Sc. III are hereby informed that their Internal Examination of Mathematics will be conducted from 9th October 2019 to 12th October 2019. Syllabus and timetable for examination is given below:

Sr. No.	Name of Paper	Units	Date	Time
1	Real Analysis	UNIT I, II, III	09/10/2019	3.00 -4.00
2	Modern Algebra	UNIT I, II, III	10/10/2019	3.00 -4.00
3	Partial Differential Equations	UNIT I, II, III	11/10/2019	3.00 -4.00
4	Numerical Methods I	UNIT I, II, III, IV	12/10/2019	3.00 -4.00

*** Nature of question paper:**

Time: 1hour **Total Marks: 30**

Q.1 Attempt any four **(16 Marks)**

Five questions

Q.2 Attempt any two **(14 Marks)**

Three questions

Venue: Roome No. 39

ESTD. JUNE 1964

(S. P. Patankar)
HEAD
Department of Mathematics
Vivekanand College, Kolhapur

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-V Internal Examination 2019-20

MATHEMATICS

Real Analysis

Subject Code: **Date: 09/10/2019**

Total Marks: 30 **Time: 3.00 - 4.00**

Q.1 Attempt any four of the following **[16]**

i) If $a, b \in R, then$ prove that $||a| - |b|| \leq |a - b|$.

ii) Prove that $\lim_{n\to\infty} (\sqrt{n+1} - \sqrt{n}) = 0$

iii) Show that series $\sum_{n=1}^{\infty} \frac{1}{n^P}$ converges for P > 1

iv) Show that $f(x) = \begin{cases} 1 \ when \ x \in Q \\ -1 \ when \ x \in Q^c \end{cases}$ is not R – integrable over [a, b].

v) If P = {0, 1, 2, 4} is partition of the interval [0, 4] and f(x) = x^2. Find

a) norm P b) U (P, f) c)L(P, f) d) U(P, f) – L(P, f)

Q.2. Attempt any two of the following: **[14]**

i) State and prove Cauchy convergence criterion theorem

ii) State and prove Monotone convergence theorem and show that the sequence

$S_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots\ldots\ldots \frac{1}{2n}$ $\forall n \in N$ is convergent.

iii) State and prove Darboux theorem.

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-V Internal Examination 2019-20

MATHEMATICS

Modern Algebra

Subject Code: **Date: 10/10/2019**

Total Marks: 30 **Time: 3.00 - 4.00**

Q.1 Attempt any four of the following **[16]**

i) Define Order of Cycle. Find the order of cycle (1 2 3 5) from S_5 defined on $S = \{1,2,3,4,5\}$

ii) Show that an non empty subset H of group G is subgroup if and only if $ab^{-1} \in H$, $\forall$ a,b $\in$ H

iii) Show that intersection of two normal subgroups is again a normal subgroup.

iv) Prove that every finite integral domain is field

v)Define sum of two ideals and Show that Sum of two ideal is again a ideal.

Q.2. Attempt any two of the following: **[14]**

i) State and prove Cayley's Theorem.

ii)Prove that, Set of all even permutations forms group called Alternating group.

iii) Prove that Infinite cyclic group have exactly two generators

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-V Internal Examination 2019-20

MATHEMATICS

Partial Differential Equations

Subject Code: **Date: 11/10/2019**

Total Marks: 30 **Time: 3.00 - 4.00**

Q.1 Attempt any four of the following **[16]**

i) *Explane Charpits method of solving non linear pde &*

hence or otherwise solve the pde $z(p^2+q^2)+px+qy=0$

ii) *Find the integral if exists of equation* $xzdx+xzdy+xydz=0$

iii) *Find the integral surface of the differential equation*

$(p^2+q^2)x=pz$ *when it passes through the curve*

$C: x_0=0, y_0=s^2, z_0=2s,$

iv) *Classify & reduce the equation*

$x^2u_{xx}+y^2u_{yy}-2xyu_{xy}=e^x$ *into canonical form*

v) *Find the integral if exists of equation* $xzdx+xzdy+xydz=0$

Q.2. Attempt any two of the following: **[14]**

i) *Find the complete integral of*

$2z+p^2+qy+2y^2=0$ *by using charpits method*

ii) *Reduce the equation* $xu_{xx}-yu_{yy}=0$ *into canonical form*

iii) *Show that* $2z=(ax+y)^2+b$ *is complete integral of*

$px+qy-q^2=0$

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-V Internal Examination 2019-20

MATHEMATICS

Numerical Methods I

Subject Code: **Date: 12/10/2019**

Total Marks: 30 **Time: 3.00 - 4.00**

Q. 1 Attempt any four of the following: **[16]**

i) Solve the given system of equation by using Gauss – Jordan method
$3x+\ 2y+4z=7,\quad 2x+y+z\ =7,\quad x+3y+5z=2$

ii) Find the eigen value and corresponding eigen vector of $A=\begin{bmatrix}3&2&2\\2&2&0\\2&0&4\end{bmatrix}$

iii) Find second largest eigen value and eigen vector of $A=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$ by method of exhaustion (largest eigen value is 8 and corresponding eigen vector is $[2\quad -1\quad 1]^T$)

iv) Explain Newton -Raphson method.

v) Explain Regula - Falsi method.

Q.2. Attempt any two of the following: **[14]**

i)Solve the following system of linear equations by Jacobi' method :

$8x_1-3x_2+2x_3=20, 6x_1+3x_2+12x_3=35, 4x_1+11x_2-x_3=33$

ii) Solve the following system of linear equations by Gauss Elimination method

$x_1+x_2-x_3=2, 2x_1+3x_2+5x_3=-3, 3x_1+2x_2-3x_3=6$

iii) Using power method find the dominant Eigen value and corresponding Eigen vector of

$A=\begin{bmatrix}1&6&1\\1&2&0\\0&0&3\end{bmatrix}$ with starting vector $X_0=(1,0,0)^T$.

Date:11/10/2019

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
B. Com. I Sem. I
Internal Examination 2019-2020

All the students of B.Com. I Mathematics are hereby informed that their Internal Examination of Mathematics will be conducted on **18^{th} October, 2019 from 10.00 am to 11.00 am.**

Syllabus for examination :

Name of Paper	Topics
Business Mathematics-I GEC-1045A	Unit 3 Matrix

Venue- Room No. 56

ESTD. JUNE 1964

Mr. S. P. Patankar
HEAD
Department of Mathematics
Vivekanand College, Kolhapur

Nature of question Paper

Time: 1 Hr. **Total Marks: 30**

Q.1) Attempt any two [20]

1)

2)

3)

Q.2) Attempt any two [10]

1)

2)

3)

ESTD. JUNE 1964

Mr. S. P. Patankar
HEAD
Department of Mathematics
Vivekanand College, Kolhapur

Vivekanand College, Kolhapur (Autonomous)

B.Com. (Part-I) Semester-I

Internal Examination

Business Mathematics Paper- I

Time: 10.00 am to 11.00 am **Total Marks: 30**

Q.1) Attempt any two **[20]**

1) Find the inverse of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

2) If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$ verify that (AB)C=A(BC)

3) Solve the equations $2x + y + z = 2, x + y + z = 0, 4x - y - 3z = 20$ by using matrix

Q.2) Attempt any two **[10]**

1) State the properties of Determinant of matrix.

