

**Vivekanand College, Kolhapur (Autonomous)**  
**Department of Mathematics**  
**B. Sc. I Sem. I Mathematics**  
**Theory Internal Examination 2022-23**

All the students of B.Sc. I Mathematics (A and C) are hereby informed that their theory Internal Examination of Mathematics will be conducted on **12<sup>th</sup> December, 2022 at 2.30 pm to 3.30 pm**. The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus for examination will be as mentioned in following table.

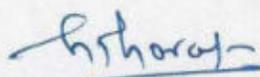
Sr. No.	Name of Paper	Topics
1	DSC-1003A: Section I-Calculus	Unit I and Unit II
2	DSC-1003A - Section-II: Algebra and Geometry	Unit I and Unit III

**\*Nature of question paper:-**

**Q.1) Attempt any two (14 marks)**

**Q.2) Attempt any four (16 marks)**



  
Mr. S. P. Thorat  
**HEAD**  
Department of Mathematics  
Vivekanand College, Kolhapur

**Vivekanand College, Kolhapur (Autonomous)**

**B.Sc. (Part-I) Semester-I Internal Examination, December-2022**

**MATHEMATICS**

**Subject Code: DSC-1003A**

**Paper Code:**

**Day and Date: Monday & 12/12/2022**

**Total Marks: 30**

**Time: 3.00 pm to 4.00 pm**

**Section-I**

**Q.1 ATTEMPT ANY ONE. (07)**

- 1) State and prove Leibnitz's Theorem.
- 2) State and prove Lagrange's mean value theorem.

**Q.2 ATTEMPT ANY TWO. (08)**

- 1) If  $y = \frac{1}{1-5x+6x^2}$  then find  $y_n$
- 2) If  $y = \sin^4 x$  then find  $y_n$
- 3) Verify Rolle's theorem for  $f(x) = x^2 - 4x + 10$  on the interval  $[0,4]$ .

**Section-II**

**Q.3 ATTEMPT ANY ONE. (07)**

- 1) State and prove De-Moivre's Theorem.
- 2) State and prove Cayley-Hamilton Theorem.

**Q.4 ATTEMPT ANY TWO. (08)**

- 1) Investigate for what value of  $\lambda$  and  $\mu$ , the equations  $x + y + z = 6$ ,  $x + 2y + 6z = 5$ ,  $x + 2y + \lambda z = \mu$  have i) No solution ii) Unique solution iii) Infinite solution.
- 2) State and prove the Factor theorem.
- 3) Solve  $x^3 - 18x - 35 = 0$  by cardan's method.



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**Vivekanand College, Kolhapur**  
**(Autonomous)**



KOLHAPUR (AUTONOMOUS)

Date:05/12/2022

**Vivekanand College, Kolhapur (Autonomous)**

**Department of Mathematics**

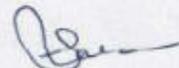
**B. Sc. II Sem. III**

**Internal Examination 2022-23**

All the students of B.Sc. II are hereby informed that their Internal Examination of Mathematics will be conducted from 12<sup>th</sup> December 2022 at time 3.00- 4.00 in Room No. 14. The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus and timetable of examination is given below:

Sr. No.	Paper	Units	Date
1	Section-I Number Theory	Unit I- Divisibility theory in the integers Unit IV- Number Theoretic function	12/12/2022
2	Section-II Integral Calculus	Unit I- Beta and Gamma function Unit IV- Fourier Series	



  
**(Prof. S. P. Patankar)**  
**HEAD**

Department of Mathematics  
Vivekanand College, Kolhapur

**Nature of Question Paper:**

Time : 1Hr.

Total Marks: 30

**Section-I**

Q.1 Attempt any One

[07]

1)

2)

Q.2 Attempt any Two

[08]

1)

2)

3)

**Section-II**

Q.1 Attempt any One

[07]

1)

2)

Q.2 Attempt any Two

[08]

1)

2)

3)

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-II) Semester-III Internal Examination, December-2022

MATHEMATICS

Subject Code: 223 2002

Time: 1.00 pm to 2.00 pm

Day and Date: 12/12/2022 (Monday)

Total Marks: 30

Section-I

Q.1 ATTEMPT ANY ONE. (07)

- 1) State and prove Division algorithm.
- 2) i) Show that for each positive integer  $n \geq 1$ ,  $n = \sum_{d|n} \phi(d)$  the sum being extended over all divisors of  $n$ .  
ii) Show that if  $n$  is the product of twin primes i.e.,  $n = p(p+2)$  where  $p$  and  $p+2$  are primes then,  $\phi(n)\sigma(n) = (n+1)(n-3)$

Q.2 ATTEMPT ANY TWO. (08)

- 1) Determine initial solution of Diophantine equation  $172x + 20y = 1000$
- 2) i) Show that for  $n > 1$ , the sum of the positive integers less than  $n$  & relatively prime to  $n$  is  $\frac{1}{2}n\phi(n)$ .  
ii) Evaluate  $\tau$  and  $\sigma$  for  $n = 3000$ .
- 3) i) Evaluate  $\phi(900)$ .  
ii) If  $n$  is an odd integer then show that  $\phi(2n) = \phi(n)$

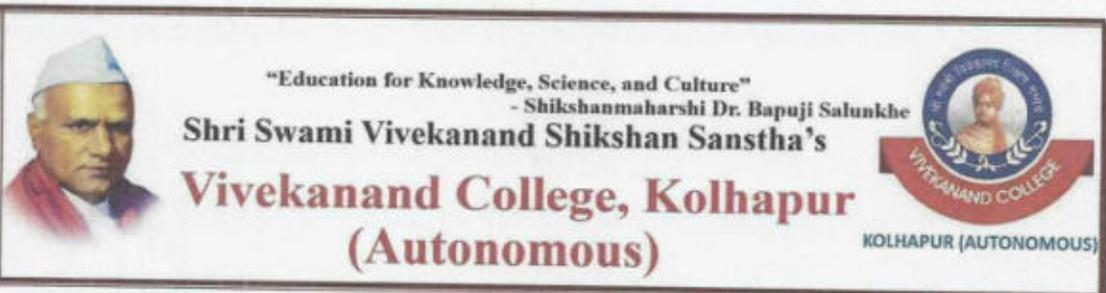
Section-II

Q.3 ATTEMPT ANY ONE. (07)

- 1) State and prove relation between Beta and Gamma function.
- 2) Change the order of integration and evaluate  $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy dx dy$ .

Q.3 ATTEMPT ANY TWO (08)

- 1) Evaluate  $\int_0^{\infty} x^m \cdot e^{-ax^n} dx$ .
- 2) Evaluate  $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ .
- 3) Expand  $f(x) = \begin{cases} 1, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$  as a Fourier series.



Date:14/11/2022

## Department of Mathematics

### B. Sc. III Sem. V

### Internal Examination 2022-23

All the students of B.Sc. III are hereby informed that their Internal Examination of Mathematics will be conducted from 23<sup>th</sup> November 2022 to 26<sup>th</sup> November 2022 at time 3.00- 4.00. The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus, timetable and nature of question paper of examination is given below:

Sr. No.	Name of Paper	Units	Date
1	Real Analysis	UNIT I, II	23/11/2022
2	Modern Algebra	UNIT I, II	24/11/2022
3	Matrix Algebra	UNIT I, II	25/11/2022
4	Numerical Methods I	UNIT I, II	26/11/2022

**\* Nature of question paper:**

**Time: 1 hour**

**Total Marks: 20**

**Q.1 Select the most correct alternative for each of the following:**  
[05]

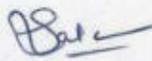
**Five questions**

**Q.2 Attempt any three** [15]

**Four questions**

**Venue: Roome No. 39**



  
**(Prof. S. P. Patankar)**  
**HEAD**  
Department of Mathematics  
Vivekanand College, Kolhapur

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-V Internal Examination, November 2022

MATHEMATICS

Real Analysis

Subject Code: DSC-1003E1

Time: 3.00-4.00

Date: 23/11/2023

Total Marks: 20

Q. 1 Select the most correct alternative for each of the following: [5]

1) Range of sequence is.....

- a) uncountable                      b) countable                      c) infinite                      d) finite

2)  $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \left( \frac{1}{\sqrt{3+\sqrt{6}}} + \frac{1}{\sqrt{6+\sqrt{9}}} + \dots + \frac{1}{\sqrt{3n+\sqrt{3n+3}}} \right)$  is.....

- a)  $\frac{1}{\sqrt{3}}$                       b)  $\sqrt{3}$                       c)  $\sqrt{3} + 1$                       d)  $\frac{1}{\sqrt{3+1}}$

3) The infimum value of  $x_n = (-1)^n + \cos \frac{n\pi}{4}$  is .....

- a) 2                      b) -1                      c)  $-1 - \frac{1}{\sqrt{2}}$                       d)  $-\frac{1}{\sqrt{2}}$

4) Which of the following set does not satisfy this  $|4x - 5| \leq 13$  inequality

- a)  $[-2, 9/2]$                       b)  $(9/2, 6)$                       c)  $[0, 4]$                       d)  $(-1/2, 12/5)$

5) The limit point of the sequence  $x_n = \cos \frac{n\pi}{2}$  is.....

- a)  $\{1, -1\}$                       b)  $\{0, 1\}$                       c)  $\{-1, 0, 1\}$                       d)  $\{-1, 0, 2\}$

Q.2. Attempt any three of the following:

[15]

1) Show that  $\lim_{n \rightarrow \infty} \left( \frac{1}{1+n^2} \right) = 0$

2) If  $a, b \in \mathbb{R}$ , then show that  $||a| - |b|| < |a - b|$

3) Prove that a sequence in  $\mathbb{R}$  can have at most one limit.

4) If  $\alpha$  is the least upper bound of  $A$  and  $c + A = \{c+x : x \in A, c > 0\}$  then show that  $c + \alpha$  is the least upper bound of  $c + A$ .

