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In Mathematics

By

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**Project name:-** **APPLICATIONS OF MATRICES**

Under the Guidance of

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Head Of The Department Of Mathematics

"Education For Knowledge ,Science& culture"
-ShikshanmaharshiDr. BapujiSalunkhe.

Shri Swami Vivekanand ShikshanSanstha's,

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DEPARTMENT OF MATHEMATICS

# CERTIFICATE

This is to certify that Mr./Ms./Mrs. **Rajkumar Baban Karape** has successfully completed the project work on topic "APPLICATIONS OF MATRICES" towards the partial fulfilment for the course of Bachelor of Science (Mathematics) work of Vivekanand College, Kolhapur (Autonomous) during the academic year 2022-2023. This report represents the bonafide work of student.

Place : Kolhapur

Date : 05 April 2023

Mr. S. P. Thorat

Head Of Dept. Of Mathematics

HEAD
Department of Mathematics
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# DECLERATION

I undersigned hereby declare that project entitled "APPLICATIONS OF MATRICES". Completed under the guidance of Mr. S.P. Thorat sir. (Department of Mathematics Vivekanand College (Autonomous), Kolhapur). Based on the experiment results and cited data. I declare that this is my original work which is submitted to Vivekanand College, Kolhapur in this academic year.

Mr./Ms. :- Rajkumar Baban Karape

# ACKNOWLEDGEMENT

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I offer sincere gratitude to all of them. I have great pleasure to express my deep sense of indebtedness and heart of full gratitude to my project guide Mr. S.P. Thorat sir. For his expert and valuable guidance and continuous encouragement given to me during the course of project work.

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Place :- Kolhapur

Date :- 05 April 2023

Mr./Ms. :- Rajkumar Baban Karape

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# INTRODUCTION

## Matrix :-

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Each element of a matrix is often denoted by a variable with two subscripts. For instance, $a_{2,1}$ represents the element at the second row and first column of a matrix A.

In mathematics, a matrix (plural matrices) is a rectangular array of numbers, symbols, or expressions arranged in rows and columns. The individual items in matrix are called its elements or entries. An example of a matrix with 2 rows and 3 columns is

$$\begin{bmatrix} 8 & 10 & -13 \\ 5 & -3 & 9 \end{bmatrix}$$

Matrices of the same size can be added or subtracted element by element. The rule for matrix multiplication, however, is that two matrices can be multiplied only when the number of columns in the first equals the number of rows in the second. A major application of matrices is to represent linear transformations, that is, generalizations of linear functions such as $f(x) = 4x$. For example, the rotation of vectors in three dimensional space is a linear transformation which can be represented by a rotation matrix R. If v is a column vector (a matrix with only one column) describing the position of a point in space, the product

Rv is a column vector describing the position of that point after a rotation. The product of two matrices is a matrix that represents the composition of two linear transformations. Another application of matrices is in the solution of a system of linear equations. If the matrix is square, it is possible to deduce some of its properties by computing its determinant. For example, a square matrix has an inverse if and only if its determinant is not zero. Eigen values and eigen vectors provide insight into the geometry of linear transformations

Applications of matrices are found in most scientific fields. In every branch of physics, including classical mechanics, optics, electromagnetism, quantum mechanics, and quantum electrodynamics, they are used to study physical phenomena, such as the motion of rigid bodies in computer graphics, they are used to project a 3-dimensional image onto a 2-dimensional screen. In probability theory and statistics, stochastic matrices are used to describe sets of probabilities; for instance, they are used within the Page Rank algorithm that ranks the pages in a Google search. Matrix calculus generalizes classical analytical notions such as derivatives and exponentials to higher dimensions

A major branch of numerical analysis is devoted to the development of efficient algorithms for matrix computations, a subject that is centuries old and is today an expanding area of research. Matrix decomposition methods simplify computations, both theoretically and practically Algorithms that are tailored to particular matrix structures, such as sparse matrices and near-diagonal matrices, expedite computations in finite element method and other computations. Infinite matrices occur in planetary theory and in atomic theory. A simple example of an infinite matrix is the matrix representing the derivative operator, which acts on the Taylor series of a function.