2) Solve the following equations using crammer's rule
$3x + 3y - z = 11, 2x - y + 2z = 9, x + y + z = 9$

3) Find the value of x if $\begin{bmatrix} 2+x & 3+x & 4+x \\ 1 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}$

Date-14/02/2020

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
B. Sc. I Sem. II Mathematics
Theory Internal Examination 2019-20

All the students of B.Sc. I Mathematics (A and C) are hereby informed that their theory Internal Examination of Mathematics will be conducted on **20th February, 2020 at 2.00 pm to 3.00 pm.** The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus for examination will be as mentioned in following table.

Sr. No.	Name of Paper	Syllabus
1	DSC-1003A-Section-I: Differential Equations-I	Unit I
2	DSC-1003A-Section-II: Differential Equations-II	Unit I

***Nature of question paper:-**
Q.1) Attempt any two (20 marks)
Q.2) Attempt any two (10 marks)

ESTD. JUNE 1964

Mr. S. P. Patankar
HEAD
Department of Mathematics
Vivekanand College, Kolhapur

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

B.SC. Part- I (Mathematics) (Sem-II)
Internal Examination- Feb 2020
Differential Equation -I and II
Course code : DSC-1003 B

Marks-30

Q.1) Attempt any two of the following. **[20]**

1) State and prove Necessary and sufficient condition for differential equation Mdx +Ndy=0 to be exact.

2) Define Clairaut's equation and explain the method of solving it. Hence solve

$$sinpx\ cosy = cospx\ siny + p$$

3) Explain method of variation of parameters.

Q.2) Attempt any two of the following **[10]**

1) Solve $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{\cos x}{1+x^2}$

2) Solve $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$

3) Solve the second order differential equation $y'' - 9y' + 20y = 0$.

Date:11/03/2020

Vivekanand College, Kolhapur (Autonomous)

Department of Mathematics

B. Sc. II Sem. IV

Internal Examination 2019-20

All the students of B. Sc. II are hereby informed that their Internal Examination of Mathematics will be conducted on **19th** March 2020 at time 3.30 p.m. To 4.30p.m. in Room No. 11. Syllabus and timetable of examination is given below.

Sr. No.	Paper	Units
1	Section I- Discrete Mathematics	Unit II- Generating and recurrence relation
2	Section II- Integral Transformation	Unit I- Laplace transformation Unit II- Inverse Laplace transformation

ESTD. JUNE 1964

(Prof. S. P. Patankar)
HEAD
Department of Mathematics
Vivekanand College, Kolhapur

Nature of question paper

Time:-1 Hours	Total Marks:30

Section -I

Q.1) Attempt any One [07]

i)

ii)

Q.2) Attempt any two [08]

i)

ii)

iii)

Section-II

Q.1) Attempt any One [07]

i)

ii)

Q.2) Attempt any two [08]

i)

ii)

iii)

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-II) Semester-IV Internal Examination 2019-2020

MATHEMATICS

Subject Code: **Time: 1 Hr.**

Date: 19/03 /2020 **Total Marks: 30**

Section-I

Discrete Mathematics

Q.1 Attempt any Two **[07]**

1) Find total solution of difference equation $a_r - 7a_{r-1} + 10a_{r-2} = 6 + 8r$ with initial conditions $a_0 = 1, a_1 = 2$.
2) Find generating function i) $(0 \times 5^0, 1 \times 5^1, 2 \times 5^2, \dots)$

   ii) $\left(1, \frac{2}{3}, \frac{3}{9}, \frac{4}{27}, \dots\right)$

Q.2 Attempt any Two **[08]**

1) Find Homogeneous solution of $a_r - 9a_{r-1} + 4a_{r-3} + 12a_{r-4} = 0$
2) Prove that generating function of the sequence 2, 4, 8, 16, 32,... is $\frac{2}{1-2x}$
3) Find the particular solution of $a_r + 6a_{r-1} + 9a_{r-2} = 2$

Section-II

Integral Transformation

Q.1 Attempt any Two **[07]**

1) If $L\{F(t)\} = f(s)$ then prove that $L\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$, Hence or otherwise

   solve $L\{sin^2 at\}$
2) If $L^{-1}\{f(s)\} = F(t)$ then prove that $L^{-1}\{f(s-a)\} = e^{at} L^{-1}\{f(s)\}$

Q.2 Attempt any two **[08]**

1) Find the Laplace transform of $F(t) = \sin 2t \cos t$
2) Find $L^{-1}\left\{\frac{3s-2}{s^{5/2}} - \frac{7}{3s+2}\right\}$
3) Using Laplace transform, Solve $4y'' + \pi^2 y = 0$, $y(0) = 2$, $y'(0) = 0$

Date: 02/03/2020

Vivekanand College, Kolhapur (Autonomous)
Department of Mathematics
B. Sc. III Sem. VI
Internal Examination 2019-20

All the students of B.Sc. III are hereby informed that their Internal Examination of Mathematics will be conducted from 11th March 2020 to 14th March 2020. Syllabus and timetable for examination is given below:

Sr. No.	Name of Paper	Units	Date	Time
1	Metric Space	UNIT I, II	11/03/2020	12.00 -1.00
2	Linear Algebra	UNIT I, II	12/03/2020	12.00 -1.00
3	Complex Analysis	UNIT I, II	13/03/2020	12.00 -1.00
4	Numerical Methods II	UNIT I, II, III	14/03/2020	12.00 -1.00

*** Nature of question paper:**

Time: 1hour	**Total Marks: 30**
Q.1 Attempt any four	**(16 Marks)**
Five questions	
Q.2 Attempt any two	**(14 Marks)**
Three questions	

Venue: Roome No. 39

ESTD. JUNE 1964

(S. P. Patankar)
HEAD
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Vivekanand College, Kolhapur

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-VI Internal Examination 2019-20

MATHEMATICS

Metric Space

Subject Code: **Date: 11/03/2020**

Total Marks: 30 **Time: 12.00 - 1.00**

Q.1 Attempt any four of the following **[16]**

i) Show that the mapping d : $R^2 \rightarrow R^2$ defined by d (x, y) $= |x_1 - y_1| + |x_2 - y_2|$ where $x = (x_1, x_2), y = (y_1, y_2)$ is metric on R^2

ii)Prove that in a metric space, every open sphere is an open set

iii) Consider the usual metric space (R, d). Find the closure of the following sets

a) A = {1/n: n ∈ N} b) Z c) Q d) (2, 6)

iv) Prove that, in a metric space, every convergent sequence is a Cauchy sequence

v) Prove that, in a metric space (X, d) the union of an arbitrary family of open set is open.