Subject Code: DSC-1003E1

Time: 3.00-4.00

Date: 24/11/2023

Total Marks: 20

**Q. 1 Select the correct alternative for each of the following:** [5]

1] A mapping  $f : G \rightarrow G'$  between two groups is onto and homomorphism then

$f$  is .....

a] Epimorphism   b] Endomorphism   c] Monomorphism   d] Automorphism

2]  $\langle \mathbb{Z}, \circ \rangle$  where  $\circ$  is usual multiplication is.....

a] Group   b] Abelian group   c] Not a group   d] Quotient group of  $\mathbb{Z}$

3]  $G$  is ..... if and only if  $Z(G) = G$ .

a] Group   b] Abelian group   c] Semi group   d] None

4] With usual notation, In  $\mathbb{Z}_5$ , Order of 3 [ $O(3)$ ] is ...

a] 4   b] 3   c] 2   d] 5

5] i) Cyclic group is always abelian. ii) Abelian group is always cyclic. Then

a] Only ii is true.   b] Both i and ii are true

c] Both i and ii are false   d] i is true and ii is false

**Q.2. Attempt any three of the following:** [15]

1] Define kernel of homomorphism  $\theta$  where,  $\theta : G \rightarrow G'$ . Show that,  $\theta$  is one to one if and only if  $\ker \theta = \{e\}$ .

2] Define normaliser of element  $a$  in group  $G$ . Show that, Centre of group  $G$  is subgroup of  $G$ .

3] Prove that, An infinite cyclic group has precisely two generators.

4] Define cyclic group. Show that, Subgroup of cyclic group is cyclic.

5] Define even permutation, Odd permutation. Find order of  $f \in S_8$ ,

where  $f = (1\ 2\ 5)(6\ 7)$

Vivekanand College, Kolhapur (Autonomous)  
B.Sc. (Part-III) Semester-V Internal Examination, November 2022

MATHEMATICS  
Matrix Algebra

Subject Code: DSC-1003E2  
Date: 25/11/2022

Time: 3.00-4.00  
Total Marks: 20

Q. 1 Select the correct alternative for each of the following: [5]

1) which of the following matrix is not diagonalizable over R?

a)  $\begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$

b)  $\begin{bmatrix} 2 & 4 & -6 \\ 4 & 2 & -6 \\ -6 & -6 & -15 \end{bmatrix}$

c)  $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

d)  $\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

2) If  $A = \begin{bmatrix} 2 & 0 & 3 & 2 & 0 & -2 \\ 0 & 1 & 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 & 4 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$ ,  $b_1 = \begin{bmatrix} 5 \\ 1 \\ 1 \\ 4 \end{bmatrix}$  and  $b_2 = \begin{bmatrix} 5 \\ 1 \\ 3 \\ 3 \end{bmatrix}$  then which of the

following is true?

- a) Both system  $AX = b_1$  and  $AX = b_2$  are consistent
- b) The system  $AX = b_1 - b_2$  consistent
- c) The system  $AX = b_1 - b_2$  inconsistent
- d) The system  $AX = b_1$  inconsistent and  $AX = b_2$  are consistent

3) Rank of  $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$  is \_\_\_\_\_.

- a) 1
- b) 2
- c) 3
- d) None of these

4) For the system of equations  $x + 2y + z = 2$ ,  $2x + 4y + 3z = 3$ ,  $3x + 6y + 5z = 4$ , if  $x = 1$ ,  $y = m$ ,  $z = n$  then  $m =$  \_\_\_\_\_ and  $n =$  \_\_\_\_\_.

- a) -1, 1
- b) 1, -1
- c) 1, 1
- d) -1, -1

5) If A is a 3 X 3 matrix with the trace 3 and determinant 2. If 1 is an eigen value of A, then the eigen value of the matrix  $A^2 - 2I$  are

a)  $1, 2(i-1), -2(i+1)$

b)  $-1, 2(i-1), 2(i+1)$

c)  $1, 2(i+1), -2(i+1)$

d)  $-1, 2(i-1), -2(i+1)$

**Q.2. Attempt any three of the following:**

**[15]**

1) Find the rank of the Matrix  $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$

2) Solve the System of Linear equation

$$x + 3y + z = 5, \quad 2x + 3y + z = 2,$$

$$x + y + 5z = -7, \quad 2x + 3y - 3z = 14$$

3) Find inverse by elementary row transformation for  $A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 2 & 0 & 1 & 4 \end{bmatrix}$

4) Determine The non-singular matrix P such that  $P^{-1}AP$  is diagonal matrix

where  $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$

**Vivekanand College, Kolhapur (Autonomous)**  
**B.Sc. (Part-III) Semester-V Internal Examination 2022-2023**

**MATHEMATICS**  
**Numerical Methods I**

**Subject Code: DSC-1003E2**  
**Day and Date: 26/11/2022**

**Time: 3.00-4.00**  
**Total Marks: 20**

**Q.1. Select the most correct alternative for each of the following: [05]**

- 1) In Bisection method, the new interval length is -----  
A)  $\frac{a+b}{2}$     B)  $\frac{a+b}{2^n}$     C)  $\frac{|b-a|}{2}$     D)  $\frac{|b-a|}{2^n}$
- 2) For the system of equations,  $6x_1 + 3x_2 + x_3 = 12$ ,  
 $x_1 + 5x_2 + 2x_3 = 3$ ,  $2x_1 + 4x_2 + 7x_3 = 21$ , the values of  $x_1, x_2, x_3$   
by Gauss-Jordan method are -----  
A) -2, -1, 3    B) 2, 1, -3    C) 2, -1, -3    D) 2, -1, 3
- 3) The root of the equation  $\cos x - xe^x = 0$  in the interval (0, 1) by  
Secant method in second iteration is -----  
A) 0.5168    B) 0.5318    C) 0.3167    D) 0.4467
- 4) In ----- method, elements above and below diagonal are  
simultaneously made zero.  
A) Gauss-Elimination    B) Gauss-Jordan  
C) Jacobi's method    D) Gauss-Seidel
- 5) The largest eigen values of matrix  $\begin{bmatrix} 1 & 4 & 16 \\ 4 & 16 & 1 \\ 16 & 1 & 4 \end{bmatrix}$  is -----  
A) 16    B) 21    C) 48    D) 64

**Q.2. Attempt any three of the following: [15]**

- 1) Explain Newton -Raphson method.
- 2) Explain Regula - Falsi method.
- 3) Solve the following equation by Jacobi method ( Do three iterations).  
 $27x + 6y - z = 85$ ,  $6x + 15y + 2z = 72$ ,  $x + y + 54z = 110$   
(Initial value  $x = 0, y = 0, z = 0$ ).

4) Determine the largest eigen value and the corresponding eigen vector of the matrix  $A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}$  by using power method. (Initial vector  $X = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ )

Date: 28/12/2022

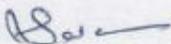
**Vivekanand College, Kolhapur (Autonomous)**  
**Department of Mathematics**  
**B. Com. I Sem. I**  
**Internal Examination 2022-2023**

All the students of B.Com. I are hereby informed that the Internal Examination of Mathematics will be conducted in online manner (Google Forms) on **5<sup>th</sup> January, 2023 at 12.00 pm to 1.00 pm**. The google form link will be provided on WhatsApp group and Telegram Group 5 min before examination time.

Sr. No.	Name of Paper	Topics
1	Business Mathematics-I GEC-1045A	Unit 1 Arithmetic Progration And Geometric Progration Unit 2 Compound Interest, Ratio, Percentage, Proportion and Partnership Unit 3: Matrix

**\*Nature of question paper:- 15 MCQs of Two mark each**



  
**Mr. S. P. Patankar**  
**HEAD**  
Department of Mathematics  
Vivekanand College, Kolhapur

Date-10/04/2023

**Vivekanand College, Kolhapur (Autonomous)**  
**Department of Mathematics**  
**B. Sc. I Sem. II Mathematics**  
**Theory Internal Examination 2022-23**

All the students of B.Sc. I Mathematics (A and C) are hereby informed that their theory Internal Examination of Mathematics will be conducted on **18<sup>th</sup> April, 2023 at 12.15 pm to 1.15 pm**. The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus for examination will be as mentioned in following table.

Sr. No.	Name of Paper	Topics
1	DSC-1003B: Section I-Multivariable Calculus	Unit I and Unit III
2	DSC-1003B: Section II-Ordinary Differential Equation	Unit I and Unit III

**\*Nature of question paper per section:-**

**Q.1) Attempt any One (07 marks)**

**Q.2) Attempt any two (08 marks)**



*S. P. Thorat*  
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**Vivekanand College, Kolhapur (Autonomous)**  
**B.Sc. (Part-I) Semester-II Internal Examination**  
**MATHEMATICS (DSC-1003B)**

**Day and Date: Monday & 18/04/2022**

**Total Marks: 30**

**Time: 2.00 pm to 3.00 pm**

**Section-I**

**Q.1 ATTEMPT ANY ONE.**

**(07)**

1) Evaluate repeated and simultaneous limit of the following function if exist.

$$f(x, y) = \frac{2xy^2}{x^2 + y^4}, x^2 + y^4 \neq 0, f(0,0) = 0.$$

2) If  $u = \log(\tan x + \tan y + \tan z)$  Prove that,  $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$ .

**Q.2 ATTEMPT ANY TWO.**

**(08)**

1) By using chain rule Show that,  $\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = \frac{u-v}{u^2+v^2}$  if  $z = \tan^{-1}\left(\frac{x}{y}\right)$  and  $x = u + v, y = u - v$ .

2) If  $u = e^{xy^2}$  then find the value of  $\frac{\partial u}{\partial x}$ .

3) If  $f(x) = 3x^3 + 2xy^2 + y^3$  then find the value of  $f_{xx}$ .

**Section-II**

**Q.3 ATTEMPT ANY ONE.**

**(07)**

1) Explain the method of solving Linear differential equation  $\frac{dy}{dx} + Py = Q$  where P and Q are functions of x only.

2) With usual notations prove that  $\frac{1}{f(D)} e^{ax} v = e^{ax} \frac{1}{f(D+a)} v$ .

**Q.4 ATTEMPT ANY TWO.**

**(08)**

1) Solve  $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$

2) Solve  $p^2 - 5p + 6 = 0$

3) Solve  $(D^2 - 12D + 35)y = 0$



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(Autonomous)**



KOLHAPUR (AUTONOMOUS)

Date: 15/04/2023

**Vivekanand College, Kolhapur (Autonomous)**

**Department of Mathematics**

**B. Sc. II Sem. IV**

**Internal Examination 2022-23**

All the students of B.Sc. II are hereby informed that their Internal Examination of Mathematics will be conducted from 24<sup>th</sup> April 2022 at time 3.00- 4.00 in Room No. 56. The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus and timetable of examination is given below:

Sr. No.	Paper	Units	Date
1	Section-I Discrete Mathematics	Unit II- Generating function and recurrence relation Unit III- Graph Theory	24/04/2023
2	Section-II Integral Transform	Unit I- Laplace Transform Unit IV- Inverse Laplace Transform	



*S. P. Thorat*  
(Prof. S. P. Thorat)  
**HEAD**

Department of Mathematics  
Vivekanand College, Kolhapur.