# Matrix

Definition:-

A matrix is a rectangular array of numbers or other mathematical objects, for which operations such as addition and multiplication are defined. Most commonly, a matrix over a field F is a rectangular array of scalars from F. Most of this article focuses on real and complex matrices, i.e., matrices whose elements are real numbers or complex numbers respectively. More general types of entries are discussed below. For instance, this is a real matrix :

$$A=\begin{bmatrix} 2.6 & -4.7 \\ 1.9 & 3.6 \\ -0.9 & 5.5 \end{bmatrix}$$

The numbers, symbols or expression in the matrix are called its entries or its elements. The horizontal and vertical lines of entries in a matrix are called rows and columns, respectively.

**Size:-**

The size of a matrix is defined by the number of rows and columns that it contains. A matrix with m rows and n columns is called an mxn matrix or m-by-n matrix while m and n are called its dimensions. For example, the matrix A above is a 3 x 2 matrix.

Matrices which have a single row are called row vectors, and those which have a single column are called column vectors. A matrix which has the same number of rows and columns is called a square matrix. A matrix with an infinite number of rows or columns (or both) is called an infinite matrix. In some contexts, such as computer algebra programs, it is useful to consider a matrix with no rows or no columns, called an empty matrix.

| Name | Size | Example | Description |
|---|---|---|---|
| Row vector | 1 x n | [ 4 8 3 ] | A matrix with one row, sometimes used to represent a vector |
| Column vector | n x 1 | $\begin{bmatrix} 8 \\ 3 \\ 7 \end{bmatrix}$ | A matrix with one column, sometimes used to represent a vector |
| Square matrix | n x n | $\begin{bmatrix} 5 & 9 & 6 \\ 1 & 13 & 3 \\ 4 & 6 & 11 \end{bmatrix}$ | A matrix with the same number of rows and columns, sometimes used to represent a linear transformation from a vector space to itself, such as reflection, rotation of shearing |

**Notation:-**

Matrices are commonly written in box brackets

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

An alternative notation uses large parentheses instead of box brackets.

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

The specifics of symbolic matrix notation varies widely, with some prevailing trends. Matrices are usually symbolized using upper-case letters (such as A in the examples above), while the corresponding lower- case letters, with two subscript indices (e.g., $a_{11}$, or $a_{1,1}$), represent the entries. In addition to using upper-case letters to symbolize matrices, many authors use a special typographical style, commonly boldface upright (non-italic), to further distinguish matrices from other mathematical objects.

An alternative notation involves the use of a double-underline with the variable name, with or without boldface style, (e.g. $\underline{\underline{A}}$ ).

The entry in the i-th row and j-th column of a matrix A is sometimes referred to as the ij. (i,j), or $(i,j)^{th}$ entry of the matrix, and most commonly denoted as $a_{i,j}$ or $a_{ij}$ . Alternative notations for that entry are A[i,j] or $A_{ij}$. For example, the (1,3) entry of the following matrix A is 5 (also denoted as, $a_{13}$. A[1,3] or $A_{1,3}$

$$A=\begin{bmatrix} 4 & 7 & -5 & 0 \\ -9 & 0 & 3 & -1 \\ 6 & 2 & 7 & 5 \end{bmatrix}$$

Sometimes, the entries of a matrix can be defined by a formula such as $a_{ij} = f(i,j)$. For example, each of the entries the following matrix A is determined by $a_{ij} = i-j$.