Q.2 Attempt any two of the following **[14]**

i) Prove that a metric space (X, d) is totally bounded if every sequence in X contains a Cauchy sequence

ii) Prove that if T is a contraction mapping on a complete metric space (X, d) then there exist a unique point x in X such that T(x) = x

iii) Prove that A mapping f: X → Y is continuous on X iff $f^{-1}(G)$ is open in X for all open subsets of Y.

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-VI Internal Examination 2019-20

MATHEMATICS

Linear Algebra

Subject Code: **Date: 12/03/2020**

Total Marks: 30 **Time: 12.00 - 1.00**

Q.1 Attempt any four of the following **[16]**

i)A non empty subset W of vector space V(F) is subspace of V iff
$ax + by \in$ W for a, b $\in$ F , x, y $\in$ W

ii) Let T be linear operator on FDVS V(F) then show that c $\in$ F is an characteristic value of T iff T-CI is singular, With usual notation

iii) Find Range, Rank , Kernel, Nullity of following
T : $R^2 \to R^3$ such that , T (x , y) = (x, x+y, y)

iv) Define the range of a homomorphism. Prove that the range of a homomorphism $T: V \to U$ is a subspace of V.

v) Define Characteristic values of matrix, Characteristic equation of matrix.
Find Characteristic values of A = $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

Q.2 Attempt any two of the following **[14]**

i) Define Linear span of nonempty subset of Finite dimensional vector space V(F). Suppose S is finite subset of vector space V(F) such that L(S) = V. then prove that their exist subset of S which is basis of V

ii) State and prove Sylvester's law of nullity.

iii) Let V be a Non Zero IPS of dimension n then show that V has Orthonormal Basis.

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-VI Internal Examination 2019-20

MATHEMATICS

Complex Analysis

Subject Code: **Date: 13/03/2020**

Total Marks: 30 **Time: 12.00 - 1.00**

Q.1 Attempt any four of the following [16]

i) Show that the function $u = x^4 - 6x^2y^2 + y^4$ satisfies Laplaces equation and find its corresponding analytic function $f(z) = u + iv$

ii) If $f(z) = u + iv$ is an analytic and $u - v = e^x(\cos y - \sin y)$ find $f(z)$ in terms of z.

iii) Find the Laurent's series for $f(z) = \frac{1}{(z+1)(z+3)}$ in powers of z when $1 < |z| < 3$

v) Write a short note on Milne's Thomson's method for constructing analytic function

vi) Evaluate $\int_c \frac{z^3+3}{z^2-1} dz$ where C is the circle $|z + 1| = 1$.

Q.2 Attempt any two of the following [14]

i) Define analytic function. If $f(z) = u + iv$ is an analytic function and $z = re^{i\theta}$ where u, v, r, θ are all real, show that the Cauchy-Riemann equations are

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

ii)Explain Milne's Thomson's method for constructing of an analytic function. Hence find $f(z)$ when $u = e^x cosy$.

ii) If $f(z)$ is analytic in simply connected domain D bounded by rectifiable curve C and is continuous on C then prove that $f(z) = \frac{1}{2\pi i}\int_C \frac{f(\xi)}{\xi - z} d\xi$ where z is any point of domain D.

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-VI Internal Examination 2019-20

MATHEMATICS

Numerical Methods II

Subject Code: **Date: 14/03/2020**

Total Marks: 30 **Time: 12.00 - 1.00**

Q.1 Attempt any four of the following **[16]**

i) Show that a) $\Delta - \nabla = \nabla\Delta$ b) $\Delta\nabla = \nabla\Delta$

ii) Using Newton's divided difference formula, find the value of f (8) from the following table:

x	4	5	7	10	11	13
y	48	100	294	900	1210	2028

iii) Find $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ at x = 5from the following data

x	5	5.1	5.2	5.3
y	6.2146	6.2344	6.2538	6.2729

iv) Evaluate the integration $\int_0^2 e^x sinxdx$ using Trapezoidal rule with 4 subdivisions.

v)Using Lagrange's formula of interpolation find the interpolating polynomial and the value of f (2.5) from the following table:

x	1	2	3	4
y	1	4	9	16

Q.2 Attempt any two questions **[14]**

i) Derive the relation between operator E^{-1} and ∇ and find the values of y at x = 0.25 and x = 0.35 from the following data

X	0.1	0.2	0.3	0.4	0.5
Y	1.40	1.56	1.76	2.00	2.28

ii) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by a) Simpson's 3/8 rule and b) Simpson's 1/3 rule take n = 6

iii) Derive the formula for $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ by using Newton's forward interpolation.

Date-22/02/2020

Vivekanand College, Kolhapur (Autonomous)

Department of Mathematics

B. Com. I Sem. II

Internal Examination 2019-2020

All the students of B.Com. I Mathematics are hereby informed that their Internal Examination of Mathematics will be conducted on **28^{th} February, 2020 from 10.00 am to 11.00 am.**

Syllabus for examination:

Name of Paper	Topics
Business Mathematics-II GEC-1045B	Unit 1: Functions Unit 4: Integration

Venue:- Room No. 56

ESTD. JUNE 1964

Mr. S. P. Patankar
HEAD
Department of Mathematics
Vivekanand College, Kolhapur

Nature of question Paper

Time: 1 Hr. **Total Marks: 30**

Q.1) Attempt any two [20]

1)

2)

3)

Q.2) Attempt any two [10]

1)

2)

3)

ESTD. JUNE 1964

Mr. S. P. Patankar
HEAD
Department of Mathematics
Vivekanand College, Kolhapur

Vivekanand College, Kolhapur (Autonomous)
B.Com. (Part-I) Semester-II
Internal Examination
Business Mathematics Paper- II

Time: 1 Hr. **Total Marks: 30**

Q.1) Attempt any two **[20]**

1) Discuss the continuity of the function f(x) in [0, 10] defined by,

$$f(x) = \begin{cases} 3x+5, & for\ 0 \le x < 3 \\ 2x+8, & for\ 3 \le x < 5 \\ x+13, & for\ 5 \le x \le 10 \end{cases}$$

2) (a) Find $\lim\limits_{x \to 2} \frac{\sqrt{x^2+1}-\sqrt{5}}{x-2}$. (b) Find $\lim\limits_{x \to 3} \frac{\sqrt{x^2+1}-\sqrt{10}}{x-3}$.