**Nature of question paper:**

**Section-I**

**Q.1 Choose the correct alternatives**

**[3]**

- 1)
- 2)
- 3)

**Q.2 Attempt any three**

**[12]**

- 1)
- 2)
- 3)
- 4)

**Section-II**

**Q.1 Choose the correct alternatives**

**[3]**

- 1)
- 2)
- 3)

**Q.2 Attempt any three**

**[12]**

- 1)
- 2)
- 3)
- 4)

**Vivekanand College, Kolhapur (Autonomous)**  
**B.Sc. (Part-II) Semester-IV Internal Examination, April 2023**  
**MATHEMATICS**

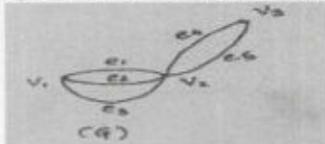
**Subject Code:** \_\_\_\_\_  
**Day and Date:** 24/04/2023(Monday)

**Time:** 1 Hr.  
**Total Marks:** 30

**Section-I**  
**Discrete Mathematics**

**Q.1 Select the correct alternatives for each of the following:** [03]

- 1) Order of the recurrence relation  $a_n = a_{n-1} + a_{n-2}$  is....  
 A) 3                      B) 2                      C) 1                      D) Not Defined
- 2) Which of the following is non-homogeneous difference equation  
 A)  $a_n - 3a_{n-1} = 0$                       B)  $a_n + 4a_{n-1} + 4a_{n-2} = 0$   
 C)  $a_n + 4a_{n-1} - 4 = 0$                       D)  $a_n + 3a_{n-1} = 0$
- 3) In the following graph, number of even and odd vertices are .... Respectively



- A) 1,2                      B) 2,1                      C) 2,2                      D) 1,1

**Q.2) Attempt any three of the following:** [12]

- 1) Solve the recurrence relation  $a_n = 3a_{n-1} + 2n$
- 2) Obtain the generating function of  $a_r = 2^r + 3^r$ ,  $r \geq 0$
- 3) State and prove Hand-Shaking lemma.
- 4) Define the following terms with example  
 i) Graph    ii) Edge and loop    iii) Simple graph    iv) Multigraph

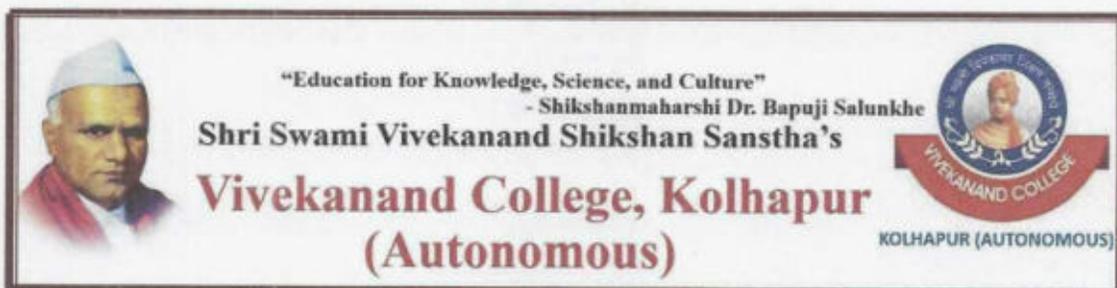
**Section-II**  
**Integral Transform**

**Q.1 Select the correct alternatives for each of the following:** [03]

- 1)  $L\{t^n\} = \dots$   
 A)  $\frac{n!}{s^n}$                       B)  $\frac{n!}{s^{n-1}}$                       C)  $\frac{n!}{s^{n+1}}$                       D)  $\frac{1}{s^{n+1}}$
- 2)  $L\{\sin at\} = \dots$   
 A)  $\frac{s}{s^2-a^2}$                       B)  $\frac{a}{s^2-a^2}$                       C)  $\frac{s}{s^2+a^2}$                       D)  $\frac{a}{s^2+a^2}$
- 3)  $L^{-1}\left\{\frac{1}{(s-a)^2+b^2}\right\} = \dots$   
 A)  $\frac{1}{b}e^{at} \sin bt$                       B)  $\frac{1}{b}e^{-at} \sin bt$                       C)  $\frac{1}{b}e^{at} \cos bt$                       D)  $\frac{1}{b}e^{-at} \cos bt$

**Q.2) Attempt any three of the following:** [12]

- 1) If  $L\{F(t)\} = f(s)$  then show that  $L\{F(at)\} = \frac{1}{a}f\left(\frac{s}{a}\right)$
- 2) Evaluate  $L\left\{\frac{1-e^{-t}}{t}\right\}$
- 3) Find the inverse Laplace transform of  $\frac{4}{(s+1)(s+2)}$
- 4) Find  $L^{-1}\left\{\frac{s+2}{s^2-2s+5}\right\}$



Date:03/04/2023

**Department of Mathematics**

**B. Sc. III Sem. VI**

**Internal Examination 2022-23**

All the students of B.Sc. III are hereby informed that their Internal Examination of Mathematics will be conducted from 10<sup>th</sup> April 2023 to 13<sup>th</sup> April 2023 at time 2.00- 3.00. The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus, timetable and nature of question paper of examination is given below:

Sr. No.	Name of Paper	Units	Date
1	Metric Space	UNIT I, II	10/04/2023
2	Linear Algebra	UNIT I, II	11/04/2023
3	Complex Analysis	UNIT I, II	12/04/2023
4	Numerical Methods II	UNIT I, II	13/04/2023

\* Nature of question paper:

Time: 1 hour

Total Marks: 20

Q.1 Select the most correct alternative for each of the following:

Five questions

[05]

Q.2 Attempt any three of the following

[15]

Four questions

Venue: Roome No. 39



*S. P. Thorat*  
(Prof. S. P. Thorat)  
HEAD

Department of Mathematics  
Vivekanand College, Kolhapur

**Vivekanand College, Kolhapur (Autonomous)**  
**B.Sc. (Part-III) Semester-VI Internal Examination 2022-2023**  
**MATHEMATICS**  
**Metric Spaces**

**Subject Code: DSC-1003F1**  
**Day and Date: 10/04/2023**

**Time: 2.00-3.00**  
**Total Marks: 20**

**Q.1. Select the most correct alternative for each of the following: [05]**

1. In ..... metric space every open set as well as closed.  
A. usual      B. Discrete      C. Euclidean      D. None of them
2. Which of the following metric space is not connected?  
A. Real line      B.  $[0, 1]$       C. Discrete      D. None of them
3. Which of the following set is compact in usual metric space?  
A.  $(0, 3)$       B.  $\mathbb{R} - \mathbb{N}$       C.  $\mathbb{Z}$       D.  $\{1/n: n \in \mathbb{N}\}$  in  $[0, 1]$
4. If  $F$  is finite set of  $\mathbb{R}$  then  $\bar{F}$  is.....  
A.  $\emptyset$       B.  $F$       C.  $\mathbb{R}-F$       D.  $\mathbb{R}$
5. A metric space  $(X, d)$  is said to be..... if every Cauchy sequence in  $X$   
.....  
A. compact, converges      B. Complete, converges  
C. Complete, diverges      D. compact, diverges

**Q.2 Attempt any three of the following [15]**

1. Show that the mapping  $d : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined by  $d(x, y) = \max(|x_1 - y_1|, |x_2 - y_2|)$  where  $x = (x_1, x_2), y = (y_1, y_2)$  is metric on  $\mathbb{R}^2$
2. Prove that in a metric space, every open sphere is an open set
3. Define the following terms  
a) Cluster point      b) exterior point      c) Adherent point  
d) Interior point      e) isolated point
4. Prove that, in a metric space  $(X, d)$  the intersection of a finite number of open set is open set.

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-VI Internal Examination 2022-23

MATHEMATICS

Linear Algebra

Subject Code: DSC-1003F1

Time:2.00-3.00

Date: 11/04/2023

Total Marks: 20

Q.1. Select the correct alternative for each of the following: [05]

- 1) If  $W$  is a subspace of  $V$  then  $L(W) =$  \_\_\_\_\_.  
A)  $W$     B)  $V$     C)  $\{0\}$     D)  $\phi$
- 2) If  $\dim V = n$  and  $S = \{v_1, v_2, \dots, v_n\}$  spans  $V$  then  $S$  is \_\_\_\_\_ of  $V$ .  
A) a subspace    B) a basis  
C) a linearly dependent subset    D) the smallest subspace
- 3) A linear transformation  $T : V \rightarrow W$  is non singular if \_\_\_\_\_.  
A)  $T$  is not one- one    B)  $T$  is not onto  
C)  $\text{Ker } T = \{0\}$     D)  $\text{Range } T = \{0\}$
- 4) If  $\dim V = n$  and  $S = \{v_1, v_2, \dots, v_n\}$  spans  $V$  then  $S$  is \_\_\_\_\_ of  $V$ .  
A) a subspace    B) a basis  
C) a linearly dependent subset    D) the smallest subspace
- 5) If  $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$  then the characteristic polynomial of  $A$  is \_\_\_\_\_.  
A)  $x^2 + 1$     B)  $x^2 + 2x + 1$     C)  $x^2 + x$     D)  $x^2 - 1$

Q.2 Attempt any Three of the following: [15]

- 1) A non empty subset  $W$  of vector space  $V(F)$  is subspace of  $V$  iff  $ax + by \in W$  for  $a, b \in F, x, y \in W$
- 2) Let  $T$  be linear operator on FDVS  $V(F)$  then show that  $c \in F$  is a characteristic value of  $T$  iff  $T - cI$  is singular, With usual notation
- 3) Find Range, Rank, Kernel, Nullity of following  
 $T : R^2 \rightarrow R^3$  such that,  $T(x, y) = (x, x+y, y)$
- 4) Define the range of a homomorphism. Prove that the range of a homomorphism  $T: V \rightarrow U$  is a subspace of  $U$ .