$$A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \end{bmatrix}$$

In this case, the matrix itself is sometimes defined by that formula, within square brackets or double parenthesis. For example, the matrix above is defined as A=[i-j], or A= ((i-j)). If matrix size is m x n, the above-mentioned formula f(i, j) is valid for any i=1,... m and any j = 1,..., n. This can be either specified separately, or using m x n as a subscript. For instance, the matrix A above is 3 x 4 and can be defined as $A=[i-j]$ ($i=1,2,3$; $j = 1, .., 4$), or $A = [i-j]_{3x4}$

Some programming languages utilize doubly subscripted arrays (or arrays of arrays) to represent an m-x-n matrix. Some programming languages start the numbering of array indexes at zero, in which case the entries of an m-by-n matrix are indexed by $0 \leq 1 \leq m-1$ and $0 \leq j \leq n-1$. This article follows the more common convention in mathematical writing where enumeration starts from 1. The set of all m- by-n matrices is denoted (m, n).

# Types of Matrices

i. Diagonal matrix
ii. Lower triangular matrix
iii. Upper triangular matrix
iv. Symmetric or skew-symmetric matrix
v. Orthogonal matrix
vi. Singular matrix
vii. Idempotent matrix
viii.Nilpotent matrix

**1}Diagonal matrix :** \

A square matrix in which every element except the principal diagonal elements is zero is called a Diagonal Matrix.

**2}Upper triangular matrix :**

A square matrix in which all the elements below the principal diagonal are zero is called an upper triangular matrix.

**3}Lower triangular matrix :**

A lower-triangular matrix is a matrix which only has nonzero entries on the downwards-diagonal and below it.

**4} Symmetric or skew-symmetric matrix :**

A matrix is symmetric if and only if it is equal to its transpose. All entries above the main diagonal of a symmetric matrix are reflected into equal entries below the diagonal.

A matrix is skew-symmetric if and only if it is the opposite of its transpose. All main diagonal entries of a skew-symmetric matrix are zero.

**5} Orthogonal matrix :**

A square matrix with real numbers or elements is said to be an orthogonal matrix if its transpose is equal to its inverse matrix.

**6} Singular matrix :**

A square matrix ($m = n$) that is not invertible is called singular or degenerate. A square matrix is singular if and only if its determinant is 0.

**7} Idempotent matrix :**

Idempotent matrix is a square matrix which when multiplied by itself, gives back the same matrix.

**8} Nilpotent matrix :**

Nilpotent Matrix is a square matrix such that the product of the matrix with itself is equal to a null matrix.

## ❖ Invertible matrix and its inverse:-

A square matrix A is called invertible or non-singular if there exists a matrix B such that

$$AB = BA = I_n$$

If B exists, it is unique and is called the inverse matrix of A, denoted $A^{-1}$.

# APPLICATIONS

Matrices find many applications in scientific fields and apply to practical real life problems as well, thus making an indispensable concept for solving many practical problems. Some of them merely take advantage of the compact representation of a set of numbers in a matrix. For example, in game theory and economics, the payoff matrix encodes the payoff for two players, depending on which out of a given (finite) set of alternatives the players choose

Some of the main applications of matrices are briefed below-

## ❖ Physics :

In physics related applications, matrices are applied in the study of electrical circuits, quantum mechanics and optics. In the calculation of battery power outputs, resistor conversion of electrical energy into another useful energy, these matrices play a major role in calculations. Especially in solving the problems using Kirchoff's lae of voltage and current, the matrices are essential.

## ❖ Chemistry :

Chemistry makes use of matrices in various ways, particularly since the use of quantum theory to discuss molecular bonding and spectroscopy. Examples are the overlap matrix and the Fock matrix used in solving the Roothaan equations to obtain the molecular orbitals of the Hartree-Fock method.

## ❖ Electronics :

Traditional mesh analysis in electronics leads to a system of linear equations that can be described with a matrix The behaviour of many electronic components can be described using matrices. Let A be a 2-dimensional vector with the component's input voltage $v_1$ and input current $i_1$ as its elements, and let B be a 2-dimensional vector with the component's output voltage $v_2$ and output current $i_2$ as its elements. Then the behaviour of the electronic component can be described by B = H.A , where H is a 2 x 2 matrix containing one impedance element ($h_{12}$), one admittance element ($h_{21}$) and two dimensionless elements ($h_{11}$ and $h_{22}$). Calculating a circuit now reduces to multiplying matrices.

