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श्री स्वामी विवेकानंद शिक्षण संस्थेचे

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In

Mathematics

BY

Kalyani Pandurang patil

Roll No :-7890

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Under the Guidance of

Mr. Patankar S.P (H.O.D) Department of Mathematics

Mr. Thorat S.P (Department of Mathematics

"Education for Knowledge, Culture and Science"
... Shikshanmaharshi Dr. Bapuji Salunkhe.

Shri. Swami Vivekanand Shikshan Sanstha , kolhapur's

VIVEKANAND COLLEGE (AUTONOMOUS), KOLHAPUR

DEPARTMENT OF MATHEMATICS

CERTIFICATE

This is to certify that Miss.Kalyani Pandurang patil has successfully completed the project work on topic "HARMONICS ANALYSIS IN FOURIER SERIES " towards the partial fulfilment for the course of Bachelor of Science (Mathematics) work of Vivekanand College (Autonomous) Kolhapur affilited to Shivaji University, Kolhapur during the academic year 2021-2022 .This report represents the bonafide work of student.

Place :- Kolhapur

Date :-

Teacher Incharg   Examiner   Mr. S.P. Patankar
Head Dept. of
Mathematics

HEAD
Department of Mathematics
Vivekanand College, Kolhapur

DECLERATION

I undersigned hereby declare that project entitled "HARMONICS ANALYSIS IN FOURIER SERIES ". Completed under the guidance of Mr. S.P. Patankar sir and Mr. S. P. Thorat sir (Department of Mathematics, Vivekanand College Kolhapur). Based on the experiment results and cited data . I declare that this is my original work which is submitted to , Vivekanand College Kolhapur in this academic year.

Kalyani

28|05|2022

Miss.Kalyani Pandurang patil

Acknowledgement

On the day of completion of this project , the numerous memories agreeing rushed in my mind with full of gratitude to this encouraged and helped me a lot at various stages of this work.

I offer sincere gratitude to all of them . I have great pleasure to express my deep sense of indebtedness and heart of full gratitude to my project guide Mr. S.P. Patankar sir . For his expert and valuable guidance and continuous encouragement given to me during the course of project work.

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I would like to thank all my teachers Mr. S.P. Thorat sir , Mr. G.B. Kolhe sir and Miss. S.M. Malavi mam for co-opration, help and maintaining cheerful environment during my project.

I would also like to thanks non-teaching staff Mr. Birnale.

I would like to thanks my entire dear friends for their constant encouragement and co-opration .I am indebted to my parents who shaped me to this status with their blunt less vision and selfness agenda.

Place :- Kolhapur

Date :- 28|05|2022

Miss.Kalyani Pandurang patil

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INTRODUCTION

In mathematics, a **Fourier series** is a way to represent a function as the sum of simple sine waves. More formally, it decomposes any periodic function or periodic signal into the sum of a (possibly infinite) set of simple oscillating functions, namely sines and cosines. The discrete time Fourier transform is a periodic function, often defined in terms of a Fourier series. The Z-transform, another example of application, reduces to a Fourier series for the important case $|z|=1$. Fourier series are also central to the original proof of the Nyquist–Shannon sampling theorem. The study of Fourier series is a branch of Fourier analysis.

The Fourier series is named in honour of **Jean-Baptiste Joseph Fourier (1768–1830)**, who made important contributions to the study of trigonometric series. Fourier introduced the series for the purpose of solving the heat equation in a metal plate. Through Fourier's research the fact was established that an arbitrary (continuous) function can be represented by a trigonometric series. The first announcement of this great discovery was made by Fourier in 1807, before the French Academy.

The heat equation is a partial differential equation. Prior to Fourier's work, no solution to the heat equation was known in the general case, although particular solutions were known if the heat source behaved in a simple way, in particular, if the heat source was a sine or cosine wave. These simple solutions are now sometimes called Eigen solutions. Fourier's idea was to model a complicated heat source as a superposition (or linear combination) of simple sine and cosine waves, and to write the solution as a superposition of the corresponding Eigen solutions. This superposition or linear combination is called the Fourier series.

The Fourier series has many applications in electrical engineering, vibrational analysis, acoustics, optics, signal processing, image processing, quantum mechanics, econometrics, thin-walled shell theory etc.

PERIODIC FUNCTIONS

Let f(x) be a function defined for all real values of x. A function f(x) is said to be periodic if there exists some positive number T such that,