**Vivekanand College, Kolhapur (Autonomous)**  
**B.Sc. (Part-III) Semester-VI Internal Examination 2022**

**MATHEMATICS**

**Complex Analysis**

**Subject Code: DSC-1003F2**

**Time: 2.00-3.00**

**Date: 12/04/2023**

**Total Marks: 20**

**Q.1. Select the correct alternative for each of the following: [05]**

- 1) If  $u = r^2 \cos 2\theta$  is harmonic function then  $v =$  \_\_\_\_\_.  
A)  $r \cos 2\theta$       B)  $r \sin 2\theta$       C)  $r^2 \sin 2\theta$       D)  $r^3 \sin 2\theta$
- 2) The value of  $\int_C \frac{dz}{z}$ , Where C is the circle with centre at the origin and radius r is \_\_\_\_\_.  
A)  $\log r$       B)  $\pi i$       C)  $2\pi i$       D)  $\pi i/2$
- 3) If  $f(z) = \log \sqrt{x^2 + y^2} + i \tan^{-1} \frac{ky}{x}$  is analytic then  $k =$  \_\_\_\_\_.  
A) 0      B) -1      C) 1      D) 2
- 4) The value of  $\int_C \frac{e^z}{z-2} dz$  at  $|z| = 1$  is \_\_\_\_\_.  
A) 0      B)  $2\pi i$       C)  $4\pi i$       D)  $-2\pi i$
- 5) The analytic functions are called \_\_\_\_\_.  
A) Isomorphic      B) Homomorphic      C) Holomorphic      D) conformal

**Q.2 Attempt any Three of the following: [15]**

- 1) Find the analytic function whose imaginary part is  $v(x, y) = \cos x \cosh y$ .
- 2) Show that analytic function with constant modulus is constant.
- 3) Evaluate  $\int_0^{1+i} z^2 dz$  along the line  $y = x$
- 4) Use contour integration to prove that  $\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}$ .

**Q.2 Attempt any three questions****(15)**

- 1) Evaluate  $\int_0^1 e^{-x} dx$  with 10 equal interval by trapezoidal rule
- 2) Derive the expression for basic Simpson's (1/3)<sup>rd</sup> rule and define the Newton cote's formula.
- 3) Using Lagrange's formula, find polynomial for a given data

X	0	1	3	4
Y	-12	0	6	12

Also find y at  $x = 2$ .

- 4) Derive the formula for  $\frac{dy}{dx}$ ,  $\frac{d^2y}{dx^2}$  at  $x=x_0, x_1$  by using Newton's forward interpolation

Vivekanand College, Kolhapur (Autonomous)

B.Sc. (Part-III) Semester-V Internal Examination, 2022-2023

MATHEMATICS

Numerical Methods II

Subject Code: DSC-1003F2

Time: 2.00-3.00

Date: 13/04/2023

Total Marks: 20

Q.1. Select the correct alternative for each of the following: [05]

1) ----- is not useful for numerical integration.

A) Euler's method

B) Trapezoidal rule

C) Simpson's  $1/3^{rd}$  rule

D) Simpson's  $3/8^{th}$  rule

2) For a given initial value problem  $y' = y - x$ ,  $y(0) = 2$  the value of  $y(0.1)$  by

Runge - Kutta second order method is:

A) 2.2100

B) 2.0050

C) 2.2050

D) 2.1900

3) If  $\frac{dy}{dx} = xy$ ,  $y(0) = 1$ ,  $h = 0.1$ , then by Euler's method  $y(0.2) =$  -----

A) 1

B) 1.01

C) 1.0302

D) 1.0611

4) If  $y = f(x)$  takes the values  $y_0, y_1, \dots, \dots, y_n$  for  $a = x_0, x_1 = x_0 + h, \dots, \dots,$

$x_n = x_0 + nh = b$ . Then the value of  $\int_a^b f(x) dx$  by Trapezoidal rule is:

A)  $\frac{h}{3} [2(y_0 + y_n) + (y_1 + \dots + y_{n-1})]$

B)  $\frac{h}{2} [(y_0 + y_n) + 2(y_1 + \dots + y_{n-1})]$

C)  $\frac{h}{2} [2(y_0 + y_1) + (y_2 + \dots + y_{n-1})]$

D)  $h[(y_0 + y_n) + 2(y_1 + \dots + y_{n-1})]$

5)  $f(x_0, x_1, x_2) =$  -----

A)  $\frac{f(x_1, x_2) - f(x_0, x_1)}{x_0 - x_1}$

C)  $\frac{f(x_1, x_2) - f(x_0, x_1)}{x_0 - x_2}$

C)  $\frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$

D)  $\frac{f(x_1, x_2) - f(x_0, x_1)}{x_1 - x_0}$

**Q.2 Attempt any three questions****(15)**

- 1) Evaluate  $\int_0^1 e^{-x} dx$  with 10 equal interval by trapezoidal rule
- 2) Derive the expression for basic Simpson's (1/3)<sup>rd</sup> rule and define the Newton cote's formula.
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Also find y at x = 2.

- 4) Derive the formula for  $\frac{dy}{dx}$ ,  $\frac{d^2y}{dx^2}$  at  $x=x_0, x_1$  by using Newton's forward interpolation

Date: 09/06/2023

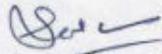
**Vivekanand College, Kolhapur (Autonomous)**  
**Department of Mathematics**  
**B. Com. I Sem. II**  
**Internal Examination 2022-23**

Internal Examination of Business Mathematics-II will be conducted in online manner (Google Forms) on **17<sup>th</sup> June, 2023 at 12.00 pm to 1.00 pm**. The google form link will be provided on WhatsApp/Telegram group 10 min before examination time.

Sr. No.	Name of Paper	Topics
1	Business Mathematics-II GEC-1045B	Unit 1: Functions Unit 2: Differentiation

**\*Nature of question paper:- 15 MCQs of Two mark each**



  
Mr. S. P. Patankar  
**HEAD**  
Department of Mathematics  
Vivekanand College, Kolhapur

Shri Swami Vivekanand Shikshan Sanstha's  
**VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)**  
 2130, E Ward, Tarabai Park, Kolhapur, Maharashtra 416003

**Subject Wise Student Blank Marks Entry**

Session: MAY-JUNE 2023

Subject: BUSINESS MATHEMATICS -II  
(GEC-1045B)

Stream: B.Com.

Standard: B.COM. - FY

Sub-Subject: CIE

Semester: SEM - II

Max Marks: 15

Print Date : 28-05-2023

Page No :Page 1 of 1

SrNo	PRN	SeatNo	GRNos	StudentName	Marks
1			1705978	SONTAKKE AISHWARYA SANTOSH	AB
2	2022025304	559026	3002940	ADURE TANAYA RAHUL	12
3	2022025308	559030	3002751	AMBI SHRADHA BALAPPA	10
4	2022025309	559031	3004280	AMKAR RASHMI SANTOSH	9
5	2022025313	559035	3002347	AWALE SHRIYAL PRALHAD	14
6	2022025320	559042	3003011	BANDI ANCY RAJU	12
7	2022025321	559043	3002426	BANDIVADEKAR SANTOSHI RAJESH	10
8	2022025326	559048	3002649	BARASKAR SAITA SANTOSH	12
9	2022025327	559049	3003459	BARGALE DIYA AJIT	9
10	2022025338	559060	3002264	BHOSALE PRANALI AMAR	11
11	2022025339	559061	3002379	BHOSALE SNEHAL SHIVAJI	9
12	2022025340	559062	3002270	BHUYEKAR SNEHA SANTOSH	13
13	2022025341	559063	3003508	BOBHATE GAYATRI RAMESH	12
14	2022025347	559069	3002971	CHAVAN ROHAN RAVINDRA	13
15	2022025351	559073	3002929	CHOPADE JESIKA PRAVIN	10
16	2022025354	559076	3006524	CHOUGALE HARSHVARDHAN EKANATH	12
17	2022025355	559077	3003112	CHOUGULE SAMRUDDHI SACHIN	10
18	2022025356	559078	3002324	CHOUGULE SANIKA SATISH	9
19	2022025359	559081	3003111	CHOUGULE VAISHNAVI VILAS	10
20	2022025362	559084	3002349	DALVI SAHIL SAMBHAJI	12
21	2022025363	559085	3003247	DHONE VIRAJ AJAY	11
22	2022025364	559086	3002257	DESAI SAKSHI RAJENDRA	8
23	2022025367	559089	3003004	DHERE AMRUTA JAYWANT	10
24	2022025378	559100	3003348	EKSHINGE AMRUTA PANDURANG	10
25	2022025380	559102	3002258	GAIKWAD MANSI SANJAY	7
26	2022025384	559106	3003127	GAUD SONI SHYAMAJI	5
27	2022025388	559110	3002436	GHADAGE SANIKA SUNIL	10
28	2022025389	559111	3002224	GHARAL AKSHTA SHIVAJI	10
29	2022025391	559113	3002343	GHODE DHANASHRI HAMBIRRAO	11
30	2022025394	559116	3002370	GUHAGARKAR JULEKHA JAMIR	12
31	2022025397	559119	3002272	GURAV SNEHAL PRAKASH	13
32	2022025401	559123	3002901	HUNDEKAR AKSHATA SHITAL	10
33	2022025402	559124	3002346	INGALE SAMRUDDHI NARAYAN	8
34	2022025404	559126	3004380	JADHAV ANIKET SHANTARAM	10
35	2022025405	559127	3003436	JADHAV ATHARV RAJENDRA	7
36	2022025408	559130	3002371	JADHAV NAKSHATRA SOMNATH	8
37	2022025410	559132	3002256	JADHAV RUTUJA DEEPAK	8
38	2022025412	559134	3002301	JADHAV VALLABH SHIVAJI	12
39	2022025415	559137	3004276	JAGDALE KASHISH UMESH	9
40	2022025416	559138	3005153	JAGOJE LEHER BIPIN	11
41	2022025418	559140	3002858	JAIN SAKSHAM RAKESH	9
42	2022025421	559143	3004297	JANGNURE SUHANI MAHESH	12
43	2022025422	559144	3003005	JESRANI RADHIKA SUNEELDAT	11
44	2022025432	559154	3002274	KARADKAR KARTIK SANJAY	7
45	2022025439	559161	3003087	KAWADE CHINMAY RAMAKANT	10
46	2022025440	559162	3002898	KESARKAR SAKSHI MADHUKAR	10
47	2022025442	559164	3003948	KHADAKE ROHIT PRAVEEN	8
48	2022025443	559165	3003622	KHADAKE VAISHNAVI SAMBHAJI	13
49	2022025444	559166	3003486	KHADE HRISHIKESH CHETAN	13
50	2022025449	559171	3002900	KHAVARE VAISHNAVI SACHIN	10
51	2022025452	559174	3002231	KHURANDAL ARYA ABHIJIT	8
52	2022025454	559176	3002234	KOTHARI TANISHKA NITIN	11
53	2022025459	559181	3002228	KULKARNI RAJLAXMI CHANDRAKANT	10
54	2022025461	559183	3002546	KUMAWAT JYOTI NANDLAL	12
55	2022025463	559185	3002904	KUMBHAR PRATIKSHA DINKAR	10