## ❖ Computer :

In computer based applications, matrices play a vital role in the projection of three dimensional image into a two dimensional screen. creating the realistic seeming motions Stochastic matrices and Eigen vector solvers are used in the page rank algorithms which are used in the ranking of web pages in Google search. The matrix calculus is used in the generalization of analytical notions like exponentials and derivatives to their higher dimensions. One of the most important usages of matrices in computer side applications are encryption of message codes. Matrices and their inverse matrices are used for a programmer for coding or encrypting a message. A message is made as a sequence of numbers in a binary format for communication and it follows code theory for solving. Hence with the help of matrices, those equations are solved. With these encryptions only, internet functions are working and even banks could work with transmission of sensitive and private data's. Computer graphics

## ❖ Geometrical optics :

Geometrical optics provides further matrix applications. In this approximative theory, the wave nature of light is neglected. The result is a model in which light rays are indeed geometrical rays. If the deflection of light rays by optical elements is small, the action of a lens or reflective element on a given light ray can be expressed as multiplication of a two- component vector with a two-by-two matrix called ray transfer matrix: the vector's components are the light ray's slope and its distance from the optical axis, while the matrix encodes the properties of the optical element. Actually, there are two kinds of matrices, viz. a refraction matrix describing the refraction at a lens surface, and a translation matrix, describing the translation of the plane of reference to the next refracting surface, where another refraction matrix applies. The optical system, consisting of a combination of lenses and/or reflective elements, is simply described by the matrix resulting from the product of the components' matrices.

## ❖ Normal modes :

A normal application of matrix in physics is to the description of linearly coupled harmonic systems. The equations of motion of such systems can be described in matrix form, with a mass matrix multiplying a displacement vector to characterize the interactions. The best way to obtain solution is to determine the system's eigen vectors, its normal modes, by diagonalizing the matrix equation. Techniques like this are crucial when it comes to the internal dynamics of molecules the internal vibration of systems consisting of mutually bound component atoms. They are also needed for describing mechanical vibrations, and oscillation in electrical circuits.

## ❖ Geology :

In geology, matrices are used for taking seismic surveys. They are used for plotting graphs, statistics and also to do scientific studies in almost different fields.

## ❖ In GDP :

Matrices are used in calculating the gross domestic products in economics which eventually helps in calculating the goods production efficiently. Matrix is nothing but measurements. There are three types of matrix.

**1. QAM--** Quality Assurence Measurement that means how much quality required in the project

**2. TMM--** Test Manager Measurement that means to estimate how work is completed and how much work yet to complete.

**3. PCM--** Process compatability mesurement that means to estimate testing process upcoming project depending on current project experienced.

## ❖ In robotics and automation:-

In robotics and automation, matrices are the base elements for the robot movements. The movements of the robots are programmed with the calculation of matrices rows and columns. The inputs for controlling robots are given based on the calculations from matrices.

