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VIVEKANAND COLLEGE, KOLHAPUR

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DEPARTMENT OF MATHEMATICS

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Project Name :**SOME FAMOUS NUMBERS**

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" ज्ञान, विज्ञानआणिसुसंस्कारयांसाठीशिक्षणप्रसार"

- शिक्षणमहर्षीडॉ. बापूजीसाळुंखे.

Shri Swami Vivekanand Shikshan Santha's

VIVEKANAND COLLEGE, KOLHAPUR

CERTIFICATE

Department of Mathematics

This is to certify that,

Miss.JOSHI PRANALI RAVINDRA Student of Vivekanand college ,Kolhapur has successfully completed a project on **"SOME FAMOUS NUMBERS"** in B.Sc. -III year at Department of Mathematics in the year 2018-19.

DATE :

PLACE : Kolhapur.

Teacher in charge **Examiner** **Head of Department**

HEAD
Department of Mathematics
Vivekanand College, Kolhapur

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DECLARATION

I, the undersigned hereby declare that project report entitled, "SOME FAMOUS NUMBERS" submitted by me to Shivaji University Kolhapur in a partial fulfillment of the requirement for the award of Degree of "B.Sc.III" in Mathematics Department is a record of bonafide project work carried out by me under the guidance of Prof. S.P. Patankar [HOD] , Prof. Sutar [Asso. Prof.], Prof. A.A. Patil [Asst. Prof.] is my original work and interpretations drawn there in are based on material collected by my ownself.

Place: Kolhapur

Date:

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Miss.JOSHI PRANALI RAVINDRA

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SOME FAMOUS NUMBERS

The professor went to the board one day
And asked his class, just for fun
What is the only conceivable way
To combine e, i, π , zero and one
A brilliant young scholar, he proved quite a hero
Who knew the professor just loved to tease
Replied : $e^{i\pi} + 1 = 0$
Then he requested : the next question, please.

Many years ago, the writer received a magnificent $15 for submitting this poem, which not long afterward, appeared in the mathematical nursery rhyme corner of a trade journal. Though the poem is pathetic enough, the real misfortune is that space did not allow acknowledgement to the famous Swiss mathematician, Leonhard Euler. In any event, the episode serves as a prologue to our next endeavor an examination of several of the important numerical constants of mathematics.

Without questions, the most famous of these "numbers" is the one we call π ; it has the approximate numerical value $\pi = 3.14159$. Although its basic definition relates the circumference of a circle to its diameter. π appears in a large number of mathematical problems that have nothing whatsoever to do with circles.

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Another extremely important number is the one identified as 'e'. Its approximate value is $e = 2.71828$. It serves as the basis for so-called natural logarithms & also for things like exponential growth in demography, radioactive decay in physics, & bell-shaped curves in probability theory.

Another famous number is the one called the golden ratio, $\phi = 1.61803$. This number shows up in the strangest places, including the architecture of the Parthenon in Greece and the Great pyramids of Egypt.

Yet another important number is Euler's constant. Its numerical value is $\gamma = 0.57721$. Again, this number appears in many problems of mathematics including the theory of heat conduction &the theory of extreme value distribution in statistics.

Our fifth &final number is also the newest one it is the Feigenbaum number, $\delta = 4.6692$. This numerical constant makes its appearance in the relatively new area of mathematics called chaos theory.

The First Famous Number Of All : π

The most ancient, most familiar, and most important number in all of mathematics indeed in all human civilization is the one we designate with the symbol. It represents the ratio of the circumference of a circle and its diameter of a circle, then $C = \pi D$. Also, if A is the area of a circle and $R = D/2$ is its radius, then $A = \pi R^2$.

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More than 4000 years ago, the Babylonians had established an approximate value for this important number π = 25/8 (that is, 3.125). At about the same time, the Egyptians had determined that π = 256/81(3.1605) but less than 22/7 (3.14286).

