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By

(Miss. Suryavanshi Priyanka Govinda)

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Project name:- " Permutation And Combination "

Under the Guidance of

Mr. Thorat S.P.

Head Of The Department Of Mathematics

"Education For Knowledge ,Science & culture"
-Shikshanmaharshi Dr. Bapuji Salunkhe.

Shri Swami Vivekanand Shikshan Sanstha's,

Vivekanand College, Kolhapur (Autonomous)

DEPARTMENT OF MATHEMATICS

CERTIFICATE

This is to certify that Miss. "Suryavanshi Priyanka Govinda " has successfully completed the project work on topic "Permutation & Combination" towards the partial fulfilment for the course of Bachelor of Science (Mathematics) work of Vivekanand College , Kolhapur(Autonomous) during the academic year 2022-2023. This report represents the bonafide work of student.

Place : Kolhapur

Date : 5/04/2023

Mr. S. P. Thorat

Head Of Dept. Of Mathematics

HEAD
Department of Mathematics
Vivekanand College, Kolhapur

DECLERATION

I undersigned hereby declare that project entitled "Permutation & Combination". Completed under the guidance of Mr.S.P. Thorat sir. (Department of Mathematics Vivekanand College (Autonomous), Kolhapur). Based on the experiment results and cited data. I declare that this is my original work which is submitted to Vivekanand College, Kolhapur in this academic year.

Miss :- Suryavanshi Priyanka Govinda

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Place :- Kolhapur

Date :- 5/04/2023

Miss :- "Suryavanshi Priyanka Govinda"

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PERMUTATION

History:

The study of group originally grew out of an understanding of permutation groups. Permutations had themselves been intensively studied by Lagrange in 1770 in his work on the algebraic solutions of polynomial equations. This subject flourished and by the mid 19th century a well-developed theory of permutation groups exited. Codified Camille Jordan in his book Traite des Substitutions et des Equations Algebriques of 1870.Jordan's book was, in turn, based on the papers that were left by Evariste Galois in 1832.

When Cayley introduced the concept of an abstract group. It was not immediately clear whether or not this was a larger collection of objects than the known permutation groups (which had a definition different from the modern lone). Cayley's went on to prove that the two concepts were equivant in Cayley's theorem.

Another classical text containing several chapters on permutation groups is Burnside's Theory of Groups of finite order of 1911. The first half of the twentieth century was a fallow period in the study of group theory in general,but interest in permutation groups was revived in the 1950s by H. Wielandt whose German lecture notes were reprinted as Finite Permutation groups was reviewed in the 1950s by H.Wielandt whose German lecture notes were reprinted as Finite Permutation Groups in 1916.

Definition and notations:

The term permutation refers to a mathematical calculation of the number of ways a particular set can be arranged. Put simply, a permutation is a word that describes the number of ways things can be ordered or arranged. With permutations, the order of the arrangement matters

Permutation Formula

$$^nP_r = \frac{n!}{(n-r)!}$$

NOTATIONS:

$_nP_r$ = permutation

n = total number of objects

r = number of objects selected

Types of Permutation

- Permutation of n different objects (when repetition is not allowed)
- Repetition, where repetition is allowed.
- Permutation when the objects are not distinct (Permutation of multi sets)

Evaluate the following:

(i) $\frac{7!}{6!}$ (ii) $\frac{8!}{5!}$ (iii) $\frac{9!}{6!3!}$

Solution

(i) $\frac{7!}{6!} = \frac{7 \times 6!}{6!} = 7.$

(ii) $\frac{8!}{5!} = \frac{8 \times 7 \times 6 \times 5!}{5!} = 336.$

(iii) $\frac{9!}{6!3!} = \frac{9 \times 8 \times 7 \times 6!}{6! \times 3!} = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84.$

PROPERTIES OF PERMUTATIONS

1. Products of Disjoint Cycles

Every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.

Proof:

Let a be a permutation on A= (1, 2, ..., n). To write a in disjoint cycle form, we start by choosing any member of A, say a,. and let

a=a(a). a,=a(a(a)) = (a), and so on, until we arrive at a -a" (a) for some m. We know that such an m exists because the sequence a,, a (a), a' (a),... must be finite; so there must eventually be a repetition, say a' (a)-a' (a) for some i and / with i<j. Then a =a" (a), where m=1-1. We express this relationship among 4,4,4, as

The three dots at the end indicate the possibility that we may not have exhausted the set A in this process. In such a case, we merely choose any element b, of A not appearing in the first cycle and proceed to create a new cycle as before. That is, we let b=a (b), b, a^2 (b), and so on, until we reach b=a' (b) for some k. This new cycle will have no elements in common with the previously constructed cycle. For, if so, then a' (a)=a' (b) for some i and j. But then a (a)=b, and therefore b=a, for some t. This contradicts the way b, was chosen. Continuing this process

until we run out of elements of A, our permutation will appear as

In this way, we see that every permutation can be written as a product of disjoint cycles.