## ❖ Cryptography :

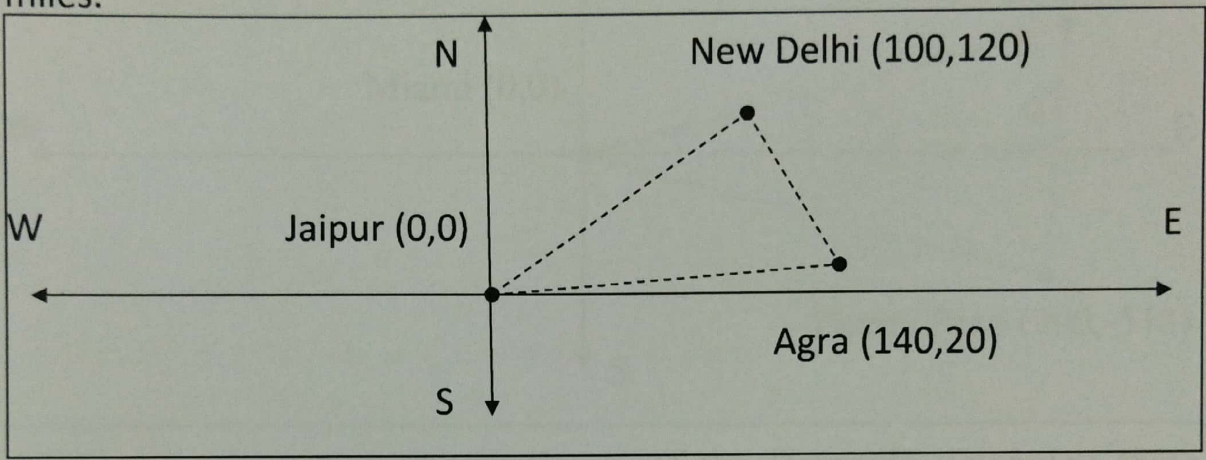
When a programmer encrypts or codes a message, he can use matrices and their inverse. The internet function could not function without encryption, and neither could banks since they now use these same means to transmit private and sensitive data.

Cryptography is concerned with keeping communications Today governments use sophisticated methods of coding and private. decoding messages. One type of code, which is extremely difficult to break, makes use of a large matrix to encode a message. The receiver of the message decodes it using the inverse of the matrix. This first matrix is called the encoding matrix and its inverse is called the decoding matrix.

## ❖ In Real Life :

We can have application for matrices of any dimension in real-life. Most things when changed have an impact on others, or change as a result of many factors. Take something simple - price of bread. Price can be determined by supply against demand - however can also be made very complicated. Price will be affected by price of grain, economic conditions, location. This can then be taken further price of grain is affected by weather, demand for other products, and even price of oil to fill the tractor on the farm. If you want to see how something changes in a what-if situation then you need a dimension for each factor.

**1.** The Golden Triangle is a large triangular region in the India. The Taj Mahal is one of the many wonders that lie within the boundaries of this triangle. The triangle is formed by the imaginary lines that connect the cities of New Delhi, Jaipur, and Agra. Use a determinant to estimate the area of the Golden Triangle. The coordinates given are measured in miles.




The approximate coordinates of the Golden Triangle's three vertices are: (100,120), (140,20), and (0,0). So the area of the region is as follows:

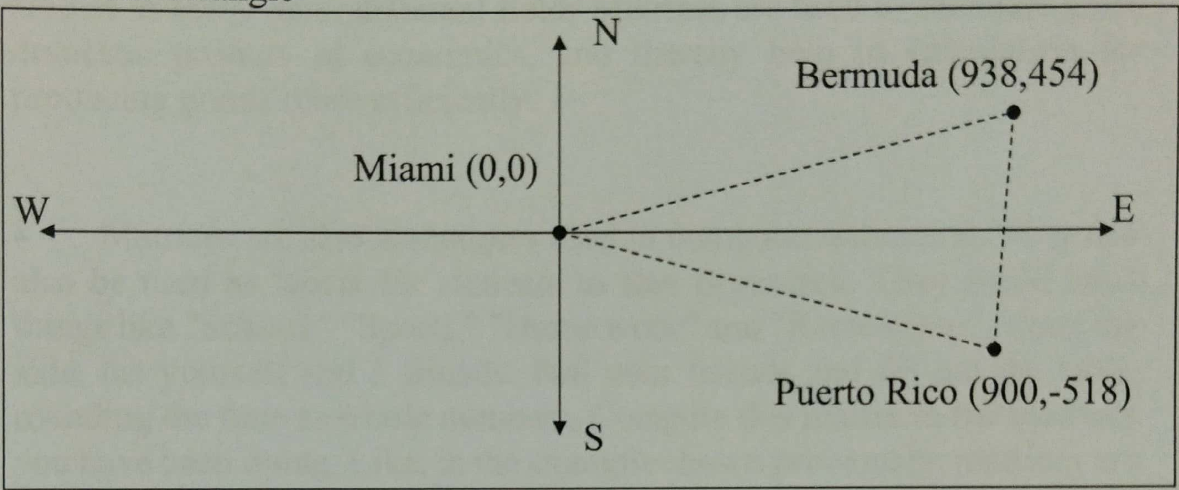
$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 100 & 120 & 1 \\ 140 & 20 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\text{Area} = \pm\frac{1}{2}\,[(2000 + 0 + 0) - (0 + 0 + 16800)]$$