Surprisingly accurate numerical values of π were also known in ancient Chinese, Hindu & Mayan civilizations. The Bible, however, missed it by quite a bit. A verse in the old Testament 1 Kings 7:23, implies the value π = 3. The bill passed the House of Representatives by a 63 to 0 vote. Fortunately, a mathematics professor from Purdue University arrived on the scene in the nick of time. Thanks to his invention, the π = 3 bill was withdrawn from deliberation by the Indian Senate & to date, has not been considered further.

It turns out that π is an irrational number, which means that is, it cannot be expressed as a ratio of two integers, like 22/7. In addition, it is a transcendental number, which means that it is not the root of an algebraic equation with integer coefficients, such as $x^2-7x+12=0$. The consequence of this transcendental property of π is that the numbers to the right of the decimal point (i.e. 3.1415…) go on forever and ever without any apparent order of pattern.

It has long been a kind of contest or competitive game among mathematicians and now computer specialists to enlarge the number of decimal places of π. In the year 1560, it had been established that π = 3.141592, i.e., it was known to an accuracy of six numbers to the right of the decimal point. By the end of the sixteenth century, π had been calculated to thirty decimal places. A summary of the growth of our knowledge of the number of the known decimal places π prior to the twentieth century is presented in table.7

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Table 7.1

(Number 'N' of known decimal places of 'π' before the twentieth century.)

Year	N
1560	6
1600	30
1621	35
1700	71
1706	100
1717	127
1800	140
1824	208
1847	248
1853	440
1855	500
1874	707

With the invention and very fast development of electronic computers during the twentieth century, the number of known decimal places of the π has increased rapidly and enormously. In 1947, π was known to 808 decimal places. By 1947, the number had increased to over 10,000 and by 1967 to 5,00,000. Twenty years later in 1987, the number had grown to 25 million. In 1997, professor Kanada and his colleagues at the University of Tokyo computed to an incredible 51.5 billion decimal places.

One wonders why they want to have all this information about the number of known decimal places of π. Well, for mathematicians involved in number theory, the extensive list of decimal places provides very useful information concerning patterns, distributions, randomness and other properties and features of number sequences.

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Over the years, many analytical methods and mathematical equations have been developed and utilized to calculate π. For example, in the past the following expression has been employed to determine the value of π :

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\ldots\ldots \qquad (7.1)$$

This kind of equation is called an infinite series. Another well-known expression utilized for the computation of π is

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots\ldots \qquad (7.2)$$

You might want to try calculating π from equations (7.1) and (7.2). You will quickly discover that they are very slow in producing an answer. Indeed, a good many terms must be employed obtain even a rough estimate of π. An equation that is much more suitable is the one employed by the noted German mathematician Carl Friedrich Gauss (1777-1855) :

$$\frac{\pi}{4} = \tan^{-1}(1/2) + \tan^{-1}(1/5) + \tan^{-1}(1/8) \qquad (7.3)$$

With regard to the use of infinite series for calculating the value of π, a remarkable advance was made in1995 when the following expression was given by Baily et al (1997) :

$$\pi = \sum_{k=0}^{\infty} \frac{1}{16}\left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6}\right)$$

(7.4)

Although this equation is only slightly more complicated than the preceding expressions, it yields the value of π much more quickly. You might want to convince yourself that π is correctly computed to six decimal places by using the terms corresponding to k= 0,1,2,3 in equation (7.4).

A charming little book by Beckmann (1977) gives a brief history of π and descriptions of numerous things about IT.The number of letters in each word gives the respective number in the sequence (i.e. = 3.14159265358979) :

"How I want a drink, alcoholic of course, after the heavy lectures involving quantum mechanics."

If you prefer a shorter mnemonic device for π, here is one that will give you seven decimal places (3.1415926) :

"May I have a large container of coffee?"

A substantial contribution was made to mathematics literature with the publication of Pi: A source Book by Berggren, Borwein & Borwein(1997). This voluminous work presents the history of this important number over the past 4000 years. Included in its contents are seventy representative documents on the subject. Most of the contents, of course, deal seriously with the mathematical & computational aspects. For example, the contributions of the Indian mathematical genius *Srinivasa Ramanujan*(1887-1920) are included in the book.