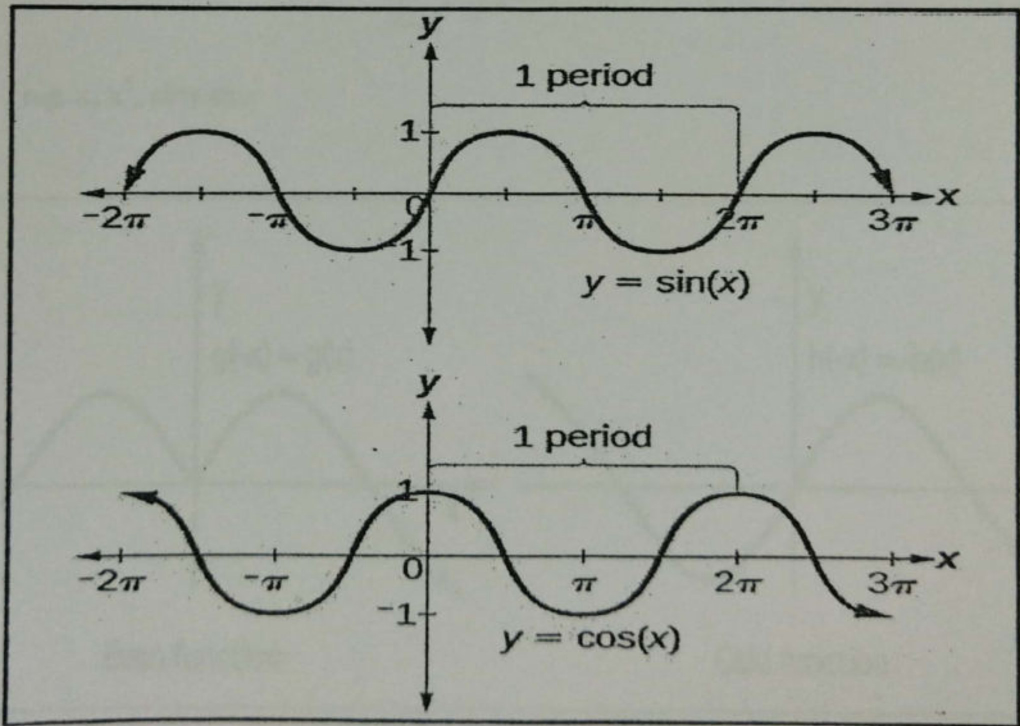
$$f(x+T) = f(x) \quad \forall x$$

The positive number T is called the period. The smallest positive value of T is called primitive period of least period or some times period of f(x).

e.g. f(x) = sinx and f(x) = cosx are periodic functions with period 2π as

$\sin(2\pi+x) = \sin x$;

$\cos(2\pi+x) = \cos x$.



If any function f(x) can be expressed in terms of sine and cosine, then it is called Fourier series expansion of f(x).

EVEN & ODD FUNCTIONS

❖ Even Function :

A function $y = f(x)$ is said to be **even** if $f(-x) = f(x)$ for all values of x. The graph of an even function is always symmetrical about the ***y*-axis** (i.e. it is a mirror image).

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

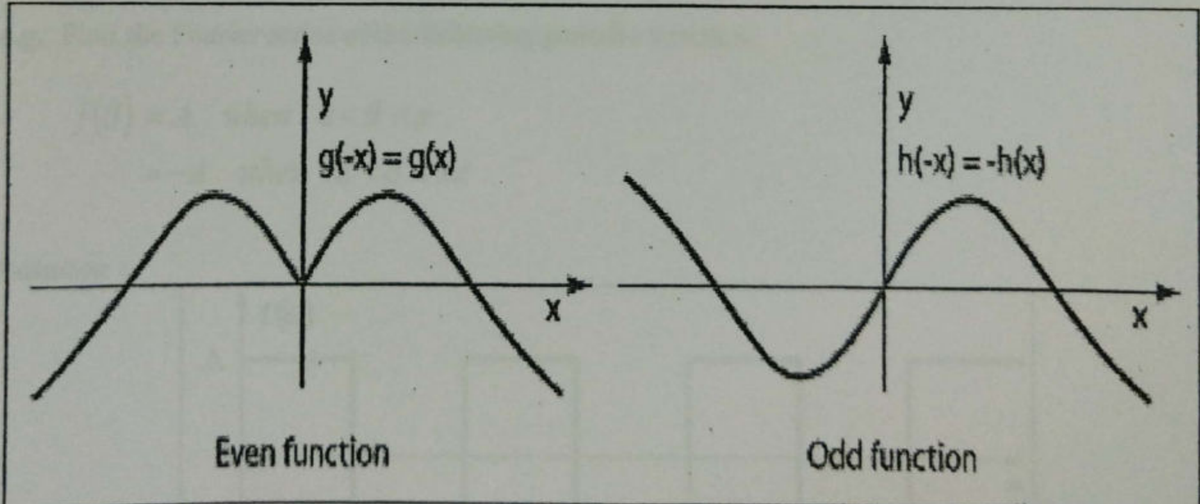
e.g. k, x^2, cosx etc.

❖ Odd Function :

A function $y = f(x)$ is said to be **odd** if $f(-x) = -f(x)$ for all values of x. The graph of an odd function is always symmetrical about the **origin**.

$$\int_{-a}^{a} f(x)dx = 0$$

e.g. x, x^5, sinx etc.