Area = 7400

Hence, area of the Golden Triangle is about 7400 square miles.

**2.** The Bermuda Triangle is a large triangular region in the Atlantic ocean. Many ships and airplanes have been lost in this region.
The triangle is formed by imaginary lines connecting Bermuda, Puerto Rico, and Miami, Florida. Use a determinant to estimate the area of the Bermuda Triangle




The approximate coordinates of the Bermuda Triangle's three vertices are: (938,454), (900,-518) and (0,0). So the area of the region is as follows:

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 938 & 454 & 1 \\ 900 & -518 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\text{Area} = \pm\frac{1}{2}\,[(-458{,}884 + 0 + 0) - (0 + 0 + 408{,}600)]$$

$$\text{Area} = 447{,}242$$

Hence, area of the Bermuda Triangle is about 447,000 square mile

## Matrices are also used in following :

- Matrices are also used in graphs and statistics for doing scientific studies in many other different fields Matrices are used to calculate gross domestic product in economics, and thereby help in calculation for producing goods more efficiently.

- Matrices are also sometimes used in computer animation. They can also be used as labels for students to stay organized. They could label things like "School," "Sports," "Home work" and "Recreation." Along the side, list yourself and 3 friends. Poll your friends and fill out the table, rounding the time to whole numbers, Compare this matrix to the matrices you have been doing. Like, in the example shown previously, matrices are useful for polls.

- In architecture, matrices are used with computing. If needed, it will be very easy to add the data together, like we do with matrices in mathematics. Like in some problems of our homework. matrices could be useful to figuring out things like price and quantity, like with the foods and prices in our homework. As you can see, there are many and very useful ways matrices could be applied in our everyday lives and even in the future.

- Most things when changed have an impact on others, or change as a result of many factors. Take something simple - price of bread. Price can be determined by supply against demand -however can also be made very complicated. Price will be affected by price of grain, economic conditions, location. This can then be taken further price of grain is affected by weather, demand for other products, and even price of oil to fill the tractor on the farm. If you want to see how something changes in a what-if situation then you need a dimension for each factor.

- Matrices are used in representing the real world data's like the traits of people's population, habits, etc. They are best representation methods for plotting the common survey things.

- Matrices are used in many organizations such as for scientists for recording the data for their experiments.

- Matrices are very useful for organization, like for scientists who have to record the data from their experiments if it includes numbers. In engineering, math reports are recorded using matrices.

- Matrices are used in calculating the gross domestic products in economics which eventually helps in calculating the goods production efficiently.
- Matrices are used in many organizations such as for scientists for recording the data for their experiments.
- Matrices are used to cover channels, hidden tent within web pages, hidden files in plain sight, null ciphers and steganography.
- In recent wireless internet connection through mobile phone, known as wireless application protocol also utilize matrices in the form of stenography.
- Cryptography also utilize matrices, cryptography is science of information security. These technologies hide information in storage or transits.

# REFERENCE


## ❖ Books :

1. Matrix Theory And Applications - Denis Serre.

2. Matrix Methods of Strustural Analysis – C. Natarajan . P. Revathi.

## ❖ Website :

1. en.wikipedia.org