A good many of the documents in the work deal with strictly historical studies thus, considerable attention is given to studies carried out one ago in Egypt, Greece, India, China & medieval Islam. And finally, a number of presentations are somewhat whimsical or even amusing selections. These include a 402 –

mnemonic for π, constructed in the format of a circle, and a display of the numerous documents presented to the Indian legislature in 1897 to decrease the legal value of π.

The second famous number : e

Although nearly everyone knows about the number π, its nearest rival in fame & importance, the number 'e', is virtually unknown outside of mathematics, science and engineering. The main reason for this is that 'e', which has the approximate value e = 2.72828, is not really encountered or utilized-except in natural logarithms-until we get involved in calculus and other areas of more advanced mathematics. In these subjects, 'e' is an extremely important numerical constant.

The Swiss mathematician Leonhard Euler(1707-1783)was one of the most prolific in all of history. Among a great many other major contributions, he was the one who assigned the symbol 'e' to this famous number & proved the relationship

$$\lim_{n\to\infty} (1 + \frac{1}{n})^n = e. \qquad (7.5)$$

This expression says that as the whole number 'n' increases in magnitude, the value of the quantity in parenthesis say as S approaches the number, e = 2.81828. For example, if n = 10, then S = 2.59374; if n = 100, S = 2.70481; & so on to n = ∞.

Another great mathematician, England's Isaac Newton(1643-1727), showed that

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$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots \qquad (7.6)$$

This expression is another example of an infinite series. From either equation (7.5) or equation (7.6), we can easily compute the value of 'e'. As is true π, the quantity 'e' is an irrational and also transcendental number.

The topic of logarithms was mentioned above. We probably remember logarithms from your course in elementary algebra though you may not recall that you probably dealt only with so-called "common" logarithms. This is the system in which the number 10 is used as the *base* of the logarithm probably because humans had 10 fingers.

Now any number can serve as the *base* of an arithmetic system including operations involving logarithms. In information theory & computer science, the number 2 ("binary") is generally used although sometimes the numbers 8 ("octal") & 16 ("hexadecimal") are employed. In contrast to the use of the number 10 as the *base* for "common" logarithms, the number 'e' is used as the *base* for so-called "natural" logarithms.

Suppose we have the simple equation $y = c^x$, where 'c' is a positive constant. If we set c = e, then clearly $y = e^x$. This is called the exponential function. Here comes the calculus, the derivative of this equation is $dy/dx = e^x$. In other words, the derivative of the exponential function is equal to the function itself. By the same way, the integral of e^x takes on the same form, i.e. $\int y\, dx = e^x$. Thus, by using this particular numerical value for 'e', the function e^x, its derivative, & its integral are all equal. For calculus operations, this represents an enormous simplification.

Further more, with $y = e^x$, then taking logarithms, we have $x = \log_e y$, where the subscript 'e' means that the base of the logarithm is 'e'. This is a "natural" logarithm. Incidentally, $\log_e y$ is sometimes written in y to avoid confusion with $\log_{10} y$, the so-called logarithm. Your calculator probably has keys for both types of logarithms.

An Example: Earning Interest on your savings account

We take a quick look at a topic in which all of us are interested : How much money can you earn on your savings account? To answer the question, we utilize equation (7.5) but change it slightly to the form,

$$P = P_0 \left(1 + \frac{r}{n} \right)^n, \qquad (7.7)$$

In which P_0 = Rs. 1,000 is the amount of money you have in your savings account at the beginning of the year, that is, the original principal. r = 6% = 0.06 is the annual interest rate paid by our bank. 'n' is the number of "compound periods" during the year & 'p' is the amount of money in your account at the end of the

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year, i.e. 12 months later. The difference between P & P_0 is the amount of interest you earned during the year.