3) Define the following terms

(a) Explicit Function (b) Parametric Function (c) Composite function

(d) Algebraic and Transcendental function (e) Increasing and Decreasing Function

Q.2) Attempt any two **[10]**

1) Find the value of λ, if $\int_0^1 (x^4 + x^3 + \lambda)\, dx = \frac{49}{20}$

2) Evaluate $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$

3) If $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$ then evaluate the values of a and b.

section - (I)

" ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार "

-शिक्षणमहर्षी डॉ. बापूजी साळुंखे

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Suppliment No. :

Subject : Mathematics

Roll No. : 7248

Test / Tutorial No. :

Class : B.S C FY

Div. : A

Q1)

10 + 14 = 24

2) state and

→ To obtain the n^{th} derivative of product of two functions.

statement : Leibnitz's theorem show that n^{th} derivation of $(u.v)$ where u and v are functions of x then

$$y_n = (u.v)_n = {}^nC_0 u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + \dots\dots {}^nC_r u_{n-r} v_r + \dots\dots + {}^nC_n u v_n.$$

where suffixes denotes w.r.t. x

$$y_{n+1} = (u.v)_{n+1} = {}^nC_0 u_{n+1} v + {}^nC_0 u_n v_1 + {}^nC_2 u_{n-1} v_2 + \dots\dots {}^nC_r u_{n-r+1} v_r + \dots\dots + {}^nC_n u v_{n+1}$$

$$y_{n+1} = {}^nC_0 u_{n+1} v + u_n v_1 ({}^nC_0 + {}^nC_1) + u_{n-1} v_2 ({}^nC_1 + {}^nC_2) + ({}^nC_{r-1} + {}^nC_r) u_{n-r+1} + v_{r+1} + \dots + {}^nC_n u_{n+1}.$$

we know ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

${}^nC_1 + {}^nC_2 = {}^{n+1}C_2$ and so on.

${}^nC_0 = 1 = {}^{n+1}C_0$ and ${}^nC_n = 1 = {}^{n+1}C_{n+1}.$

Explanaton B,

$$y_{n+1} = {}^{n+1}C_0 \, u_{n+1} v + {}^{n+1}C_1 \, u_n v_1 + {}^{n+1}C_2 \, u_{n-1} v_2$$

$$\ldots\ldots + {}^{n+1}C_n \cdot u \cdot v_{n+1} .$$

~~Hence proved~~.

put $n+1 = n$.

$$y_n = {}^{n}C_0 \, u_n v + {}^{n}C_1 \, u_{n-1} v_1 + {}^{n}C_2 \, u_{n-2} v_2 .$$

4

$$\ldots + {}^{n}C_n \, u \cdot v_n$$

Hence proved.

Q.2) ① If $y = \frac{x}{1+3x+2x^2}$

$\longrightarrow$ $y = \frac{x}{2x^2+2x+1x+1}$

$$y = \frac{x}{2x(x+1)+1(x+1)}$$

$$y = \frac{x}{(2x+1)(x+1)}$$

$$x = \frac{1/2}{(2\cancel{x}+1)(x+1)} + \frac{+1}{(2x+1)(+1+1)}$$

2

2

$$= \frac{\cancel{1}/2}{\cancel{2}(x+1)} + \frac{1}{2(2x+1)}$$

$$= \frac{1}{2}\left[\frac{1}{x+1} + \frac{1}{(2x+1)}\right]$$

$$= \frac{1}{2}\left[\frac{(-1)^n \cdot n!(1)^n}{(x+1)^{n+1}} + \frac{(-1)^n \cdot n!(2)^n}{(2x+1)^{n+1}}\right]$$

Q2) ③ $\longrightarrow$ $f(x) = 2x^3 - 5x^2 + 3x + 2 \quad x \in [0, 3/2]$

(i) $2x^3 - 5x^2 + 3x + 2$ is polynomial so it is continious function in $x \in [0, 3/2]$

(ii) $f(x) = 2x^3 - 5x^2 + 3x + 2.$
is differentiable in open interval $x \in (0, 3/2).$

(iii) $f(a) = f(b)$
$f(a) = f(0) = 2(0)^3 - 5(0)^2 + 3(0) + 2.$
$\therefore f(0) = 2 = f(a).$

$\therefore f(b) = f(3/2) = 2\left(\frac{3}{2}\right)^3 - 5\left(\frac{3}{2}\right)^2 + 3(3/2) + 2$
$= \cancel{2} \times \frac{27}{\cancel{8}\,4} - 5 \times \frac{9}{4} + \frac{9}{2} + 2$
$= \frac{-189}{42} + 9/2 + 2 = 2$

∴ ~~B~~

∴ $f(a) = f(b)$ that is $f(0) = f(3/2) = 2$.

~~Hence, this $f(x) =$~~ $2x^3 - 5x^2 + 3x + 2$ verify Rolle's theorem.

It satisfies all conditions of R.M.V.T.

$f'(c) = 0.$

$f'(c) = 6x^2 - 10x + 3$

$= 6c^2 - 10c + 3.$ 18

$= \dfrac{\cancel{1}-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-10 \pm \sqrt{100-18\times 4}}{2\times 6}$ or $= \dfrac{-10 - \sqrt{100-18\times 4}}{12}.$

$= \dfrac{-10 + \sqrt{\cancel{82}\,100-72}}{12}$ or $= \dfrac{-10 - \sqrt{\cancel{82}\,100-72}}{12}$

$= \dfrac{-10 + \sqrt{28}}{12}$ or $= \dfrac{-10 - \sqrt{28}}{12}$

$= \dfrac{-10 + 2\sqrt{7}}{12}$ or $= \dfrac{-10 - 2\sqrt{7}}{12}.$

$= \dfrac{-5}{6} + \dfrac{\sqrt{7}}{6}$ $= \dfrac{-5}{6} - \dfrac{\sqrt{7}}{6}$

$= \dfrac{-5+\sqrt{7}}{6}$ $= \dfrac{-5-\sqrt{7}}{6}$

$\left(\dfrac{-5+\sqrt{7}}{6}, \dfrac{-5-\sqrt{7}}{6}\right) \in C.$

∴ It satisfies all conditions of R·M·VT. so its D verify all condition.

