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Shri.Swami Vivekanand Shikshan Sanstha's

VIVEKANAND COLLEGE,

KOLHAPUR

Department Of Mathematics

A project on

Fibonacci sequence and golden ratio

By

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(B.Sc.III)

Under the guidance of

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Prof. Sutar S. T.

(Year 2019-2020)

CERTIFICATE

ROLL NO. :8157 SEAT NO. :

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VIVEKANAND COLLEGE,KOLHAPUR

DEPARTMENT OF MATHEMATICS

This is to certify that,

Miss Sathe Ankita Mahipati student of vivekanand college Kolhapur has successfully completed a project on "Fibonacci Sequence and golden ratio "in B.Sc. III In the year 2019-2020.

Date: 19/03/20

Place: Kolhapur

14/3/2020

Teacher in charge Examiner Head of department

HEAD
Department of Mathematics
Vivekanand College, Kolhapur

DECLARATION

I undersigned hereby declare that the project report entitled **"Fibonacci sequence and golden ratio".**

Written and submitted to vivekanand college Kolhapur a partial fulfillment of B.Sc. III(mathematics)under the guidance of **Prof. Patankar s. p.** are my original work .The empirical results in project report are based on data collected by myself .

I understand that any such copying is liable to be published as authorities deem fit.

Date: 19/03/20

Place: KOLHAPUR

Miss Ankita Mahipati Sathe.

ACKNOWLEDGEMENT

I owe a great many thanks to all who helped and supported me during the writing of this project.

I am thankful to Prof.Patankar sir, who is the guide of the project for guidance and correcting various documents of my project carefully. He has taken pain to go through the project and make necessary correction as and when needed.

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I would also thank my college and my faculty members without whom this project would have been a distant reality.I also extend my heartfelt thanks to my family and well wishers.

Submitted by

Miss .Ankita Mahipati Sathe.

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HISTORY

The Fibonacci sequence was invented by Italian Leonardo pisano Bigollo (1180-1250), who is known in mathematical history by several names : Leonardo of pisa & Fibonancci.

In the 1202 AD, Leaonardo Finonancci wrote in his book " Liber Abaci" of a simplumerical sequence this the foundation for an incredible mathematical relationship behind phi. This sequence was known as early as 6th century AD by Indian mathematicians, but it was Fibnonacci who introduced it to the west after his travels throughout the Mediterranean world & North Africa.

As well as being famous for the Fibonacci sequence, he helped spread Hindu-Arabic Numerals (Like our present number 0,1,2,3,4,5,6,7,8,.9) through Europe in place of Roman Numerals (I, II, III, IV, V, etc.) that has saved us all a lot of trouble!

INTRODUCTION

Golden ratio is a number and is equal to 1.618 . There is close connection between Fibonacci sequence and golden ratio. Golden ratio is a ratio of sum of two quantities to that of larger quantity .Golden ratio is denoted by ϕ or Ψ . PHIDIAS is inventor of golden ratio . "Mona Lisa "

is the famous painting painted by leonardo da vinci ,is painted according to golden ratio. Any object which shows golden ratio is more attractive and beautiful.Many face surgeries done by doctors are based on golden ratio.It is found in nature also. Sunflowers , DNA ,shell of snails are natural examples of golden ratio. In this project we discussed in detail about golden ratio, its natural examples and use in different fields.

Fibonacci Sequence :

It is the set of number that starts with zero followed by one & proceeds based on the rule that each number is equal to the sum of preceding two numbers & it is 0,1,1,2,3,5,8,13,21,34... i.e.

- The 2 is found by adding the two numbers before it (1+1).
- The 3 is found by adding the two numbers before it (1+2).
- And the 5 is (2+3) & so on.

Example :

The next number in the sequence above is 21+34=55.

Here is a long list :

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, ... & so on.