FOURIER SERIES

The Fourier series is an infinite series expansion involving trigonometric functions. Consider a function f(x) which is defined in the interval [-c, c] and outside this interval by f(x+2c) = f(x) i.e. the function f(x) has a period 2c. Now suppose f(x) can be expanded in a series of the form given by,

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{c}+b_n\sin\frac{n\pi x}{c}\right)$$

This expansion of f(x) is called '**Fourier expansion** corresponding to f(x)' or '**Fourier series** for f(x)'. The coefficients a_n and b_n are called '**Fourier Coefficients**' and their values are given by,

$$a_0=\frac{1}{c}\int_{-c}^{c}f(x)dx$$

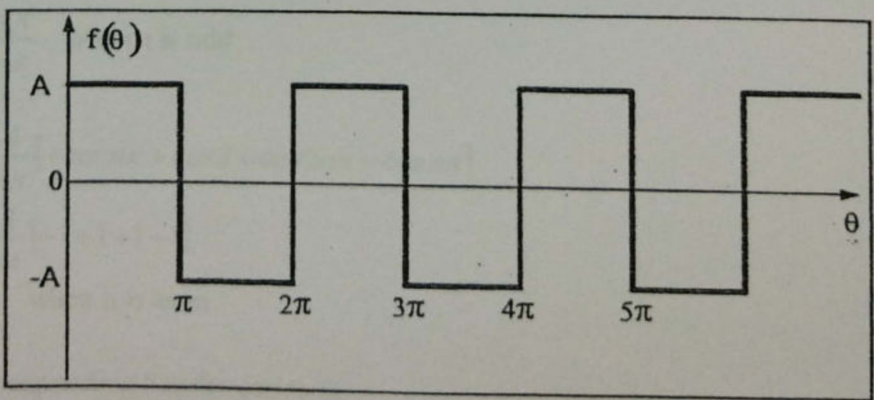
$$a_n=\frac{1}{c}\int_{-c}^{c}f(x)\cos\frac{n\pi x}{c}dx$$

$$b_n=\frac{1}{c}\int_{-c}^{c}f(x)\sin\frac{n\pi x}{c}dx \qquad \text{where, } n=0,1,2,3,\ldots$$

e.g. Find the Fourier series of the following periodic function.

$$f(\theta)=A \quad when \quad 0<\theta<\pi$$
$$=-A \quad when \quad \pi<\theta<2\pi$$

Solution :



The given function is periodic over the period of 2π.

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)d\theta$$
$$= \frac{1}{2\pi}\left[\int_0^{\pi} f(\theta)d\theta + \int_{\pi}^{2\pi} f(\theta)d\theta\right]$$
$$= \frac{1}{2\pi}\left[\int_0^{\pi} A\,d\theta + \int_{\pi}^{2\pi} -A\,d\theta\right]$$
$$= 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos n\theta\,d\theta$$
$$= \frac{1}{\pi}\left[\int_0^{\pi} A\cos n\theta\,d\theta + \int_{\pi}^{2\pi}(-A)\cos n\theta\,d\theta\right]$$
$$= \frac{1}{\pi}\left[A\frac{\sin n\theta}{n}\right]_0^{\pi} + \frac{1}{\pi}\left[-A\frac{\sin n\theta}{n}\right]_{\pi}^{2\pi} = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\sin n\theta\,d\theta$$
$$= \frac{1}{\pi}\left[\int_0^{\pi} A\sin n\theta\,d\theta + \int_{\pi}^{2\pi}(-A)\sin n\theta\,d\theta\right]$$
$$= \frac{1}{\pi}\left[-A\frac{\cos n\theta}{n}\right]_0^{\pi} + \frac{1}{\pi}\left[A\frac{\cos n\theta}{n}\right]_{\pi}^{2\pi}$$
$$= \frac{A}{n\pi}[-\cos n\pi + \cos 0 + \cos 2n\pi - \cos n\pi]$$

$$= \frac{A}{n\pi}[-\cos n\pi + \cos 0 + \cos 2n\pi - \cos n\pi]$$
$$= \frac{A}{n\pi}[1+1+1+1]$$
$$= \frac{4A}{n\pi} \quad \text{when n is odd}$$

$$= \frac{A}{n\pi}[-\cos n\pi + \cos 0 + \cos 2n\pi - \cos n\pi]$$
$$= \frac{A}{n\pi}[-1+1+1-1]$$
$$= 0 \quad \text{when n is even}$$

Therefore, the corresponding Fourier series is,

$$\frac{4A}{\pi}\left(\sin\theta + \frac{1}{3}\sin 3\theta + \frac{1}{5}\sin 5\theta + \frac{1}{7}\sin 7\theta + \cdots\right)$$

DIRICHLET CONDITION

Any periodic waveform f(x) of period $T = 2c$, can be expressed in a Fourier series provided that,

(a) It has a finite number of discontinuities within the period $2c$;

(b) It has a finite average value in the period $2c$;

(c) It has a finite number of positive and negative maxima and minima.

When these conditions, called the Dirichlet conditions, are satisfied, the Fourier series for the function $f(x)$ exists.