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**Subject Wise Student Blank Marks Entry**

**Session:** MAY-JUNE 2023

**Subject:** BUSINESS MATHEMATICS -II  
(GEC-1045B)

**Stream:** B.Com.

**Standard:** B.COM. - FY

**Sub-Subject:** CIE

**Semester:** SEM - II

**Max Marks:** 15

**Print Date :** 28-05-2023

**Page No :**Page 1 of 1

56	2022025467	559189	3003874	KUMBHAR SIDDHESH UDAY	13
57	2022025469	559191	3002248	KUMBHAR YASH NANDKUMAR	9
58	2022025476	559198	3002374	LOHAR KRUSHNAHARI PRAKASH	9
59	2022025479	559201	3003072	LUGADE VAISHNAVI ASHOK	8
60	2022025481	559203	3002358	MAJGAONKAR SALONI SAMBHAJI	7
61	2022025483	559205	3002250	MALI SAKSHI MADHUKAR	7
62	2022025488	559210	3002398	MANE AMRUTA ASHOK	AB
63	2022025489	559211	3003354	MANE NANDINI ARUN	AB
64	2022025492	559214	3003082	MANE SIMRAN SANJAY	5
65	2022025494	559216	3002678	MANE SNEHAL PANDURANG	12
66	2022025496	559218	3002429	MANJAREKAR AMAN PRAKASH	12
67	2022025500	559222	3002362	MERWADE ASHUTOSH RAJENDRA	12
68	2022025503	559225	3002956	MISAL SRUSHTI SATISH	12
69	2022025509	559231	3002241	MORCHI SAMARTH KIRAN	9
70	2022025510	559232	3003244	MORE YASHRAJ KIRAN	13
71	2022025511	559233	3003085	MUKE JANHAVI MAHAVIR	13
72	2022025512	559234	3002326	MULLA ATIFA AMIR	9
73	2022025513	559235	3003088	MURALE SAKSHI CHANDRAPRABHU	11
74	2022025519	559241	3002968	NAIKAWADI PAWAN BHIMRAO	10
75	2022025520	559242	3003134	NALAWADE SUREKHA DINESH	9
76	2022025528	559250	3003171	OSWAL NISHA AMRUT	10
77	2022025530	559252	3002490	PAHUJA SNEHA SUNIL	11
78	2022025531	559253	3003164	PALSANDE PRASAD SURYKANT	13
79	2022025532	559254	3003140	PANKE DIKSHA SAMBHAJI	7
80	2022025537	559259	3002338	PATANKAR SAKSHI SANJAY	10
81	2022025545	559267	3002425	PATIL ARPITA PRAKASH	10
82	2022025547	559269	3002317	PATIL DHRUV RANJIT	9
83	2022025549	559271	3002266	PATIL JAGRUTI ARUN	7
84	2022025551	559273	3002937	PATIL MANAS PRIYA	13
85	2022025554	559276	3002428	PATIL PIYUSH MADHUKAR	9
86	2022025556	559278	3002351	PATIL POOJA SUNIL	10
87	2022025557	559279	3002318	PATIL PRACHI BHAGAVANT	11
88	2022025565	559287	3002389	PATIL RANVEER SAMADHAN	9
89	2022025566	559288	3002233	PATIL RIYA KIRAN	10
90	2022025568	559290	3002352	PATIL SAKSHI CHANDRAKANT	8
91	2022025573	559295	3002260	PATIL SHIVANI NIVRUTTI	12
92	2022025574	559296	3002271	PATIL SHIVTEJ SATPAL	11
93	2022025575	559297	3002268	PATIL SHRADHA PRAMOD	10
94	2022025577	559299	3002410	PATIL SHRUSHTI SHIVAGONDA	11
95	2022025578	559300	3003641	PATIL SHRUTI SANATKUMAR	12
96	2022025582	559304	3002945	PATIL SUHAS UTTAM	9
97	2022025586	559308	3003589	PATIL VARUN VIJAY	AB
98	2022025589	559311	3002981	PATIL VIVEK ASHOK	10
99	2022025590	559312	3002943	PATIL YOGESHWAR VINAYAK	8
100	2022025592	559314	3002391	PAWAR PRERANA SANJAY	10
101	2022025595	559317	3004121	PENDHARKAR OM ANUP	9
102	2022025600	559322	3004091	POWAR CHETAN RAJENDRA	12
103	2022025606	559328	3003023	POWAR SHREYA PRATAP	10
104	2022025610	559332	3003183	PUKALE VEDANTIKA VISHWAS	12
105	2022025611	559333	3002247	RAJEGHORPADE RAJGAURI SHIVAJIRAO	7
106	2022025614	559336	3003618	RAMBHIA KHUSHI NIKHIL	11
107	2022025615	559337	3002922	RANADE VAIDEHI AJIT	6
108	2022025623	559345	3002300	REVANKAR PRITTY SHANTARAM	11
109	2022025624	559346	3002480	SACHDEV MAHEK SHAM	11
110	2022025625	559347	3003144	SAKATE PRATIK SANJAY	12
111	2022025634	559356	3003145	SATUSE TEJAS ARUN	10

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**Max Marks:** 15

**Semester:** SEM - II

**Print Date :** 28-05-2023

**Page No :**Page 1 of 1

112	2022025635	559357	3002941	SATWEKAR MITALI MANOJ	10
113	2022025644	559366	3002916	SHAIKH NUMA SAMEER	11
114	2022025651	559373	3003549	SHENDAGE SUHANI PRAMOD	9
115	2022025653	559375	3004087	SHINDE ADITI VIJAY	9
116	2022025654	559376	3002354	SHINDE KOMAL SANJAY	8
117	2022025656	559378	3002915	SHINGE SAKSHI TANAJI	13
118	2022025659	559381	3002254	SUTAR DIKSHA PRAKASH	13
119	2022025666	559388	3002435	SUTAR YASH SUNIL	10
120	2022025677	559399	3003510	WADKAR KAUSTUBH PRAVIN	11
121	2022025686	559408	3006307	NIRANKARI SNEHA GOVARDHAN	7
122	2022025688	559410	3006481	SUTAR ABHISHEK RAJENDRA	AB
123	2022025875	559488	3002406	MANE DARSHAN TULSHIDAS	10
124	2022025915	559528	3002890	UPADHYE KARTIK KAMLAKAR	AB
125	2021026205	559544	2950577	LANGARKAR NAMRATA NILESH	AB
126	2021026240	559549	2950766	MOHITE AMISHKA AMAR	14
127	2021026049	632282	2950307	CHOUGULE SHIVRAJ SUDHIR	AB
128	2021026533	632604	2953840	ANURAG RAMKRISHNA SUBHEDAR	7
129	2020026062	705089	2696832	GAIKWAD PADMASINH VIJAYSINH	13

Section - I

" ज्ञान, विज्ञान आणि सुरांस्कार यांसाठी शिक्षण प्रसार "

- शिक्षणमहर्षी डॉ. बापूजी साळुंखे

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Subject : Mathematics

Test / Tutorial No. :

Div. : A

Suppliment No. :

Roll No. : 7248

Class : B.S.C FY

Q1)

2) state and

10 + 14 = 24 hr

→ To obtain the  $n^{\text{th}}$  derivative of product of two functions :

statement of Leibnitz's theorem show that  $n^{\text{th}}$  derivation of  $(u.v)$  where  $u$  and  $v$  are functions of  $x$  then

$$y_n = (u.v)_n = {}^n C_0 u_n v + {}^n C_1 u_{n-1} v_1 + {}^n C_2 u_{n-2} v_2 + \dots + {}^n C_r u_{n-r} v_r + \dots + {}^n C_n u v_n$$

where suffixes denotes w.r.t.  $x$

$$y_{n+1} = (u.v)_{n+1} = {}^{n+1} C_0 u_{n+1} v + {}^{n+1} C_1 u_n v_1 + {}^{n+1} C_2 u_{n-1} v_2 + \dots + {}^{n+1} C_r u_{n-r+1} v_r + \dots + {}^{n+1} C_n u v_{n+1}$$

$$y_{n+1} = {}^{n+1} C_0 u_{n+1} v + u_n v_1 ({}^n C_0 + {}^n C_1) + u_{n-1} v_2 ({}^n C_1 + {}^n C_2) + \dots + ({}^n C_{r-1} + {}^n C_r) u_{n-r+1} v_r + \dots + u_{r+1} v_{r+1} + \dots + {}^n C_n u_{n+1} v_{n+1}$$

we know  ${}^n C_{r-1} + {}^n C_r = {}^{n+1} C_r$

${}^n C_1 + {}^n C_2 = {}^{n+1} C_2$  and so on.

${}^n C_0 = 1 = {}^{n+1} C_0$  and  ${}^n C_n = 1 = {}^{n+1} C_{n+1}$

Explanation B,

$$y_{n+1} = {}^{n+1}C_0 \mu_{n+1} v + {}^{n+1}C_1 \mu_n v_1 + {}^{n+1}C_2 \mu_{n-1} v_2 + \dots + {}^{n+1}C_n \mu \cdot v_{n+1}.$$

~~Hence proved.~~

put  ~~$n+1 = n$ .~~

$$y_n = {}^n C_0 \mu_n v + {}^n C_1 \mu_{n-1} v_1 + {}^n C_2 \mu_{n-2} v_2 + \dots + {}^n C_n \mu \cdot v_n.$$

Hence proved.