Disjoint Cycles Commute ✓

If the pair of cycles a (aaa) and (bb) have no entries in common, then aß - Ba.

Proof:

For definiteness, let us say that a and ß are permutations of the set where the c's are the members of S left fixed by both a and ß (there may not be any c's). To prove that aß= Ba, we must show that (af) (x)=(Ba)(x) for all x in S. If x is one of the a elements, say a,, then

(a)(a)=a(B(a))=a(a) = a,... since ß fixes all a elements. (We interpret a,, as a, if i=m). For the same reason,

Hence, the functions of aß and Ba agree on the a elements. A similar argument shows that aß and Ba agree on the b elements

as well. Finally, suppose that x is a c element, say c,. Then, since both a and ß fix c elements, we have (ap) (c)=a(B(c))=a(c)=c,

and

(a)(c)-(a(c))-(0)-

This completes the proof.

In demonstrating how to multiply cycles, we showed that the product (13)(27)(456)(8)1237)(648)(5) can be written in disjoint cycle form as (1732)(48)(56). Is economy in expression the only advantage to writing a permutation in disjoint cycle form? No. The next theorem shows that the disjoint cycle form has the enormous advantage of allowing us to "eyeball" the order of the permutation

Applications of Permutation Groups:

1. Permutations are used in almost every branch of mathematics, and in many other fields of science. In computer science, they are used for analyzing sorting algorithms; in quantum physics, for describing states of particles; and in biology, for describing RNA sequences.

2. Permutation involves arranging a set of objects or data in sequential order and determining the number of ways it can be arranged. An important point to remember here is that the order of arrangement of objects/ data matters in permutation

3. Arranging people, digits, numbers, alphabets, letters, and colours are examples of permutations. Selection of menu, food, clothes, subjects, the team are examples of combinations

4. A combination lock: is a useful item that helps safeguard our belongings when we are out and about. Now, we all know that one can open a combination lock only when the perfect code, in the correct sequential order, is entered. If the code is not in the right order, it won't open. To give you an example, if the lock code is 321, you will have to enter the code in the same sequence to open it. Entering 231 or 123 will not help.

5. PASSWORD: Every day we find ourselves typing passwords on our laptops or mobile phones. Passwords also help in keeping our digital information safe. Similar to a combination lock, passwords are also an arrangement of alphabets, digits, and characters in a particular order. These work only when we enter all the characters of the password in the correct sequence. A tiny misplacement here in there, and you cannot access your device

6. PHONE NUMBERS: Phone numbers typically consist of different parts. For instance, in the phone number +1- 603-452-5521, +1 is the country code, 603 is the area code, 452 is the exchange code, and 5521 is the subscriber code. The first three sets of numbers are fixed depending on which area the phone number belongs. The last four numbers are permutations of digits to form a unique subscriber number. In its entirety, a complete phone number is also a permutation, as one must follow the same order to dial and connect to the right person.

7. A permutation is an arrangement of objects in a definite order

COMBINATION

HISTORY OF COMBINATION

In the 19th century, the subject of partially ordered sets and lattice theory originated in the work of Dedekind, Peirce, and Schröder. However, it was Garrett Birkhoff's seminal work in his book Lattice Theory published in 1967,[26] and the work of John von Neumann that truly established the subjects.[27] In the 1930s, Hall (1936) and Weisner (1935) independently stated the general Möbius inversion formula.[28] In 1964, Gian-Carlo Rota's On the Foundations of Combinatorial Theory I. Theory of Möbius Functions introduced poset and lattice theory as theories in Combinatorics.[27] Richard P. Stanley has had a big impact in contemporary combinatorics for his work in matroid theory,[29] for introducing Zeta polynomials,[30] for explicitly defining Eulerian posets,[31] developing the theory of binomial posets along with Rota and Peter Doubilet,[32] and more. Paul Erdős made seminal contributions to combinatorics throughout the century, winning the Wolf prize in-part for these contributions.

The mathematical field of combinatorics was studied to varying degrees in numerous ancient societies. Its study in Europe dates to the work of Leonardo Fibonacci in the 13th century AD, which introduced Arabian and Indian ideas to the continent. It has continued to be studied in the modern era.

DE TIONS AND NOTATIONS

A co on is a combination of n things taken k at a time without repetition. To re ombinations in which repetition is allowed, the terms k-combination with on, k-multiset,[2] or k-selection.[3] are often used.[4] If, in the above exam were possible to have two of any one kind of fruit there would be 3 more tions: one with two apples, one with two oranges, and one with two pears.