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Suppliment No. :		Subject :	Mathematics.
Roll No. :	7248	Test / Tutorial No. :	
Class :	B.SC FY	Div. :	Ⓐ

Q.3) ②.

Statement : Every square matrix - satisfies characteristic equation.

In other words,

$|A-xI| = 0$ will be of form.

$a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n = 0$.

$a_0 + a_1 A + a_2 A^2 + \dots + a_n A^n = 0$.

This equation satisfies by matrix A.

Proof : Let A be the given square matrix of order n.

Then its characteristic equation ~~is~~ ~~that is~~ $|A-xI| = 0$.

let, $Adj[A-xI] = B_0 + B_1 x + B_2 x^2 + \dots + B_{n-1} x^{n-1}$.

The matrix of Degree $(n-1)$.

$A \cdot Adj(A) = |A| I$.

now, $[A-xI] Adj [A-xI] = [A-xI] I$.

$= [a_0 + a_1 x^1 + a_2 x^2 + a \dots \dots + a_n x^n] I$. —— ①.

But

$= [A-xI] \cdot Adj [A-xI]$

$= [A-xI] \cdot [B_0 + B_1 x + B_2 x^2 + \dots + B_{n-1} x^{n-1}]$

$= AB_0 + AB_1 x + AB_2 x^2 + \dots + AB_{n-1} x^{n-1} -$

$B_0 xI - B_1 x^2 I - B_2 x^3 I \dots - + B_{n-1} x^n I$

$= AB_0 + xI[AB_1 - B_0) + x^2 I[AB_2 - B_1] + x^3 I[AB_2 - B_2]$
$+ x^3 I + (AB_2 - B_2) + \cdots B_{n-1} \, x_n$ — ②

= Comparing power of z from ① and ② we get

$a_0 = AB_0, \quad a_1 A = A^2 B_1 - AB_0$

$a_2 A^2 = A^3 B_2 - A^2 B.$

$a_n A^n = A^n \cancel{AA} + B_{n-1}$

~~And add these equalities.~~

~~$a_0 + a_1 A + a_2 A^2 + \cdots + a_n A^n = AB.$~~

~~$AB_0 + A$~~

6

$= a_0 + a_1 n + a_2 A^2 + \cdots + a_n A^n + AB_0 + A^2 B_1 - AB_0 +$
$A^3 B_2 - A^2 B_1 + \cdots + A^n B_{n-1}.$

$= a_0 + a_1 A + a_2 A^2 + \cdots + a_n A^n = 0.$

which prove the theorem.

Q.4)

2) $\longrightarrow \quad x + y + z = 6$

$x - 2y + 3z = 1$

$x + 2y + \lambda z = \mu.$

$$A \cdot X = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ \mu \end{bmatrix}$$

$R_2 \longrightarrow R_2 - R_1$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 2 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ \mu \end{bmatrix}$$

$$R_3 \longrightarrow R_3 - R_1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 2 \\ 0 & 1 & \lambda-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ \mu-6 \end{bmatrix}$$

~~(i) No solution.~~

~~$\lambda \neq 1$~~ $\lambda =$

(ii) ~~Unique solution.~~

~~$\lambda \neq 1$ and $\mu \neq 6$.~~

~~$\rho(A) = \rho(B)$~~

(iii) ~~No solution.~~ ~~Infinite solution~~

~~$\lambda = 1$ and $\mu = 6$~~

$\rho(A) = 3$, $\rho(AB) = 3$. $n = 3$.

(i) unique solution

$\rho(A) = \rho(AB) = \cancel{3}n$

when $\lambda \neq 1$ and $\cancel{\mu} \neq 6$.

~~then this unique solution~~

4

(ii) No solution.

$\lambda = 1$ and $\mu \neq 6$.

$\rho(A) \neq \rho(AB)$

$2 \neq 3$

(iii) Infinite solution.

$\rho(A) = \rho(AB) < n$.

$\lambda = 1$ and $\mu = 6$

~~$\rho(A)$~~ $2 = 2 < 3$ Hence then infinite solution obtain

③ $\longrightarrow x^3 - 18x - 35 = 0.$

$\cancel{x} = \frac{-a_1}{n(a_0)} = \frac{0}{3(1)} = 0$

$t = u^{1/3} + v^{1/3}$

$t^3 = (u+v) + 3u^{1/3}.v^{1/3} \times t.$

~~$t^3 = u+v + 3u$~~

$t^3 - 3u^{1/3}.v^{1/3} \times t - (u+v) = 0.$

~~$t = 1$~~ $x^3 = t^3.$ —

$(3u^{1/3}.v^{1/3})t = (+18x)$ — ①

$(u+v) = +35.$ — ②

$\cancel{3}\, u^{1/3}.v^{1/3}\, \cancel{t} = \cancel{18}\ \cancel{3}\times 6.$

$u^{1/3}.v^{1/3} = 6.$

$u.v = 216.$

$u+v = 35.$

$t^2 - (u+v)t + (u.v) = 0.$

$t^2 - 35t + 216 = 0.$

216
^

~~$t =$~~

$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-35 \pm \sqrt{1225 - 4\times 1\times 216}}{2\times 1}$ or $= \frac{-35 - \sqrt{361}}{2}$

$= \frac{-35 + \sqrt{361}}{2}$

25
64
361

2
16
4
64

5
5
5
50

7748

Q 4) ① If α is root of equation $f(x) = 0$ than the polynomial $f(x)$ is divisible by $(x - \alpha)$.

proof : Given α is root of $p(x) = x$.

$p(\alpha) = 0$ — ①.

now divide $f(x)$ by $(x - \alpha)$ we get quotient (Q) and remainder (R).

$\therefore f(x) = (x - \alpha)\, Q + R$ — ②

~~putting $(x - \alpha) Q + R$~~ .

putting $x = \alpha$ on the both side

$f(\alpha) = (\alpha - \alpha) Q + R$

$f(\alpha) = 0$.

4

both for equation ①.

$R = 0$.

that is $f(x) = (x - \alpha) \cdot Q$

$\therefore f(x)$ is divisible by $(x - \alpha)$.

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Signature of Supervisor :

15+15 = 30

Supplement No. :
Roll No. : 7709
Class : B.Sc-II
Subject : Maths [Number Theory]
Test / Tutorial No. :
Div. :

(Section - I

Q.1.

② Fermat's Theorem :-

If p is prime number and suppose that $p \nmid a$ then $a^{p-1} \equiv 1 \pmod{p}$.

Proof:- let $(p-1)$ has positive multipliers of a

i.e $a, 2a, 3a, \ldots\ldots, (p-1)a$

Consider, Integer $(p-1)$ are congruent to p- modulo p.

let if possible $ra \equiv sa \pmod{p}$

$1 \le r < s < p$

$p \nmid a \Longrightarrow \gcd(a,p) = 1$

and also p is prime.