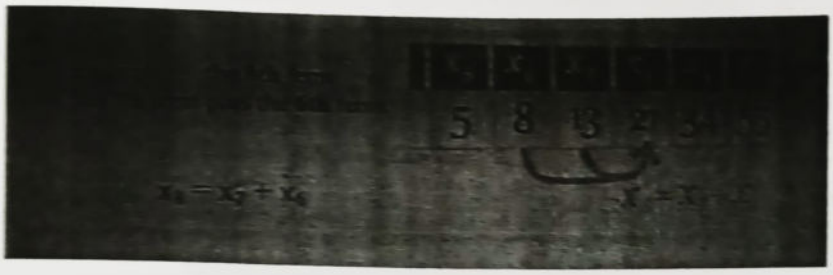
THE RULE :

The Fibonacci Sequence can be written as a "Rule".

First, the terms are numbered from 0 onwards like this :

n =	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	...
X_n =	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	...

So term number 6 is called X_6 (which equals 8).



So we can write the rule :

The Rule is $\mathbf{X_n = X_{n-1} + x_{n-2}}$

$F_n = F_{n-1} + F_{n-2}$

Where,

- $\mathbf{X_n}$ is term number "n".
- $\mathbf{X_{n-1}}$ is the previous term (n-1).
- $\mathbf{X_{n-2}}$ is the term before that (n-2).

Example : Term 9 is calculated like this :

$$X_9 = X_{9-1} + X_{9-2}$$

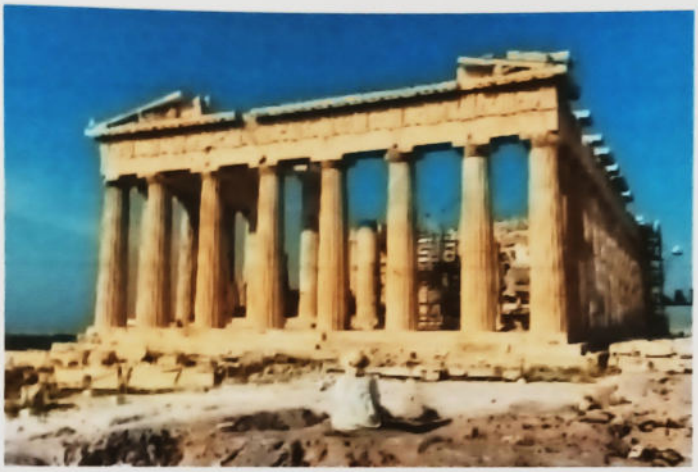
$$= X_8 + X_7$$

$$= 21+13$$

$$= 34$$

Inventor of golden ratio:

Throughout history, ratio for length to width of rectangles of 1.61803 has been considered the most pleasing to the eye. This ratio was named the Golden ratio by Greeks. In the world of mathematics, the numeric value is called "phi", named for the Greek sculptor ***phidias***. The space between the column form golden rectangles throughout this structure which is found in Athens, Greece.



He sculped many things including the bands of sculpture that run above the columns of the *parthenon*.

Phidias widely used the golden ration in this works of sculpture. The exterior diamensions of the parthenon in Athens, built in about 440 BC form a perfect golden rectangle.

Golden Ratio(ϕ or Ψ)

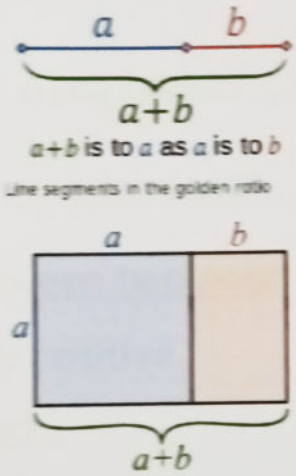
Defination of golden ratio :

In mathematics two quantities are in golden ratio if their ratio is the same as the ratio of their sum to the larger of two quantities.

i.e. Let a>b.

then, $\frac{a}{b} = \frac{a+b}{a}$

Derivation of obtaining golden ratio :-



We have, by definition of golden ratio

$\frac{a+b}{a} = \frac{a}{b} = \Psi$ ...(1)

Consider,

L.H.S. = $\frac{a+b}{a}$

$= 1+\frac{b}{a}$

$= 1+\frac{1}{\psi}$

From (1),

$1+\frac{1}{\psi} = \psi$

$\psi+1 = \psi^2$

$\psi^2 - \psi - 1 = 0$ ...(2)

As equation (2) is quadratic equation, so it has two solutions.