Combination Formula

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

NO ONS:

n = number of combinations

n = total number of objects in the set

r = number of choosing objects from the set

DERIVATION OF FORMULA OF COMBINATION

Let us take (r) number of boxes and each box can hold one thing.

Then, the number of possibilities for selecting the first object from a set of n objects= n

The number of possibilities to choose the second object from among (n-1) distinct objects is as follows= (n-1)

Number of possibilities to choose the third object from (n-2) different options: (n-2)

Number of possible ways to choose the rth object from a set of [n-(r-1)] different objects= [n-(r-1)]

An ordered subset of r elements is created by completing the selection of r things from the initial set of n things.

∴ The number of ways to choose r elements from a set of n elements is n (n – 1) (n – 2) (n-3)… (n – (r – 1) or n (n – 1) (n – 2)… (n – r + 1) n – r + 1 n – r + 1

Let's take a look at the ordered subset of r elements, as well as all of their permutations. This subset's total number of permutations equals r! Because every combination of r things can be rearranged in r! different ways.

As a result, (nCr * r!) is the total number of permutations of n different items taken r at a time. It's all about nPr.

nPr= nCr*r!

nCr =nPr/r! = n!/(n-r)!r!

Example:

Since there are 52 cards in a deck, n=52

As the order of cards in a hand in not important, number of different pokers hands may be computed as :

Solution:

$${}_{52}C_5$$

$$= \frac{52!}{(52-5)!\ 5!}$$

$$= \frac{52!}{47!\ 5!}$$

$$= \frac{(52)(51)(50)(49)(48)}{(5)(4)(3)(2)(1)}$$

$$= (13)(51)(10)(49)(8)$$

$$= 2,598,960$$

PROPERTIES OF COMBINATION

The Identity Property of Combination

The identity property of combination states that the combination of any set with the empty set results in the original set. In other words, if you have a set A and you add to it the empty set, then the resulting set is still A.

The Commutative Property of Combination

The commutative property of combination states that the order in which you combine two sets does not matter. So, if you have two sets A and B, then the combination A + B is the same as B + A.

The Associative Property of Combination

The associative property of combination states that when you have three sets, you can combine them in any order and get the same result. So, if you have sets A, B, and C, then the combination (A + B) + C is the same as A + (B + C).

The Distributive Property of Combination

The distributive property of combination states that when you have two sets and a third set that is a combination of those two sets, you can combine the first set with each element of the third set and get the same result as combining the first set with the third set. So, if you have sets A and B, and C is the combination of A and B, then the combination A + (B + C) is the same as (A + B) + C.

The Empty Set Property of Combination

The empty set property of combination states that the combination of any set with the empty set is the empty set. So, if you have a set A and you add to it the empty set, then the resulting set is the empty set.

The Subset Property of Combination

The subset property of combination states that if you have two sets A and B, and A is a subset of B, then the combination A + B is also a subset of B. So, if you have sets A and B, and A is a subset of B, then the combination A + B is also a subset of B.

The Superset Property of Combination

The superset property of combination states that if you have two sets A and B, and A is a superset of B, then the combination A + B is also a superset of B. So, if you have sets A and B, and A is a superset of B, then the combination A + B is also a superset of B.

(i) $^{n}C_0 = {}^{n}C_n = 1$

(ii) $^{n}C_1 = {}^{n}C_{n-1} = n$

(iii) $^{n}C_r = \dfrac{^{n}P_r}{r!}$

(iv) $^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

(v) $^{n}C_r = {}^{n}C_{n-r}$

(vi) $r\,{}^{n}C_{r-1} = (n-r+1)\,{}^{n}C_{r-1}$

APPLICATION OF COMBINATION

1. Statistics: Permutations and combinations are used in statistical analysis to calculate the probability of certain events occurring and to determine the likelihood of specific outcomes.

2. Cryptography: Permutations and combinations are used in cryptography to create secure passwords and encryption keys.

3. Sports: Permutations and combinations can be used in sports to analyze and optimize strategies, such as determining the best lineup of players for a particular game.

4. Medicine: Permutations and combinations are used in medicine to study the effectiveness of different combinations of treatments for different medical conditions.

5. Marketing: Permutations and combinations can be used to analyze the effectiveness of different marketing strategies and tactics combinations.

6. Manufacturing: Permutations and combinations can be used in manufacturing to analyze and optimize production processes, such as determining the most efficient order for assembling products.

7. Travel: Permutations and combinations can be used in travel to analyze and optimize travel routes and schedules, such as determining the best combination of flights and hotels for a particular trip.

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