HALF RANGE SERIES

➢ Half Range Cosine Series :

If the function f(x) is even function in the range [-c, c], then b_n=0 and therefore Fourier expansion of f(x) contains a_0 and a_n i.e. cosine function only. Such series is called 'Half Range Cosine Series'. This expansion does not contain sine terms.

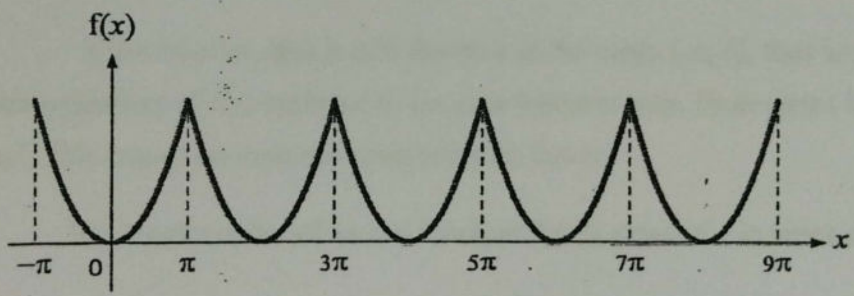
The Fourier series of an even function f(x) is expressed in terms of a cosine series.

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx)$$

e.g. Find the Fourier series of the following periodic function.

$$f(x) = x^2 \quad when \quad -\pi \le x \le \pi$$

Solution:



The given function is periodic over the period of 2π.

$$f(\theta + 2\pi) = f(\theta)$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2\, dx$$

$$= \frac{1}{2\pi}\left[\frac{x^3}{3}\right]_{x=-\pi}^{x=\pi} = \frac{\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{\pi} x^2 \cos nx dx\right]$$

By using Integration by parts, we get,

$$a_n = \frac{4}{n^2}\cos n\pi$$

$$a_n = -\frac{4}{n^2} \quad \text{when n is odd}$$

$$a_n = \frac{4}{n^2} \quad \text{when n is even}$$

This is an even function.

$\therefore\ b_n = 0$

The corresponding Fourier series is,

$$\frac{\pi^2}{3} - 4\left(\cos x - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \frac{\cos 4x}{4^2} + \cdots\right)$$

➢ Half Range Sine Series :

If the function f(x) is odd function in the range [-c, c], then a_0=0 and a_n=0 and therefore Fourier expansion of f(x) contains b_n i.e. sine function only. Such series is called 'Half Range Sine Series'. This expansion does not contain cosine terms.

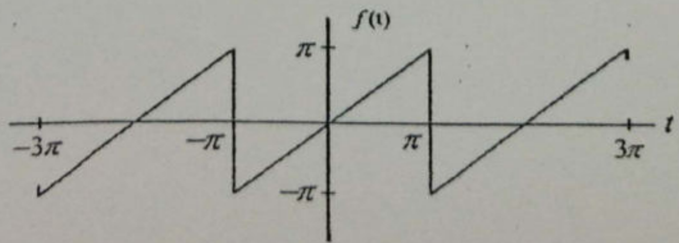
The Fourier series of an odd function f(x) is expressed in terms of a sine series.

$$f(x) = \sum_{n=1}^{\infty}(b_n \sin nx)$$

e.g. Find the Fourier series of the following periodic function.

$$f(x) = x \quad when \quad -\pi \le x \le \pi$$

Solution :



The given function is periodic over the period of 2π.

$$f(\theta+2\pi)=f(\theta)$$

Since, $f(x)=x$ is odd function because $f(-x)=-x=-f(x)$.

So, $a_0=0$ and $a_n=0$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx\,dx$$

$$=\frac{1}{\pi}\left[\int_{-\pi}^{\pi}x\sin nx dx\right]$$

By using Integration by parts, we get,

$$b_n=\frac{-1}{n}(2\cos n\pi)$$

$$b_n=\frac{-2}{n}(-1)^n$$

The corresponding Fourier series is,

$$2\left(\sin x-\frac{\sin 2x}{2}+\frac{\sin 3x}{3}-\cdots\right)$$

HARMONIC ANALYSIS

Recall the Fourier Series Expansion of the periodic function f(x) over the interval [-c, c] as,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{c} + b_n \sin\frac{n\pi x}{c}\right)$$

That is, it is explicitly written as,

$$f(x) = \frac{a_0}{2} + a_1 \cos\frac{n\pi x}{c} + b_1 \sin\frac{n\pi x}{c}$$
$$+ a_2 \cos\frac{n\pi x}{c} + b_2 \sin\frac{n\pi x}{c}$$
$$+ a_3 \cos\frac{n\pi x}{c} + b_3 \sin\frac{n\pi x}{c}$$
$$+\ldots$$

The term $a_1 \cos\frac{\pi x}{c} + b_1 \sin\frac{\pi x}{c}$ is known as **Fundamental term** or **First harmonic term.**

The term $a_2 \cos\frac{2\pi x}{c} + b_2 \sin\frac{2\pi x}{c}$ is known as **Second harmonic term.**