By solving this, we get,

$\psi = \frac{1+\sqrt{5}}{2} = 1.6180339887...$ &

$\psi = \frac{1-\sqrt{5}}{2} = 0.6180339887...$

As ψ is the ratio between two positive quantities.

Thus ψ is necessarily positive.

$\psi = 1.6180339887...$

Connection between Fibonacci sequence & golden ratio :

When we take any two successive (one after the other) Fibonacci numbers, their ratio is very close to the golden ratio ψ which is approximately 1.618034...

In fact, the bigger the pair of Fibonacci Numbers, the closer the approximation. Let us try a few :

A	B	B/A
2	3	1.5
3	5	1.666666666...
5	8	1.6
8	13	1.625
...	...	...
144	233	1.618055556...
233	377	1.618025751...
...	...	...

Note : This also works when we pick two **random** whole numbers to begin the sequence, such as 192 and 16 (we get the sequence 192, 62, 208, 224, 432, 656, 1088, 1744, 2832, 4576, 7408, 11984, 19392, 31376, ...) :

A	B	B/A
192	16	0.08333333...
16	208	13
208	224	1.07692308...
224	432	1.92857143...
...	...	...
7408	119841	.61771058...
11984	193921	.61815754...
...	...	...

It takes longer to get good values, but it shows that not just the Fibonacci Sequence can do this!

Uses of golden ratio :

Golden ratio used in different field other than maths & geometry.

It is used in design/Art, Beauty/Face, LIFE, Markets/Gamins, Cosmos, theology, Blog etc.

We will discuss some its used in field of

I. DESIGN/ART.

II. BEAUTY/FACE.

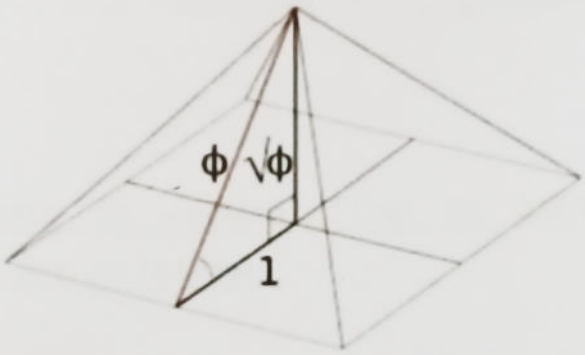
III. LIFE.

I.DESIGN/ART :-

A golden ratio pyramid is based on a triangle whose three sides represent the mathematical relationship that defines the golden ratio.

This triangle, known as a kepler triangle, has base of length 1 & a hypotenuse whose length is phi. The height of this triangle is the square root of phi. Using the pythogorean theorem, ($a^2+b^2=c^2$), this triangle represents one of the golden ratio's unique properties.

$1+phi = phi^2$



The geometry of the great pyramid of Giza varies by less than 0.025% from that of a perfect golden triangle pyramid. This seems rather amazing.

Giza pyramid

II . BEAUTY/FACE :-

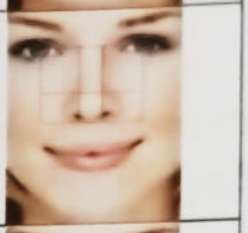
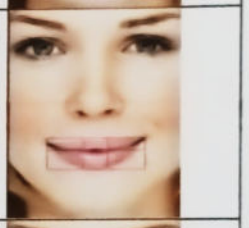
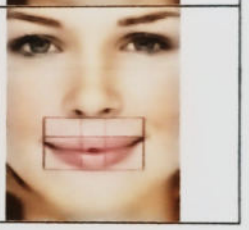
The human face abounds with examples of the golden ratio, also known as the golden section or divine proportion. It is often said that "beauty is in the eye of the beholder." Evidence shows though that the closer facial proportions are to be golden ratio, the more a face will be perceived as attractive, across race, culture or era. Most takes regarded as beautiful or attractive will have a significant number of markers whose proportions are very close to golden ratio (1.618). The guide is simple when viewed in the slides above. It includes the following list of golden ratio facial proportions:-