Now, $ra \equiv sa \pmod{p}$

$r \equiv s \pmod{p}$ $[\because \gcd(a,p) = 1]$

$p \mid r-s$

since $1 \le r < s < p$

then $p \nmid r-s$

hence it is a contradication.

So, we must have all integers (P-1) are incongruent to modulo P. i.e $ra \not\equiv sa \pmod{p}$

hence $a, 2a, 3a, ----, (P-1)a$ leaves different remainders when it is divided by P.

$$\therefore \quad a.2a.3a, ----, (P-1)a \equiv 1.2.3 ----- (P-1) \pmod{P}$$

$$a^{P-1}(P-1)! \equiv (P-1)! \pmod{P}$$

$$a^{P-1} \equiv 1 \pmod{P} \quad [\because \gcd((P-1)!, P) = 1$$

$$\therefore \quad a^{P-1} \equiv 1 \pmod{P}$$

4^{112} divided by 13

$\gcd(4, 13) = 1$

By Fermat's theorem

$$a^{P-1} \equiv 1 \pmod{P}$$

$$4^{13-1} \equiv 1 \pmod{13}$$

$$4^{12} \equiv 1 \pmod{13}$$

$$(4^{12})^9 \equiv (1)^9 \pmod{13}$$

$$4^{108} \equiv 1 \pmod{13}$$

$$4^{109} \equiv 4 \pmod{13}$$

7

$$4^{110} \equiv 16 \pmod{13}$$

8

$$4^{110} \equiv 3 \pmod{13}$$

$$4^{111} \equiv 12 \pmod{13}$$

$$4^{112} \equiv 4 \times 12 \pmod{13}$$

$$4^{112} \equiv 48 \pmod{13}$$

$$4^{112} \equiv 9 \pmod{13}$$

$\therefore$ The remainder will be 9

Q.2.

i) We have $ac \equiv bc \pmod{n}$

$n \mid ac - bc$

$ac - bc = nk$

where k is any positive integer

$c(a-b) = nk$ ——— ①

let $\gcd(c, n) = d$

then their exist integers r & s

Such that.

$d \mid c \quad d \mid n$

$c = dr \quad \cdot\ n = ds$

Such that $c = dr \quad s = \frac{n}{d}$

Now, putting value of s & n in eqⁿ ① we get

$c(a-b) = nk$

$dr(a-b) = dsk$

$r(a-b) = sk$

$s \mid r(a-b)$

$\gcd(r, s) = 1$

$\therefore \quad s \mid (a-b)$

$\therefore \quad a \equiv b \pmod{s}$

$a \equiv b \left(\bmod \frac{n}{d}\right)$

4

Q.2.

ii) $1^5+2^5+3^5+ - - - - + 100^5$ by 4.

$1^5 \equiv 1 \pmod{4}$

$2^5 \equiv 32 \pmod{4}$

$2^5 \equiv 0 \pmod{4}$

$3^5 \equiv 3^2 . 3^2 . 3 \pmod{4}$

$3^5 \equiv 1.1.3 \pmod{4}$

$3^5 \equiv 3 \pmod{4}$

$4^5 \equiv 0 \pmod{4}$

$5^5 \equiv 5^2 . 5^2 . 5 \pmod{4}$

$5^5 \equiv 1 \pmod{4}$

$6^5 \equiv (6^2)^2 . 6 \pmod{4}$

$6^5 \equiv 0 \pmod{4}$

$7^5 \equiv (7^2)^2 . 7 \pmod{4}$

$7^5 \equiv 1 . 3 \pmod{4}$

$8^5 = (8^2)^2 . 8 \pmod{4}$

$8^5 \equiv 0 \pmod{4}$

$\vdots$

$100^5 \equiv 0 \pmod{4}$

$1^5+2^5+3^5 - - - - - + 100^5 \equiv 25 [1+0+3+0] \pmod{100}$

$1^5+2^5+3^5 - - - - + 100^5 \equiv 100 \pmod{100}$

$1^5+2^5+3^5 - - - - + 100^5 \equiv 0 \pmod{100}$

$\therefore$ Remainder will be $\underline{\underline{0}}$

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Signature of Supervisor :

Suppliment No. :

Roll No. : 7709

Class : B.Sc -II

Subject : Maths [Integral Calculus]

Test / Tutorial No. :

Div. : A

07 + 08 = 15

Integral Calculus

Q.1.

1) $\int_0^\infty \frac{dx}{1+x^4}$

Gamma function :-

$\Gamma m = \int_0^\infty \cancel{x}\, e^{-x} x^{m-1}\, dx$

or

$\Gamma n = \int_0^\infty e^{-x} x^{n-1}\, dx$

put $x^2 = \tan\theta$

$x = \sqrt{\tan\theta}$

$\frac{dx}{d\theta} = \frac{1}{2\sqrt{\tan\theta}} \times \sec^2\theta$

$dx = \frac{\sec^2\theta \, d\theta}{2\sqrt{\tan\theta}}$

x	0	∞
t	0	$\pi/2$

07

Now, $= \frac{1}{2}\int_0^{\pi/2} \frac{\sec^2\theta}{(1+\tan^2\theta)} \times \frac{1}{\sqrt{\tan\theta}}\, d\theta$

$= \frac{1}{2}\int_0^{\pi/2} \frac{\sec^2\theta}{\sec^2\theta} \tan^{-1/2}\theta \, d\theta$

$= \frac{1}{2}\int_0^{\pi/2} \sin^{-1/2}\theta \cos^{1/2}\theta \, d\theta$

$= \frac{1}{2} \times' \frac{1}{2} \beta\left(\frac{1-1/2}{2}, \frac{1+1/2}{2}\right)$

$$= \frac{1}{4} \beta\left(\frac{1}{4}, \frac{3}{4}\right)$$

$$= \frac{1}{4} \frac{\Gamma 1/4 \; \Gamma 3/4}{\Gamma \frac{1}{4}+\frac{3}{4}}$$

$$= \frac{1}{4} \frac{\Gamma 1/4 \; \Gamma 3/4}{\Gamma 1}$$

we know that

$$\Gamma 1/4 \; \Gamma 3/4 = \sqrt{2}\, \pi$$

$$\Gamma 1/4 = \frac{\sqrt{2}\, \pi}{\Gamma 3/4}$$

$$= \frac{1}{4} \frac{\Gamma 3/4 \; \sqrt{2}\, \pi}{\Gamma 3/4}$$

$$= \frac{1}{4} \sqrt{2}\, \pi = \frac{\sqrt{2}\, \pi}{2 \times 2} = \underline{\underline{\frac{\pi}{2\sqrt{2}}}}$$