In general, the term $a_n \cos\frac{n\pi x}{c} + b_n \sin\frac{n\pi x}{c}$ is known as **nth harmonic term.**

The **amplitude** of nth harmonic term is given by,

$$n^{th}\ Amp = \sqrt{a_n^2 + b_n^2}$$

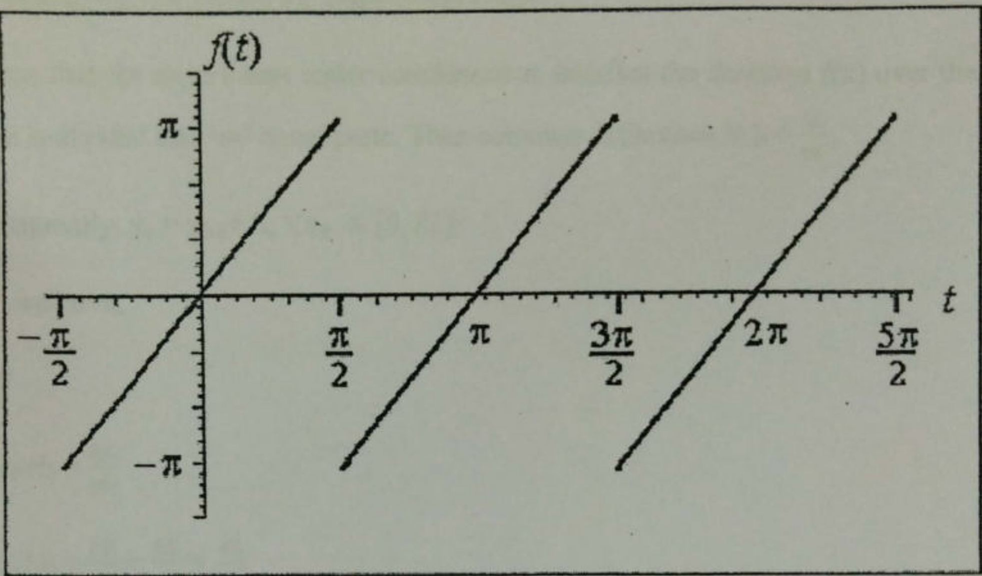
The **percentage** of nth harmonic term is given by,

$$n^{th}\,\% = \frac{n^{th}\ Amp}{1^{st}\ Amp} \times 100 = \frac{\sqrt{a_n^2+b_n^2}}{\sqrt{a_1^2+b_1^2}} \times 100$$

EVEN AND ODD HARMONICS

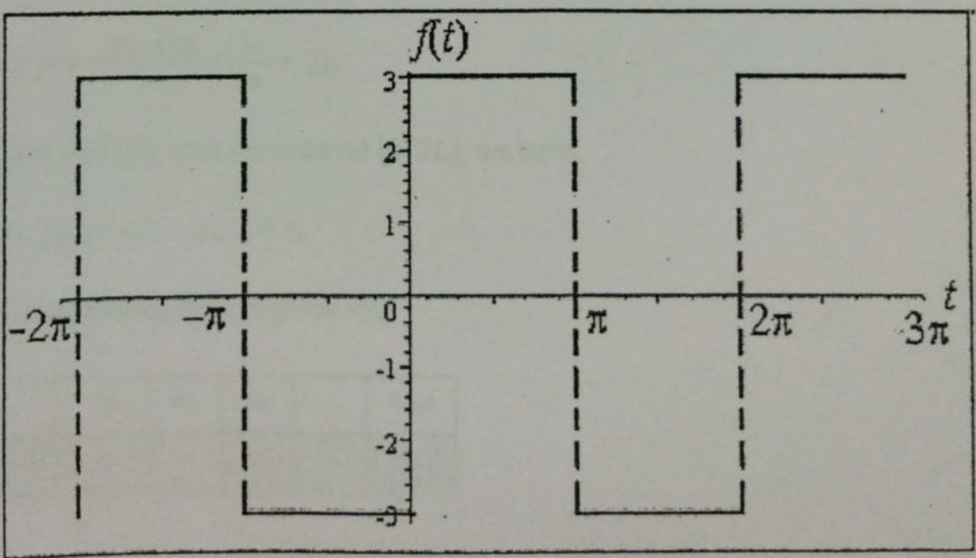
➤ Even Harmonics :

The Fourier series will contain **even harmonics** if $f(x + \pi) = f(x)$. (i.e. it has period π.)



➤ Odd Harmonics :

The Fourier series will contain **odd harmonics** if $f(x + \pi) = -f(x)$.



PRACTICAL HARMONIC ANALYSIS

In practical engineering problems, the function y=f(x) is not pre-defined but is to be formed by the obtained experimental values. When the set of experimental values is repeated after certain interval of time, the function formed is the periodic function over that interval. In such cases, we can obtain Fourier series expansion for the experiment over the interval. Thus, the Fourier coefficients of f(x) are so obtained by Trapezoidal Rule.