- **Golden ratio in vertical (Top to bottom) facial dimension are as follows:**

Sr.no.	Starting Point	Φ point	End pt.	Image
V1	Center of pupils	Center of lips	Bottom of chin	
V2	Center of pupil	Nose at nostril	Bottom of chin	-
V3	Center of pupil	Nose flair	Nose base	
V4	Top arc of eyebrows	Top of eyes	Bottom of eyes	
V5	Center of pupil	Nose at nostril	Center of lips	
V6	Top of lips	Center of lips	Bottom of lips	
V7	Nose at nostrils	Top of lips	Center of lips	-

- **Golden ratio in horizontal (side to side) facial dimension:**

Sr.no.	Starting point	Φ point	End point	Images
H1	Side of face	Outside of eye	Center of pupil	
H2	Side of face	Outside of iris	Inside of eye &inside of eye brows	
H3	Side of face	Inside of iris	Center of lips	
H4	Side of face	Inside of near eye	Inside opposite eye	
H5	Side of face	Center of face	Outside opposite eye	

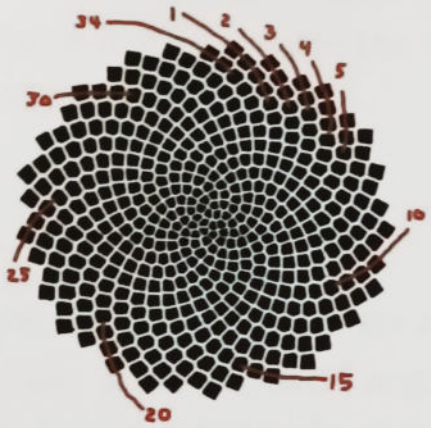
H6	Side of face	Inside opposite eye	Opposite side of face	
H7	Center of pupil	Top of nose flair	Bottom of nose	
H8	Center of pupils	Bridge of nose	Center of nostri	
H9	Inside of eye	Width of nose	Inside of nostrils	
H10	Width of mouth	Cupids bow pt. of lips	Bottom of lips	
H11	Bottom of nose	Top of lips	Bottom of lips	

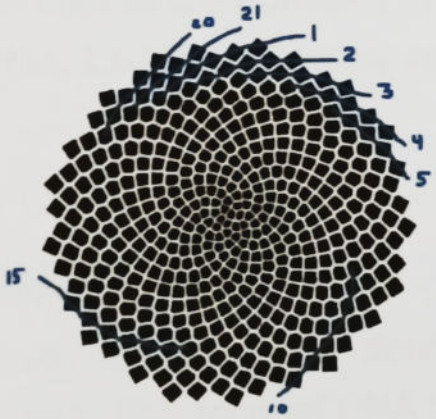
- Head height to head width is also a golden ratio .

III . LIFE :-

A)The golden ratio in Sunflowers:-

The golden ratio & the Fibonacci sequence appear in various areas of nature. The Fibonacci sequence, for example can be found when analyzing patterns of seeds on a sunflower. The individual seeds create spiral arms, curving to the right & left. The number of spirals to the left, is however not equal to those spiraling to the right.





The spiral arms to the left & to the right are always two successive numbers of the Fibonacci sequence.

Hooking at these two diagrams, one can see that there are 21 spiral arms curving to the right & 34 spiral arms curving to the left. These two numbers successive numbers in the Fibonacci sequence. Therefore, seeds in a sunflower follows the pattern of the Fibonacci sequence.

The golden angle plays an important role for the creation of this distinct alignment of seeds. The golden angle is approximately 137.5^0 & seeds in the sunflower are arranged according to it. This creates the spirals in the pattern of sunflower seeds.