Q. 2.

ii) $\int_{0}^{\pi/4} \sin^2 4x \cos^3 2x \, dx$

put $2x = t$

$dx = dt/2$

x	0	$\pi/4$
t	0	$\pi/2$

$$= \frac{1}{2} \int_{0}^{\pi/2} \sin^2 2t \; \cos^3 t \; dt$$

$$= \frac{1}{2} \int_{0}^{\pi/2} (2\sin t \cos t)^2 \; \cos^3 t \; dt$$

04

$$= \frac{1}{2} \int_{0}^{\pi/2} 4.\ \sin^2 t \; \cos^2 t \; \cos^3 t \; dt$$

$$= \frac{4}{2}\int_0^{\pi/2} \sin^2 t \cos^5 t \; dt$$

$$= \cancel{3}\int_0 2 \times \frac{1}{2}\, \beta\left(\frac{2+1}{2}, \frac{5+1}{2}\right)$$

$$= \beta\left(\frac{3}{2}, \cancel{6}\, 3\right)$$

$$= \frac{\Gamma 3/2 \;\; \Gamma 3}{\Gamma \frac{3}{2}+3}$$

$\frac{35}{3}$ 1

105

$$= \frac{\frac{1}{2}\sqrt{\pi} \times 2 \times 1}{\Gamma \frac{9}{2}}$$

$$= \frac{\sqrt{\pi}}{\frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\sqrt{\pi}}$$

$$= \frac{16}{35\times 3} = \frac{16}{105}$$

iii) $\int_0^{\pi/2} \tan^{1/2}\theta \; d\theta$

$$= \int_0^{\pi/2} \sin^{1/2}\theta \; \cos^{-1/2}\theta \; d\theta$$

$$= \frac{1}{2}\, \beta\left(\frac{1+1/2}{2}, \frac{1-1/2}{2}\right)$$

04

$$= \frac{1}{2}\, \beta\left(\frac{3}{4}, \frac{1}{4}\right)$$

$$= \frac{1}{2}\, \frac{\Gamma 3/4 \; \Gamma 1/4}{\Gamma 1} = \frac{1}{2}\, \frac{\cancel{\Gamma 3/4}\; \sqrt{2}\,\pi}{\cancel{\Gamma 3/4}}$$

$$= \frac{\pi}{\sqrt{2}}$$

i) $\int_0^{2a} x\sqrt{2ax-x^2}\ dx$

$\int_0^{2a} x.\ x^{1/2}\sqrt{2a-x}\ dx$

put $x = 2at$

$dx = 2a\,dt$

x	0	$2a$
t	0	1

01

$= 2a\int_0^1 (2at)^{3/2}$ ~~$\sqrt{1\,2a - 2at}\ dt$~~ $\sqrt{2a-2at}\ dt$

$= 2a\int_0^1 2a^{3/2}\ t^{3/2}\ (1-t)^{1/2}\ dt$

$= 2a^{1+3/2}\int_0^1 t^{3/2+1-1}\ (1-t)^{1/2+1-1}\ dt$

$= 2a^{5/2}\int_0^1 t^{5/2-1}\ (1-t)^{3/2-1}\ dt$

$= 2a^{5/2}\times\beta\left(\frac{5}{2},\frac{3}{2}\right)$

[redacted]

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Roll No. : 8289

Class : B.SC (III) Maths

Subject : MATHEMATICS

Test / Tutorial No. : Internal exam

Div. :

5 + 10 + 5 = 20/30

.1) 1) one integral of partial differential equation $\frac{y^2 z}{x} p + zxq = y^2$ is c) $x^3 - y^3 = 3c_1$

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2) The complete integral of partial differential equation $z = px + qy - 2\sqrt{pq}$ is
A) $z = ax + by - 2\sqrt{ab}$

3) The complete integral of partial differential equation $p^2 + q^2 = m^2$ is
c) $z = ax \pm \sqrt{m^2 - a^2}\, y + b$

4) The solution of partial differential equation $yzp + zxq = xy$ is c) $\phi(x^2 - y^2, x^2 - z^2) = 0$

5) The partial differential equation obtained from $z =$ $z = ax + by + ab$ by eliminating a and b is

c) $z = px + qy + pq$

3) 1. $z = y^2+2f(1/x+\log y)$ ——— ①

diff ① partially w.r.t $x \cdot y$

Consider $\frac{1}{x}+\log y = u$

$$\therefore \frac{\partial z}{\partial x} = \frac{\partial f}{\partial u}\left[\frac{\partial u}{\partial x}+\frac{\partial u}{\partial z}+\frac{\partial z}{\partial x}\right]$$

$$= 2\frac{\partial f}{\partial u}\left[\frac{1}{x^2}\right]$$

$$p = -2\frac{\partial f}{\partial u}\left(\frac{1}{x^2}\right) \text{ — ②}$$

$$\frac{\partial z}{\partial y} = 2y+2\cdot\frac{\partial f}{\partial u}\left[\frac{1}{y}\right]$$

$$q = 2y+2\frac{\partial f}{\partial u}\left[\frac{1}{y}\right]$$

5

$$q = 2y+2\frac{\partial f}{\partial u}\cdot\frac{1}{y}$$

$$q-2y = 2\cdot\frac{\partial f}{\partial y}\left(\frac{1}{y}\right) \text{ — ③}$$

dividing eq^n ② by ③ we get

$$\frac{p}{q-2y} = \frac{-1/x^2}{1/y}$$

$$\frac{p}{q-2y} = \frac{-y}{x^2}$$

$\int px^2 = (-q+2y)y$ $px^2+qy-2y^2=0$ it is requ

Pd

Q.2) B.

~~Q.3)~~

① ④ Let the give P.d.f to 1st order and non linear in and ~~P~~ q be

$F(x, y, z, p, a) = 0$ —— ①.