Assume that the experiment under consideration satisfies the function f(x) over the interval [0, 2L] which is divided into 'm' equal parts. Thus common difference is $h = \frac{2L}{m}$.

Consequently, $x_k = x_{k-1} + h, \forall x_k \in [0, 2L]$

Thus we have,

$x_0 = 0$

$x_1 = x_0 + h = \frac{2L}{m}$

$x_2 = x_1 + h = \frac{2L}{m} + \frac{2L}{m} = \frac{4L}{m}$

.

.

.

$x_m = x_{m-1} + h = \frac{(m-1)2L}{m} + \frac{2L}{m} = 2L$

As f(x) is periodic over the interval [0, 2L], we have,

$x_m = x_0, x_{m+1} = x_1, \ldots, x_{m+k} = x_k$

The corresponding table is given by,

x	x_0	x_1	x_2	...	x_{m-1}
y = f(x)	y_0	y_1	y_2	...	y_{m-1}

CASE I :

If y=f(x) is defined over the interval [0, 2π] or [0, 360⁰] and the interval is divided in multiples of degree / radians, then

$$a_0 = \frac{2}{m}\sum y$$

$$a_n = \frac{2}{m}\sum y\cos nx$$

$$b_n = \frac{2}{m}\sum y\sin nx$$

Then Fourier expansion series of f(x) can be given as,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

Example : Find the first harmonic of the Fourier series for y from the following data.

x	y
0	2.84
30	3.01
60	3.69
90	4.15
120	3.69
150	2.2
180	0.83
210	0.51
240	0.88
270	0.09
300	1.09
330	1.64

Solution :

We have,

x	y	ycosx	ysinx
0	2.84	2.84	0
30	3.01	2.6067	1.5050
60	3.69	1.8450	3.1956
90	4.15	0	4.15
120	3.69	-1.8450	3.1956
150	2.20	-1.9053	1.100
180	0.83	-0.83	0
210	0.51	-0.4417	-0.2550
240	0.88	-0.44	-0.7621
270	0.09	0	-0.0900
300	1.09	0.5450	-0.9440
330	1.64	1.4203	-0.82
Σ	24.62	3.7950	10.2751

Hence,

$$a_0 = \frac{2}{12}\sum y = \frac{24.62}{6} = 4.1033$$

$$a_1 = \frac{2}{12}\sum y\,cosx = \frac{3.7950}{6} = 0.6325$$

$$b_1 = \frac{2}{12}\sum y\,sinx = \frac{10.2751}{6} = 1.7125$$

$$\therefore \mathbf{y = 2.05165 + 0.6325cosx + 1.7125sinx}$$

CASE II :

If y=f(x) is defined over the interval [0, 2π] or [0, 360^0] and the interval is divided multiples of its period T. If there are 'm' no. of divisions of the interval [0, 2L], we have,

$$2L = T$$

$$\therefore L = \frac{T}{2}$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(\frac{n\pi x}{T/2}) + b_n \sin(\frac{n\pi x}{T/2})]$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(\frac{2n\pi x}{T}) + b_n \sin(\frac{2n\pi x}{T})]$$

And the Fourier coefficients in this case are,

$$a_0 = \frac{2}{m}\sum y$$

$$a_n = \frac{2}{m}\sum y\cos(\frac{2n\pi x}{T})$$

$$b_n = \frac{2}{m}\sum y\sin(\frac{2n\pi x}{T})$$

Example : The following table gives variation of periodic current over a period.

t sec	0	T/6	T/3	T/2	2T/3	5T/6	T
A amp	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75 amp in variable current and obtain the amplitude of 1st harmonic.

Solution :

We have, m=6

t	$\frac{2\pi t}{T}$	A	$A\cos(\frac{2\pi t}{T})$	$A\sin(\frac{2\pi t}{T})$
0	0	1.98	1.98	0
T/6	60	1.30	0.65	1.1258
T/3	120	1.05	-0.525	0.9090
T/2	180	1.30	-1.30	0
2T/3	240	-0.88	0.44	0.7620
5T/6	300	-0.25	-0.125	0.2165
	Σ	1.98	1.12	3.0133

Hence,

$$a_0 = \frac{2}{6}\sum A = \frac{4.5}{3} = 1.5$$

$$a_1 = \frac{2}{6}\sum A\cos(\frac{2\pi t}{T}) = \frac{1.12}{3} = 0.373$$

$$b_1 = \frac{2}{6}\sum A\sin(\frac{2\pi t}{T}) = \frac{3.033}{3} = 1.004$$

$$\therefore A = 0.75 + 0.373\cos(\frac{2\pi t}{T}) + 1.004\sin(\frac{2\pi t}{T})$$

∴ Direct Current = 0.75 amp

Amplitude of 1st harmonic $= \sqrt{a_1^2 + b_1^2}$

$= \sqrt{0.373^2 + 1.004^2}$

= 1.07

CASE III :

If y=f(x) is defined over the interval [0, π] or [0, 180^0] to obtain half range sine or cosine series, where division of the interval is given in terms of degree/radians.