TABLE 7.2

Interest earnings on your savings account original principal P_0 = Rs.1,000; interest rate r = 6%

Compounding Frequency	n	Year-end Principal
Annually	1	Rs.1,060.00
Semiannually	2	Rs.1,060.90
Quarterly	4	Rs.1,061.36
Monthly	12	Rs.1,061.68
Daily	365	Rs.1,061.83
Instantly	0	Rs.1,061.84

Note if the bank compounds your interest earnings only once a year, then n = 1. Accordingly, from equation (7.7), at the end of the year you will have P = Rs.1,000$(1 + 0.06/1)^2$ = Rs. 1,060. This is called simple interest. Alternatively, suppose the bank compounds

In this case, n = 2 & equation (7.7) becomes P = Rs.1,000$(1+ 0.06 / 2)^2$ = Rs. 1,060.90. Next, assume that the bank compound quarterly. Therefore, n =4 ; substituting into equation (7.7) gives P = Rs.1,000$(1+ 0.06 / 4)^4$ = Rs. 1,061.36, and so on. The result of our calculations are listed in table 7.2.

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Observation no. 1: suppose that your bank advertises and applies "daily compounding'' of interest on your savings account. Then , after 12 months, according to table, your principal is Rs. 1,061.83 and you have earned $P - P_0$ = Rs. 61.84. Dividing this by the original principal and converting to a percentage gives r = 6.183%. This is called yield.

Observation no. 2: If there is instantaneous compounding equation (7.7) becomes $P = P_0 e^r$,which gives P = Rs. 1,061.84. Clearly, as far as your interest earning is concerned, it makes essentially no difference whether your bank compounds your savings account daily, hourly or instantaneously.

Three Other Famous Numbers

We have taken quick looks at the two most famous nos. in mathematics: π & e. let us see other some nos.

Golden Ratio, $\phi = 1.61803$

This no. defines the ratio of the length 'L' and width 'H' of a rectangle that allegedly gives the most esthetically attractive appearance. Its precise value is, L / H = 1- 2 (1+ $\sqrt{5}$).

Euler's Constants, $\gamma = 0.57721$

This important no. devised by Leonhard Euler around 1750, is defined by the equation $\gamma = (1+1/2+1/3 +.....+1/(n-\log_e$

n)) as ‘n’ becomes infinite. It makes its appearance in many problems in mathematics and statistics.

**Feigenbaum Number, $\delta = 4.66920$*

This is the newest of the important nos. It was discovered in 1978 by the American mathematician Mitchell Feigenbaum, in his early studies of chaos theory. This no., δ, is the ratio of the spacing of successive intervals of period doubling in process reading to chaos.

Annual Celebration Days for ‘π’ and perhaps ‘e’

In the preceding sections, we have examined two famous nos. π & e, and looked briefly at three others, ϕ,γ and δ . Noteworthy is the fact that of these five important nos. , only π can be expressed as a date if we required that three significant figures be utilized i.e., $\pi = 3.14$ which, of course, is March 14.

It is fitting, therefore, that this most important no. of all should be celebrated each year. Accordingly, shall we declare March 14 to be the π–Day? If this meets with success, we could later on, celebrate e–Day on February 7 and perhaps ϕ–Day on January 6.

Suggested first steps: Contact the White House and the greeting card manufacturers.

Some Amazing Mathematical Relationships

In 1746, that incredibly prolific mathematical genius named Leonhard Euler presented the following identity:

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad (8.1)$$

In which cosθ is the real part and sinθ is the imaginary part of complex equation. As we shall now see, this equation yields some really amazing relationships.

Amazing Relationship 1:

Let $\theta = \pi$ (i.e., 180^0) gives,

$$e^{i\pi} + 1 = 0$$

This is an extremely remarkable result. It is an equation that uniquely relates the five most important nos. in all of mathematics e, i, π, 1and 0.

Amazing Relationship 2:

Let $\theta = \pi/2$ (i.e. 90^0) in equation(8.1). Then since cos $(\pi/2) = 0$ and $\sin(\pi/2) = 1$, we get $e^{i\pi/2} = i$. Multiplying the exponents of both sides of this equation by 'i' yields $e^{-\pi/2} = i^i$. Since $e^{-\pi/2} = 0.2089$, we obtain the very remarkable result that is, $i^i = 0.2079$. This relationship says that 'i' raised to the 'i^{th}' power is equal to a real no. Totally crazy! How can this possibly be? This is much too weird to even think about.

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