We know That

$dz = pdx + pdy$ —— ②

For the next step consist another relation

$F(x, y, z, p, a) = 0$ —— ③

Such that

The values of P and q are obtaining by solving eqn ① and ③ substiting in eqn we have integrable From Finding integration of eqn ② we get complete integral of eqn ①

From Finding eqn ③ differtiate parttially eqn ① and ② W.r.t x and y resp we get

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial z} \cdot \frac{\partial z}{\partial x} + \frac{\partial F}{\partial p} \frac{\partial p}{\partial x} \frac{\partial F}{\partial q} \frac{\partial q}{\partial x} = 0 \longrightarrow ④$$

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial z} \cdot p + \frac{\partial F}{\partial p} \cdot \frac{\partial p}{\partial x} \frac{\partial F}{\partial q} \frac{\partial q}{\partial x} = 0 \text{ —— } ⑤$$

$$\frac{\partial f}{\partial x} + \frac{\partial F}{\partial z} \cdot p + \frac{\partial F}{\partial p} \frac{\partial p}{\partial x} + \frac{\partial F}{\partial q} \cdot \frac{\partial q}{\partial x} = 0 \text{ —— } ⑤$$

$$\frac{\partial f}{\partial y} + \frac{\partial F}{\partial z} \cdot q + \frac{\partial F}{\partial p} \cdot \frac{\partial F}{\partial q} \cdot \frac{\partial q}{\partial x} = 0$$

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

36335

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Suppliment No. : 02
Roll No. : 8289
Class : B.SC(III) maths

Subject : Maths
Test / Tutorial No. : Internal exam
Div. :

① Eliminating eq^n ④ we can mutiplying eq^n ④ by $\frac{\partial F}{\partial P}$

$$\left(\frac{\partial F}{\partial x} + P\frac{\partial x}{\partial P}\right)\frac{\partial F}{\partial p} \quad \left(\frac{\partial f}{\partial z} + \frac{\partial x}{\partial P}\right)$$

$$\left(\frac{\partial F}{\partial x} + P.\frac{\partial x}{\partial P}\right) + P \left(\frac{\partial f}{\partial z} + \frac{\partial x}{\partial P}\right) + \frac{\partial F}{\partial p}$$

$$\left(\frac{\partial f}{\partial q} + \frac{\partial F}{\partial q} + \frac{\partial x}{\partial q}\right) \text{———} ⑦$$

Some here eq^n ⑥ & ⑦

$$\left(\frac{\partial F}{\partial q} + q\frac{\partial y}{\partial q}\right) + \left(\frac{\partial F}{\partial x} + P\frac{\partial z}{\partial P}\right) d + \left(\frac{\partial F}{\partial P} + \frac{\partial x}{\partial P} + \frac{\partial y}{\partial P}\right)$$

eliminate $\frac{\partial P}{\partial x}$ From ④ and ⑤

we get
multiply eq^n ④ by $\frac{\partial F}{\partial P}$ and eq^n ⑤ $\frac{\partial F}{\partial P}$ and

Substituting we get

$$\frac{\partial f}{\partial p}\left[\frac{\partial F}{\partial x}+\frac{\partial f}{\partial z}p+\frac{\partial f}{\partial x}\cdot\frac{\partial q}{\partial x}\right]-\frac{\partial F}{\partial p}\left[\frac{\partial f}{\partial x}+\frac{\partial f}{\partial z}\cdot p+\frac{\partial f}{\partial q}\frac{\partial q}{\partial x}\right]=0$$

$$\frac{\partial f}{\partial p}\left[\frac{\partial f}{\partial x}+\frac{\partial f}{\partial z}p+\frac{\partial f}{\partial q}\cdot\frac{\partial q}{\partial x}\right]-\frac{\partial f}{\partial p}\left[\frac{\partial F}{\partial x}+\frac{\partial F}{\partial z}\cdot p+\frac{\partial F}{\partial q}\frac{\partial q}{\partial x}\right]=0$$

i.e $$\left(\frac{\partial f}{\partial x}\cdot\frac{\partial F}{\partial p}-\frac{\partial f}{\partial x}\frac{\partial f}{\partial P}\right)+\left(\frac{\partial F}{\partial z}\cdot\frac{\partial f}{\partial p}-\frac{\partial F}{\partial z}\cdot\frac{\partial F}{\partial p}\right)p+$$

$$\left(\frac{\partial F}{\partial q}\cdot\frac{\partial f}{\partial p}-\frac{\partial F}{\partial q}\cdot\frac{\partial F}{\partial p}\right)\frac{\partial q}{\partial x}=0 \quad ⑧$$

Simillary eliminating $\frac{\partial q}{\partial y}$ from ⑥ & ⑦ we get

$$\left(\frac{\partial F}{\partial y}\cdot\frac{\partial F}{\partial q}-\frac{\partial F}{\partial y}\cdot\frac{\partial F}{\partial q}\right)+\left(\frac{\partial F}{\partial z}\frac{\partial F}{\partial q}-\frac{\partial F}{\partial z}\frac{\partial F}{\partial q}\right)\cdot q+$$

$$\left(\frac{\partial F}{\partial p}\cdot\frac{\partial f}{\partial q}-\frac{\partial f}{\partial p}\cdot\frac{\partial F}{\partial q}\right)\cdot\frac{\partial p}{\partial y}=0 \quad ⑨$$

Since

$$\frac{\partial q}{\partial x}=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)=\frac{\partial^2}{\partial x\partial y}$$

$$\frac{\partial p}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial z}\right)=\frac{\partial^2 z}{\partial y\partial x}$$

$$\therefore \frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x}\Rightarrow\frac{\partial q}{\partial x}=\frac{\partial p}{\partial y}$$

Adding ⑧ and ⑨ and rearraging the terms we obtain

$$\left(\frac{\partial f}{\partial x}+p\cdot\frac{\partial f}{\partial z}\right)\frac{\partial f}{\partial p}+\left(\frac{\partial f}{\partial y}+q\frac{\partial f}{\partial z}\right)+\left(-p\frac{\partial f}{\partial p}-1\cdot\frac{\partial f}{\partial q}\right)\frac{\partial f}{\partial z}$$

$$+\left(-\frac{\partial f}{\partial p}\right)\frac{\partial f}{\partial x}+\left(-\frac{\partial f}{\partial q}\right)\frac{\partial f}{\partial y}=0 \longrightarrow ⑩$$

Integral of ⑩ is obtained by auxillary eq^{n}.

$$\frac{dp}{\left(\frac{\partial f}{\partial x}+p\cdot\frac{\partial f}{\partial z}\right)}=\frac{dq}{\left(\frac{\partial f}{\partial y}+q\frac{\partial f}{\partial z}\right)}=\frac{dz}{-p\frac{\partial f}{\partial p}-q\frac{\partial f}{\partial q}}$$

$$=\frac{dx}{-\frac{\partial f}{\partial p}}=\frac{dy}{-\frac{\partial f}{\partial q}}=\frac{dF}{0} \longrightarrow ⑪$$

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∴ charpit's auxillary eq^{n} ⑪ May be Written as

$$\boxed{\frac{dp}{Fx+pFz}=\frac{dq}{Fy+q\cdot Fz}=\frac{dz}{-p^{F}p-qFq}=\frac{dx}{-Fp}=\frac{dy}{-Fq}=\frac{dF}{0}}$$