1. For half range **sine** series :

$$b_n = \frac{2}{m}\sum y\, sinnx$$

$$f(x) = \sum_{n=1}^{\infty} b_n sinnx$$

2. For half range **cosine series** :

$$a_0 = \frac{2}{m}\sum y$$

$$a_n = \frac{2}{m}\sum y\, cosnx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cosnx$$

Example : A turning moment T is given for the series of values of the crank angle 'θ' in degrees is given in the following table.

θ	T
0	0
30	5224
60	8097
90	7850
120	5499
150	2626
180	0

Expand T in a half range cosine series.

Solution :

We have, m=6

θ	T	Tcos θ	Tcos2θ	Tcos3θ
0	0	0	0	0
30	5224	4524.117	2612	0
60	8097	4048.5	-4048.5	-8097
90	7850	0	-7850	0
120	5499	-2749.5	-2749.5	5499
150	2626	-2274.183	1313	0
Σ	29296	3548.934	-10723	-2598

Hence,

$$a_0 = \frac{2}{6}\sum T = \frac{29296}{3} = 9765.333$$

$$a_1 = \frac{2}{6}\sum T\cos\theta = \frac{3548.934}{3} = 1182.978$$

$$a_2 = \frac{2}{6}\sum T\cos 2\theta = \frac{-10723}{3} = -3574.333$$

$$a_3 = \frac{2}{6}\sum T\cos 3\theta = \frac{-2598}{3} = -866$$

$$\therefore \mathbf{T = 4882.667 + 1182.978\cos\theta - 3574.333\cos 2\theta - 866\cos 3\theta}$$

CASE IV :

If y=f(x) is defined over the interval [0, π] or [0, 180^0] to obtain half range sine or cosine series, where division of the interval is given in terms of division of its period T. If there are 'm' no. of divisions of the interval [0, L], we have,

L=T

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(\frac{n\pi x}{T}) + b_n \sin(\frac{n\pi x}{T})]$$

1. For half range **sine** series :

$$b_n = \frac{2}{m}\sum y \sin(\frac{n\pi x}{T})$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin(\frac{n\pi x}{T})$$

2. For half range **cosine** series :

$$a_0 = \frac{2}{m}\sum y$$

$$a_n = \frac{2}{m}\sum y \cos(\frac{n\pi x}{T})$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi x}{T})$$

Example : Obtain the 1^{st} three coefficients in the Fourier cosine series for y, where y is given as follows.

x	0	1	2	3	4	5
y	4	8	15	7	6	2

Solution :

We have, m=6, T=6

x	$\frac{2\pi x}{T}$	y	$y\cos(\frac{2\pi x}{T})$	$y\cos2(\frac{2\pi x}{T})$	$y\cos3(\frac{2\pi x}{T})$
0	0	4	4	4	4
1	30	8	6.9282	4	0
2	60	15	7.5	-7.5	-15
3	90	7	0	-7	0
4	120	6	-3	-3	6
5	150	2	-1.732	1	0
	Σ	42	13.6962	-8.5	-5

Hence,

$$a_0 = \frac{2}{6}\sum y = \frac{42}{3} = 14$$

$$a_1 = \frac{2}{6}\sum y\cos\theta = \frac{13.6962}{3} = 4.5654$$

$$a_2 = \frac{2}{6}\sum y\cos2\theta = \frac{-8.5}{3} = -2.833$$

$$a_3 = \frac{2}{6}\sum y\cos3\theta = \frac{-5}{3} = -1.6667$$

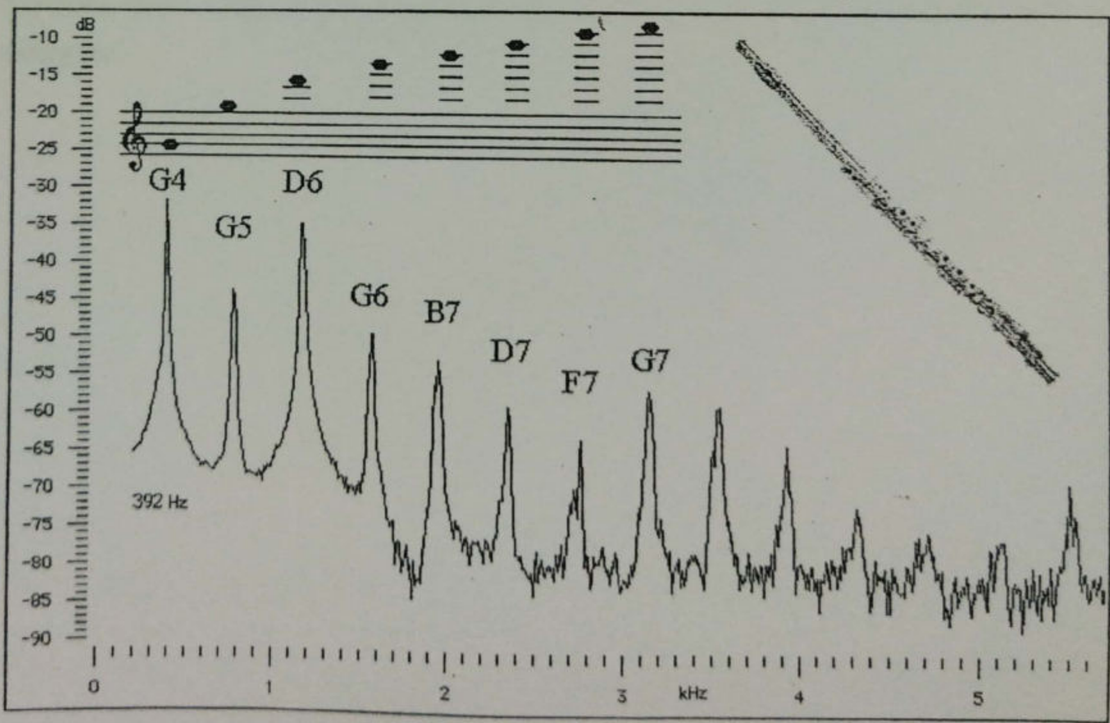
$$\therefore \mathbf{T = 7 + 4.5654\cos\theta - 2.833\cos2\theta - 1.6667\cos3\theta}$$