Q2) ① if  $y = x$

$$\rightarrow y = \frac{x}{1+3x+2x^2}$$

~~$$y = \frac{x}{2x(x+1)+1(x+1)}$$~~

$$y = \frac{x}{(2x+1)(x+1)}$$

$$x = \frac{1/2}{(2x+1)(x+1)} + \frac{+1}{(2x+1)(+1+1)}$$

~~$$= \frac{1/2}{2(x+1)} + \frac{1}{2(2x+1)}$$~~

$$= \frac{1}{2} \left[ \frac{1}{x+1} + \frac{1}{(2x+1)} \right]$$

$$= \frac{1}{2} \left[ \frac{(-1)^n \cdot n! (1)^n}{(x+1)^{n+1}} + \frac{(-1)^n \cdot n! (2)^n}{(2x+1)^{n+1}} \right]$$

Q2) ③  $\rightarrow f(x) = 2x^3 - 5x^2 + 3x + 2 \quad x \in [0, 3/2]$

(i)  $2x^3 - 5x^2 + 3x + 2$  is polynomial so it is continuous function in  $x \in [0, 3/2]$

(ii)  $f(x) = 2x^3 - 5x^2 + 3x + 2$  is differentiable in open interval  $x \in (0, 3/2)$ .

(iii)  $f(a) = f(b)$

$f(a) = f(0) = 2(0)^3 - 5(0)^2 + 3(0) + 2.$

$\therefore f(0) = 2 = f(a).$

$\therefore f(b) = f(3/2) = 2\left(\frac{3}{2}\right)^3 - 5\left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right) + 2$   
 $= 2 \times \frac{27}{8} - 5 \times \frac{9}{4} + \frac{9}{2} + 2$   
 $= \frac{-189}{4} + \frac{9}{2} + 2 = 2 //$

$\therefore$  ~~on~~

$$\therefore f(a) = f(b) \text{ that is } f(0) = f(3/2) = 2.$$

Hence, ~~the~~  $f(x) = 2x^3 - 5x^2 + 3x + 2$  verify  
Rolle's theorem.

It satisfies all conditions of R.M.V.T.

$$f(c) = 0.$$

$$f'(c) = 6x^2 - 10x + 3$$

$$= 6c^2 - 10c + 3.$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{100 - 18 \times 4}}{2 \times 6} \text{ or } = \frac{-10 - \sqrt{100 - 18 \times 4}}{12}.$$

$$= \frac{-10 + \sqrt{100 - 72}}{12} \text{ or } = \frac{-10 - \sqrt{100 - 72}}{12}$$

$$= \frac{-10 + \sqrt{28}}{12} \text{ or } = \frac{-10 - \sqrt{28}}{12}$$

$$= \frac{-10 + 2\sqrt{7}}{12} \text{ or } = \frac{-10 - 2\sqrt{7}}{12}.$$

$$= \frac{-5}{6} + \frac{\sqrt{7}}{6}$$

$$= \frac{-5}{6} - \frac{\sqrt{7}}{6}$$

$$= \frac{-5 + \sqrt{7}}{6}$$

$$= \frac{-5 - \sqrt{7}}{6}$$

$$\left( \frac{-5 + \sqrt{7}}{6}, \frac{-5 - \sqrt{7}}{6} \right) \in C.$$

$\therefore$  It satisfies all conditions of R.M.V.T. so  
it is verify all condition.

# VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

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Subject : Mathematics.

Test / Tutorial No. :

Div. : (A)

Q.3) (2)

Statement : Every square matrix satisfies characteristic equation.

In other words,

~~$|A - \lambda I| = 0$~~  will be of form.

$$a_0 + a_1 \lambda + a_2 \lambda^2 + \dots + a_n \lambda^n = 0.$$

$$a_0 + a_1 A + a_2 A^2 + \dots + a_n A^n = 0.$$

This equation satisfies by matrix A.

Proof : Let A be the given square matrix of order n.

Then its characteristic equation is ~~matrix~~  $|A - \lambda I| = 0$ .

Let,  $\text{Adj} [A - \lambda I] = B_0 + B_1 \lambda + B_2 \lambda^2 + \dots + B_{n-1} \lambda^{n-1}$ .

The matrix of degree (n-1).

$$A \cdot \text{Adj}(A) = |A| I.$$

now,  $[A - \lambda I] \text{Adj} [A - \lambda I] = [A - \lambda I] I.$

$$= [a_0 + a_1 \lambda + a_2 \lambda^2 + \dots + a_n \lambda^n] I. \quad \text{--- (1)}$$

But

$$= [A - \lambda I] \cdot \text{Adj} [A - \lambda I]$$

$$= [A - \lambda I] \cdot [B_0 + B_1 \lambda + B_2 \lambda^2 + \dots + B_{n-1} \lambda^{n-1}]$$

$$= AB_0 + AB_1 \lambda + AB_2 \lambda^2 + \dots + AB_{n-1} \lambda^{n-1} -$$

$$B_0 \lambda I - B_1 \lambda^2 I - B_2 \lambda^3 I - \dots - B_{n-1} \lambda^n I$$

$$= AB_0 + xI [AB_1 - B_0] + x^2 I [AB_2 - B_1] + x^3 I [AB_2 - B_2] + \dots + x^{n-1} I [AB_{n-1} - B_{n-2}] + x^n I [AB_n - B_{n-1}] \quad \text{--- (2)}$$

= Comparing power of  $x$  from (1) and (2) we get

$$a_0 = AB_0, \quad a_1 A = A^2 B_1 - AB_0$$

$$a_2 A^2 = A^3 B_2 - A^2 B_1$$

$$a_n A^n = A^n B_{n-1} + B_{n-1}$$

And add these equalities.

$$a_0 + a_1 A + a_2 A^2 + \dots + a_n A^n = AB_0 + A^2 B_1 - AB_0 + A^3 B_2 - A^2 B_1 + \dots + A^n B_{n-1} + B_{n-1}$$

$$= a_0 + a_1 A + a_2 A^2 + \dots + a_n A^n + AB_0 + A^2 B_1 - AB_0 + A^3 B_2 - A^2 B_1 + \dots + A^n B_{n-1} + B_{n-1}$$

$$= a_0 + a_1 A + a_2 A^2 + \dots + a_n A^n = 0$$

which prove the theorem.

Q. 4)

$$2) \quad \rightarrow \quad x + y + z = 6$$

$$x - 2y + 3z = 1$$

$$x + 2y + \lambda z = 4$$

$$A \cdot X = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 4 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 2 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ 4 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_1$$

$$\left[ \begin{array}{ccc|ccc} 1 & 1 & 1 & x_1 & & 6 \\ 0 & -3 & 2 & x_2 & & -5 \\ 0 & 1 & \lambda-1 & x_3 & & \mu-6 \end{array} \right]$$

~~(i) No solution.~~

~~$\lambda \neq 1$~~   $\lambda =$

(ii) ~~unique solution.~~

~~$\lambda \neq 1$  and  $\mu \neq 6$ .~~

~~$\rho(A) = \rho(B)$~~

~~(iii) No solution. infinite solution~~

~~$\lambda = 1$  and  $\mu = 6$~~

$\rho(A) = 3, \rho(AB) = 3. \quad n = 3.$

(i) unique solution

$\rho(A) = \rho(AB) = n$

when  $\lambda \neq 1$  and  $\mu \neq 6$ .

then this ~~is~~ unique solution

(ii) No solution.

$\lambda = 1$  and  $\mu \neq 6$ .

~~$\rho(A) \neq \rho(AB)$~~

~~$2 \neq 3$~~

(iii) infinite solution.

$\rho(A) = \rho(AB) < n$ .

$\lambda = 1$  and  $\mu = 6$

~~$\rho(A)$~~   $2 = 2 < 3$

hence there infinite solution

$$\textcircled{3} \rightarrow x^3 - 18x - 35 = 0.$$

$$\frac{h'(a_0)}{h'(1)} = \frac{0}{3(1)} = 0$$

$$t = u^{1/3} + v^{1/3}$$

$$t^3 = (u+v) + 3u^{1/3} \cdot v^{1/3} \cdot t.$$

~~$$t^3 = u+v + 3u^{1/3} \cdot v^{1/3} \cdot t$$~~

$$t^3 - 3u^{1/3} \cdot v^{1/3} \cdot t - (u+v) = 0.$$

~~$$t^3 = t^3$$~~

$$(3u^{1/3} \cdot v^{1/3})t = (+18x) \quad \text{--- (1)}$$

$$(u+v) = +35. \quad \text{--- (2)}$$

~~$$\frac{u^{1/3} \cdot v^{1/3}}{u^{1/3} \cdot v^{1/3}} = \frac{18 \cdot 6}{6}$$~~

~~$$= 6.$$~~

$$4 \cdot v = 216.$$

$$u + v = 35.$$

$$t^2 - (u+v)t + (u \cdot v) = 0.$$

$$t^2 - 35t + 216 = 0.$$

$$\frac{216}{\wedge}$$

~~$$t =$$~~

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-35 \pm \sqrt{1225 - 4 \times 1 \times 216}}{2 \times 1} \quad \text{or} \quad \frac{-35 - \sqrt{361}}{2}$$

$$= \frac{-35 + \sqrt{361}}{2}$$

2

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Q.4) ① If  $\alpha$  is root of equation  $f(x) = 0$  then the polynomial  $f(x)$  is divisible by  $(x - \alpha)$ .

proof: Given  $\alpha$  is root of  $p(x) = 0$ .

$$p(\alpha) = 0 \quad \text{--- (1)}$$

now divide  $f(x)$  by  $(x - \alpha)$  we get quotient  $Q$  and remainder  $R$ .

$$\therefore f(x) = (x - \alpha)Q + R \quad \text{--- (2)}$$

~~putting  $(x - \alpha)Q + R$ .~~

putting  $x = \alpha$  on both side

$$f(\alpha) = (\alpha - \alpha)Q + R$$

$$f(\alpha) = 0.$$

both for equation ②.

$$R = 0.$$

that is  $f(x) = (x - \alpha) \cdot Q$

$\therefore f(x)$  is divisible by  $(x - \alpha)$ .

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Roll No. : 7786

Subject : Mathematics

Class : BSc-II

Test / Tutorial No. :

Div. : A

## Section-I (Number Theory)

## 1. iii Fundamental theorem of arithmetic.

Statement : Any positive integer can be expressed as products of prime. Its representation differs from the Order of the factors which occurs.