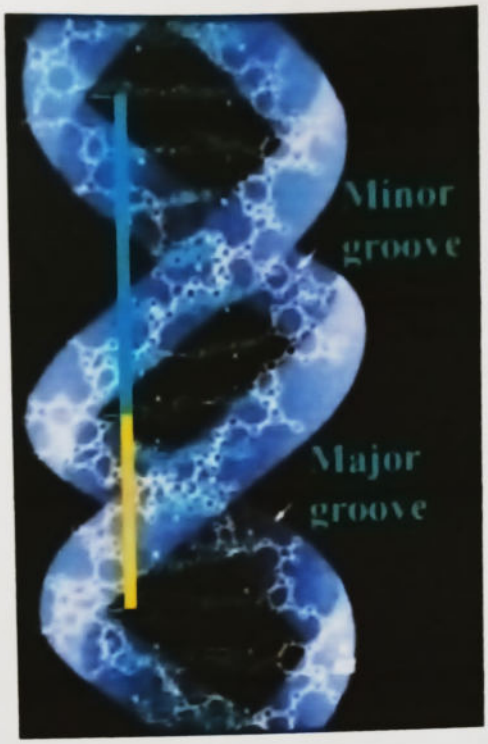
B-DNA:-

The DNA molecule, the program for all life, is based on the golden section. It measures 34 angstroms long by 21 angstroms wide for each full cycle of its double helix spiral 34 & 21, of course are numbers in the Fibonacci series & their ratio, 1.6190476 closely approximates phi, 1.6180339.

B-DNA has spirals in phi proportions :

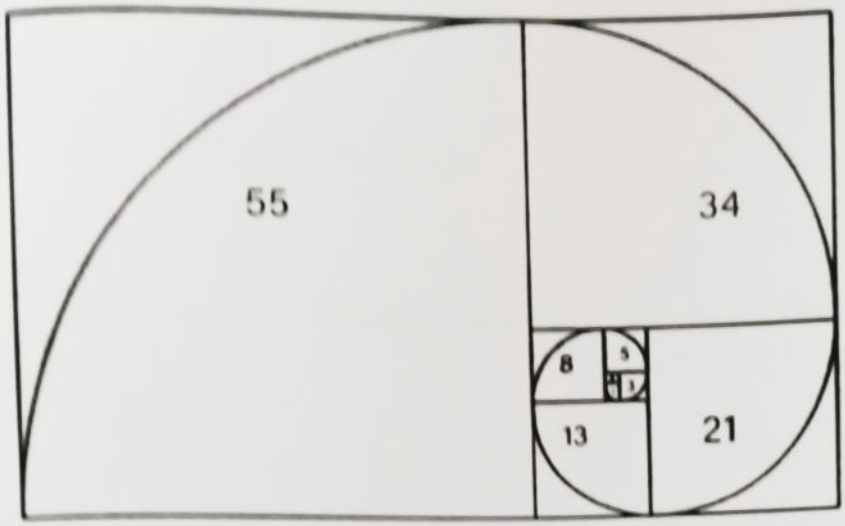
DNA in the cell appears as a double stranded helix referred to as B-DNA. This form of DNA has a two groove in its spirals with a ratio of phi in the proportion of the major

groove to the minor groove, or roughly 21 angstroms to 13 angstroms.



Insight on B-DNA proportion contributed by Melih Yazici

Golden spiral



This picture is called golden spiral because it forms a spiral .It is formed by rectangles hence this rectangles are called golden rectangles.This golden spiral plays very important role because any image or picture shows golden spiral then that image is in golden ratio.To draw the golden spiral initially form squares , of side 1,1,2,5,8,13,21,34,55,... and join them as shown in figure . Then with the help of compass draw spiral.

BIBLIOGRAPHY

1. www.wiipida.com

2. www.livescience.com

3. Fabulous Fibonacci numbers by scientist Alfred S. Posamentier

4. Golden Spiral