APPLICATIONS OF HARMONIC ANALYSIS

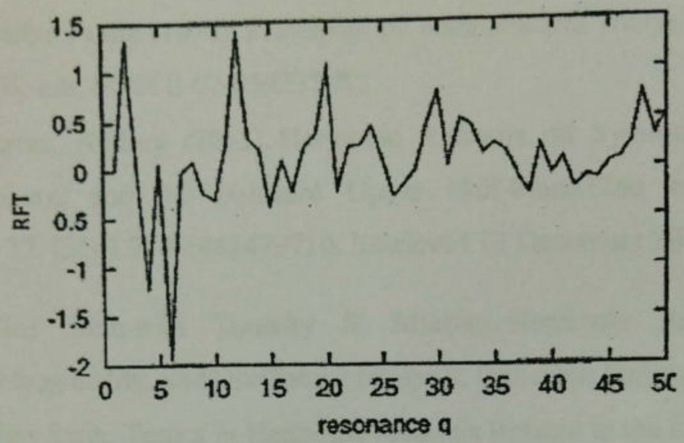
Harmonic analysis is a branch of mathematics concerned with the representation of functions or signals as the superposition of basic waves, and the study of and generalization of the notions of Fourier series and Fourier transforms (i.e. an extended form of Fourier analysis). In the past two centuries, it has become a vast subject with applications in areas as diverse as number theory, representation theory, signal processing, quantum mechanics, tidal analysis and neuroscience.

1. When we listen to different musical instruments playing the same note, they sound different to us because of the different combinations of harmonics contained in the note. For example, if a flute and a violin both play G above middle C, the harmonic spectrum is quite different. G has a frequency of 392 Hz and the harmonics are all multiples of this fundamental frequency (or about 800 Hz, 1200 Hz, 1600 Hz, etc.). **Harmony** (2 or more notes sounding at the same time) works because of harmonics (look for the chord GBD contained within the harmonics of the note G).

   We can see the relative values of the harmonics in the following sound spectrum images of a flute and a violin playing G4.



2. Harmonic analysis began as a method to assist in solving differential equations. Both the boundary conditions and solution to the system of equations would be decomposed as harmonics, a transformation that can make solving the system simpler.
3. The construction of certain explicit expander graphs, such as Ramanujan graphs, uses harmonic analysis.



4. Harmonic analysis is used in digital signal processing.
5. Harmonic analysis has many applications in electrical engineering, vibrational analysis, acoustics, optics, image processing, quantum mechanics, econometrics, thin-walled shell theory etc.

REFERENCES

1. Katznelson, Yitzhak (1976). "An introduction to harmonic analysis" (Second corrected ed.). New York: Dover Publications, Inc. ISBN 0-486-63331-4.
2. Yitzhak Katznelson, An introduction to harmonic analysis, Third edition. Cambridge University Press, 2004. ISBN 0-521-83829-0; 0-521-54359-2
3. Walter Rudin (1976). Principles of mathematical analysis (3rd ed.). New York: McGraw-Hill, Inc. ISBN 0-07-054235-X.
4. Terras, Audrey (2013). Harmonic Analysis on Symmetric Spaces-Euclidean Space, the Sphere, and the Poincaré Upper Half-Plane (2nd ed.). New York, NY: Springer. p. 37. ISBN 978-1461479710. Retrieved 12 December 2017
5. Elias Stein with Timothy S. Murphy, Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals, Princeton University Press, 1993.
6. Elias Stein, Topics in Harmonic Analysis Related to the Littlewood-Paley Theory, Princeton University Press, 1970
7. Enrique A. Gonzalez-Velasco (1992). "Connections in Mathematical Analysis: The Case of Fourier Series". American Mathematical Monthly. 99 (5): 427–441. doi:10.2307/2325087. JSTOR 2325087.