Proof: If  $n$  is prime we have nothing to prove. Let  $n$  is composite then there exist integer  $d > 1$  such that  $1 < d < n$ . Then there exist set divisor of  $n$  such that  $1 < d < n$ . Then by well order principle there is smallest integer  $P_1$  such that  $1 < P_1 < n$   
 $P_1 | n$ .  $P < P_1 + n$

Claim I:  $P_1$  is prime

suppose, then there exists divisor  $q$  such that  $1 < q < P_1$  since  $P_1 | n$  and  $q | P_1$  therefore  $q | n$ . This is contradiction to the minimality of  $P_1$   
 Hence  $P_1$  is Prime.

then there exist  $n = P_1 n_1$  where  $P_1$  is prime.

if  $n_1$  is prime then we have are done otherwise  
there exist Prime  $P_2$  such that,

$$n_1 = P_2 n_2 \quad \text{where } n < n_1 < n_2$$

if  $n_2$  is prime then we are done otherwise  
there exist Prime  $P_3$  such that,

$$n_2 = P_3 n_3 \quad \text{Where } n < n_1 < n_2 < n_3$$

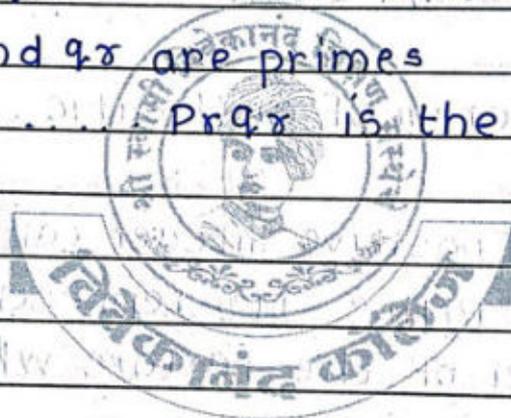
Since, There is  $n$  is Finite this process cannot be  
continuous.

Hence there exist positive integer  $r$  such that,

$$n_{r-1} = P_r q_r$$

Where  $P_r$  and  $q_r$  are primes

$\therefore n = P_1 P_2 \dots P_r q_r$  is the prime factorization  
of  $n > 0$ .



कोल्हापूर

iii) we have  $221x + 35y = 11$

→ Since  $\gcd(221, 35) = 1$  and  $1|11$

$$221 = 35 \times 6 + 11$$

$$35 = 6 \times 5 + 5$$

$$6 = 5 \times 1 + 1$$

$$5 = 1 \times 5 + 0$$

$$\therefore 1 = 6 - 5 \times 1$$

$$1 = 6 - (35 - 6 \times 5) \times 1$$

$$1 = 6 \times 6 - 35 \times 1$$

$$11 = 6 \times 66 + 35 \times (-11)$$

$$\therefore x_0 = 66 \text{ and } y_0 = -11$$

∴ By diophantine eqn.

$$x = x_0 + \left(\frac{b}{d}\right)t \text{ and } y = y_0 - \left(\frac{a}{d}\right)t$$

$$x = 66 + \left(\frac{35}{1}\right)t \text{ and } y = -11 - \left(\frac{221}{1}\right)t$$

$$x = 66 + 35t \text{ and } y = -11 - 221t$$

Hence the solution of given diophantine

eqn is  $x = 66 + 35t$  and  $y = -11 - 221t$

iii) IF  $a$  and  $b$  be integers not both zero, then prove that  $\gcd(a, b) * \text{lcm}(a, b) = a * b$

→ PROOF:

consider  $d = \gcd(a, b)$  and  $m = \frac{ab}{d}$

We have prove that  $m = \text{LCM}(a, b)$

since  $d = \gcd(a, b) \mid a$  and  $d \mid b$  then there exists integer  $x$  and  $s$  such that,  $dx = a$  and  $ds = b$ .

consider,

$$m = \frac{ab}{d} = \frac{dx \cdot b}{d} = xb = \frac{ab}{d} = m \mid d$$

$$m = \frac{ab}{d} = \frac{d \cdot s \cdot b}{d} = sb = \frac{ab}{d} = m \mid d$$

Hence first condition is satisfied.

consider multiple  $c$  hence there exist integer  $x$  and  $y$  such that  $ax = c$  and  $by = c$

since  $d = \gcd(a, b)$

then there exist integer  $u$  and  $v$ .

consider,

$$d = \frac{c}{m} = \frac{c}{\frac{ab}{d}} = \frac{cd}{ab} = \frac{c(au + by)}{ab}$$

$$d = \frac{c}{b}u + \frac{c}{a}v$$

$$d = yu + xv$$

$$m \leq c$$

Hence second condition is proved.

$$\therefore m = \text{LCM}(a, b)$$

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36148

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# VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

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Div. :

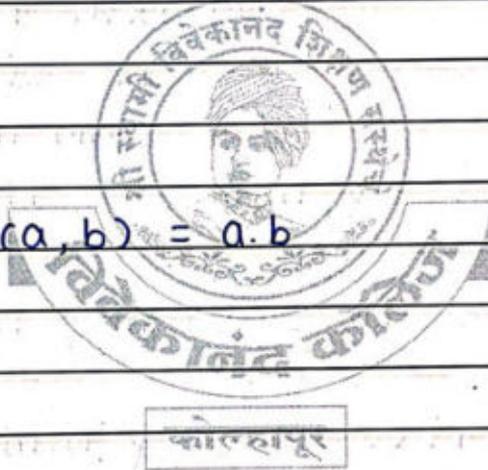
Hence Now,

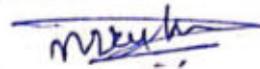
$$m = \frac{ab}{d}$$

$$\therefore md = ab$$

$$\therefore \text{lcm}(a, b) \cdot \text{gcd}(a, b) = a \cdot b$$

Hence proved.



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14  
15

## Section-II (Integral Calculus)

iii) Prove that  $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ 

As we know,

$$\Gamma(m) = 2 \int_0^{\infty} e^{-x^2} x^{2m-1} dx$$

$$\Gamma(n) = 2 \int_0^{\infty} e^{-y^2} y^{2n-1} dy$$

consider,

$$\Gamma(m)\Gamma(n) = 2 \int_0^{\infty} e^{-x^2} x^{2m-1} dx \cdot 2 \int_0^{\infty} e^{-y^2} y^{2n-1} dy$$

$$\therefore \Gamma(m)\Gamma(n) = 4 \int_0^{\infty} \int_0^{\infty} e^{-x^2} x^{2m-1} e^{-y^2} y^{2n-1} dx dy$$

$$\therefore \Gamma(m)\Gamma(n) = 4 \int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} x^{2m-1} y^{2n-1} dx dy$$

Now,

put  $x = r \cos \theta$ ,  $y = r \sin \theta$  and  $dx dy = r dr d\theta$

When  $x$  and  $y$  varies from 0 to  $\infty$   $r$  varies from 0 to  $\infty$  and  $\theta$  varies from 0 to  $\pi/2$ .

$$\therefore \Gamma(m)\Gamma(n) = 4 \int_0^{\infty} \int_0^{\pi/2} e^{-r^2} (r \cos \theta)^{2m-1} (r \sin \theta)^{2n-1} r dr d\theta$$

$$\therefore \Gamma(m)\Gamma(n) = 4 \int_0^{\infty} \int_0^{\pi/2} e^{-r^2} r^{2m-1+2n-1+1} \cos^{2m-1} \theta \sin^{2n-1} \theta dr d\theta$$

$$\therefore \Gamma(m)\Gamma(n) = 4 \int_0^{\infty} \int_0^{\pi/2} e^{-r^2} r^{2(m+n)-1} dr \cos^{2m-1} \theta \sin^{2n-1} \theta d\theta$$

$$\therefore \Gamma(m)\Gamma(n) = 2 \int_0^{\infty} e^{-r^2} r^{2(m+n)-1} dr \cdot 2 \int_0^{\pi/2} \cos^{2m-1} \theta \sin^{2n-1} \theta d\theta$$

$$\therefore \Gamma(m)\Gamma(n) = \Gamma(m+n) B(m, n)$$

$$\therefore \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} = B(m, n)$$

$$\therefore B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

This proves the beta and gamma relation.

$$\text{Evaluate } \int_0^{2a} \sqrt{2ax - x^2} \, dx$$

Here, put  $x = 2at$

$$\therefore dx = 2a \, dt$$

limits

$x$	$0$	$2a$
$t$	$0$	$1$

$$\therefore \int_0^{2a} \sqrt{2ax - x^2} \, dx = \int_0^1 \left[ \{2a(2at) - (2at)^2\}^{1/2} \right] 2a \, dt$$

$$\therefore I = \int_0^1 \left[ 4a^2t - 4a^2t^2 \right]^{1/2} 2a \, dt$$

$$\therefore I = \int_0^1 2a \left[ t - t^2 \right]^{1/2} 2a \, dt$$

$$\therefore I = \int_0^1 2a^{1+1} t^{1/2} (1-t)^{1/2} \, dt$$

$$\therefore I = \int_0^1 2a^2 t^{1/2+1-1} (1-t)^{1/2+1-1} \, dt$$

$$\therefore I = 2a^2 \int_0^1 t^{\frac{3}{2}-1} (1-t)^{\frac{3}{2}-1} \, dt$$

$$\therefore I = 2a^2 \int_0^1 t^{\frac{3}{2}-1} (1-t)^{\frac{3}{2}-1} \, dt$$

$$\therefore I = 2a^2 B\left(\frac{3}{2}, \frac{3}{2}\right)$$

$$\therefore I = 2a^2 \frac{\frac{3}{2} \frac{3}{2}}{\frac{3+3}{2}}$$

$$I = 2a^2 \frac{1}{2} \sqrt{\pi} \cdot \frac{1}{2} \sqrt{\pi}$$

$$I = 2a^2 \frac{4\pi}{2!}$$

$$I = \frac{a^2 \pi}{4}$$

iii)  $\int_0^{\pi/2} \sqrt{\cot \theta} d\theta$

$$\rightarrow \int_0^{\pi/2} \sqrt{\cot \theta} d\theta = \int_0^{\pi/2} \sqrt{\frac{\cos \theta}{\sin \theta}} d\theta$$

$$\therefore I = \int_0^{\pi/2} \cos^{1/2} \theta \sin^{-1/2} \theta d\theta$$

using formula,  $\int_0^{\pi/2} \sin^p \theta \cos^q \theta = \frac{1}{2} B\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$

$$\therefore I = \frac{1}{2} B\left(\frac{-1/2+1}{2}, \frac{1/2+1}{2}\right)$$

$$\therefore I = \frac{1}{2} B\left(\frac{1}{4}, \frac{3}{4}\right)$$

04  $\therefore I = \frac{1}{2} \frac{\frac{1}{4} \Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{4}{4}\right)}$

$$\therefore I = \frac{1}{2} \frac{\sqrt{2} \pi}{\Gamma(1)}$$

$$\therefore I = \frac{\pi}{\sqrt{2}}$$

$$\therefore \int_0^{\pi/2} \sqrt{\cot \theta} d\theta = \frac{\pi}{\sqrt{2}}$$

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

- शिक्षणमहर्षी डॉ. बापूजी साबुंखे

36516

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

# VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

## SUPLIMENT

Signature  
of  
Supervisor

Suppliment No. :

Roll No. : 8292

Class : B.Sc. III

Subject : Mathematics

Test / Tutorial No. :

Div. :

$$\frac{29+1}{30} = 30$$

Q.1

(i)

→ ~~C)  $x^2 + 4x + 2$~~

(ii)

→ ~~C)  $\frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$~~

(iii)

→ ~~D)  $f(x - mh)$~~

(iv)

→ ~~A) Equally spaced~~

(v)

→ ~~3  $[e^{x+3h} - 3e^{x+2h} + 3e^{x+h} - e^x]$~~

Q.2

(ii)

→

$$e^x \left[ \mu_0 + x \Delta \mu_0 + \frac{x^2}{2!} \Delta^2 \mu_0 + \dots \right] = \mu_0 + \mu_1 x + \frac{\mu_2 x^2}{2!} + \dots$$

$$\text{L.H.S.} = e^x \left[ \mu_0 + x \Delta \mu_0 + \frac{x^2}{2!} \Delta^2 \mu_0 + \dots \right]$$

$$= e^x \left[ 1 + x \Delta + \frac{x^2}{2!} \Delta^2 + \dots \right] \mu_0$$

$$= e^x e^{x\Delta} \cdot \mu_0$$

$$= e^{Ex} \mu_0 \quad \therefore (I + \Delta) = E$$

$$= \left[ 1 + xE + \frac{x^2 E^2}{2!} + \dots \right] \mu_0$$

$$= \left[ \mu_0 + xE\mu_0 + \frac{x^2 E^2 \mu_0}{2!} + \dots \right]$$

$$= \left[ \mu_0 + x\mu_1 + \frac{x^2 \mu_2}{2!} + \dots \right] \quad \therefore E^n = f(x+n)$$

$$= \text{R.H.S.}$$

$$= \text{Proved.}$$

Now Relation between  $\Delta$  and  $E$ .

$$\Delta f(x) = f(x+h) - f(x)$$

$$= E f(x) - f(x)$$

$$\Delta f(x) = (E - 1) f(x)$$

$$\Delta = E - 1$$

$$E = 1 + \Delta$$

Relation between  $\Delta$  and  $E$ .

3

(i)

→ Here from the data.

$$x_0 = -2, \quad x_1 = -1, \quad x_2 = 1, \quad x_3 = 3$$

$$f(x_0) = -15, \quad f(x_1) = -4, \quad f(x_2) = 0, \quad f(x_3) = 20.$$

By Lagrange's formula.

$$f(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2) + l_3(x) f(x_3).$$

Now,

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} = \frac{(x+1)(x-1)(x-3)}{(-1)(-3)(-5)}$$

$$= \frac{(x+1)(x-1)(x-3)}{-15}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} = \frac{(x+2)(x-1)(x-3)}{(1)(-2)(-4)}$$

$$= \frac{(x+2)(x-1)(x-3)}{8}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} = \frac{(x+2)(x+1)(x-3)}{(3)(2)(-2)}$$

$$= \frac{(x+2)(x+1)(x-3)}{-12}$$

$$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = \frac{(x+2)(x+1)(x-1)}{(5)(4)(2)}$$

$$= \frac{(x+2)(x+1)(x-1)}{40}$$

$$f(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2) + l_3(x) f(x_3)$$

$$= \frac{(x+1)(x-1)(x-3)}{-15} \times -15 + \frac{(x+2)(x-1)(x-3)}{8-2} \times 1 + \frac{(x+2)(x+1)(x-3)}{-12} \times 0 + \frac{(x+2)(x+1)(x-1)}{240} \times 70$$

$$f(0) = \frac{(1)(-1)(-3)}{-2} + \frac{(2)(-1)(-3)}{2} + \frac{(2)(1)(-1)}{2}$$

$$= 3 - 3 - 1$$

$$f(0) = -1$$



(iii)

→ Construct table.

$x$	$f(x)$	1st DD	2nd DD	3rd DD	4th DD
0	1	06			
2	13	21	05		
3	34	39	09	1	
4	73	78	13	1	0
6	229				

here  $y_0 = 1$ ,  $f(x_0, x_1) = 06$ ,  $f(x_0, x_1, x_2) = 5$ ,  $f(x_0, x_1, x_2, x_3) = 1$

$$y = y_0 + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3) + 0$$

$$y = 1 + (x-0) \times 06 + (x)(x-2) \times 5 + (x)(x-2)(x-3) \times 1$$

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

# VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

## SUPPLIMENT

Signature of Supervisor	
Subject:	mathematics
Test / Tutorial No.:	
Div.:	

Suppliment No. : 1  
 Roll No. : 8292  
 Class : B.Sc III

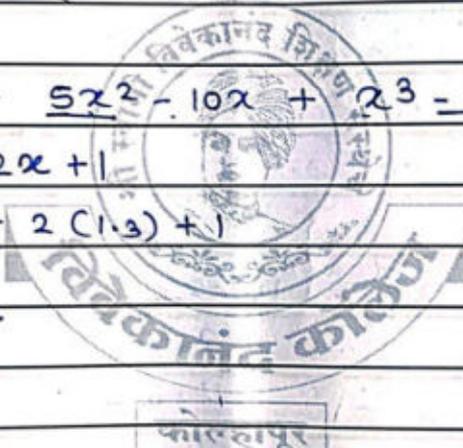
$$y = 1 + 6x + (x^2 - 2x)5 + (x^2 - 2x)(x-3)$$

$$= 1 + 6x + 5x^2 - 10x + x^3 - 2x^2 - 3x^2 + 6x$$

~~$$y = x^3 + 2x + 1$$~~

~~$$y(1.3) = (1.3)^3 + 2(1.3) + 1$$~~

~~$$y(1.3) = \underline{\underline{5.797}}$$~~



(iv)

→

We construct table.

$x$	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	0	03		
2	3	05	02	
4	8	07	02	0
6	15	09	02	0
8	24	11	02	0
10	35			

Here  $x=9$ ,  $a=10$ ,  $h=2$

We know that

$$a - nh = x$$

$$10 - n \cdot 2 = 9$$

$$10 - x = n \cdot 2$$

$$nh = \frac{10 - x}{2}$$

Now we know the Newton's backward interpolation

$$f(a - nh) = f(a) - n \nabla f(a) + \frac{n(n-1)}{2!} \nabla^2 f(a) + \dots$$

$$= 35 - \frac{10 - x}{2} \times 11 + \frac{\left(\frac{10 - x}{2}\right) \left(\frac{10 - x}{2} - 1\right)}{2} \times 2$$

$$= 35 - \frac{10 - x}{2} \times 11 + \frac{(10 - x)^2}{2} - \frac{10 - x}{2}$$

$$= 35 - 12 \left(\frac{10 - x}{2}\right) + \frac{(10 - x)^2}{(2)^2}$$

$$= \frac{35 \times 4 - 24(10 - x) + 10^2 - 20x + x^2}{4}$$

$$= \frac{140 - 240 + 24x + 100 - 20x + x^2}{4}$$

$$f(a - nh) = \frac{x^2 + 4x}{4}$$

$$f(9) = \frac{(9)^2 + 9 \times 4}{4}$$

$$= \frac{29 \cdot 25}{4}$$

$$f'(9) = \frac{2x + 4}{4}$$

$$= \frac{2 \times (9) + 4}{4} = \underline{\underline{5.5}}$$

$$\therefore f(9) = \underline{\underline{29.25}}$$

$$f'(9) = 5.5$$

Here  $x=9$ ,  $a=10$ ,  $h=2$

We know that

$$a - nh = x$$

$$10 - n \cdot 2 = 9$$

$$10 - x = n \cdot 2$$

$$n = \frac{10 - x}{2}$$

Now we know the Newton's backward interpolation

$$f(a - nh) = f(a) + (-n) \nabla f(a) + \frac{n(n-1)}{2!} \nabla^2 f(a) + 0$$

$$= 35 - \frac{10-x}{2} \times 11 + \frac{\left(\frac{10-x}{2}\right) \left(\frac{10-x}{2} - 1\right)}{2} \times 2$$

$$= 35 - \frac{10-x}{2} \times 11 + \frac{(10-x)^2 - 10-x}{2}$$

$$= 35 - 12 \left(\frac{10-x}{2}\right) + \frac{(10-x)^2}{(2)^2}$$

$$= 35 \times 4 - 24(10-x) + 10^2 - 20x + x^2$$

$$= 140 - 240 + 24x + 100 - 20x + x^2$$

$$f(a - nh) = \frac{x^2 + 4x}{4}$$

$$f(9) = \frac{(9)^2 + 9 \times 4}{4}$$

$$= \frac{29.25}{4}$$

$$f'(9) = \frac{2x+4}{4}$$

$$= \frac{2 \times (9) + 4}{4} = \underline{\underline{5.5}}$$

$$\therefore f(9) = \underline{\underline{29.25}} \quad \& \quad f'(9) = \underline{\underline{5.5}}$$

①

→ first construct Table

$x$	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	3			
1	6	3		
2	11	5	02	0
3	8	7	02	

By newton's  $a + mh = x$

$$m = \frac{x-a}{h}$$

$$m = \frac{x-0}{1}$$

By newton's forward interpolation.

$$f(x) = f(a) + m\Delta f(a) + \frac{m(m-1)}{2!} \Delta^2 f(a)$$

$$= 3 + 3x + \frac{x(x-1)}{2} \times 2$$

$$= 3 + 3x + x^2 - x$$

$$f(x) = x^2 + 2x + 3$$

