"ज्ञान, विज्ञान आणि सुसंस्कार यांच्यासाठी शिक्षणाचा प्रसार"

शिक्षण महाराशी डॉ. बापूजी साळुंखे.

Shri Swami Vivekanan Shikshan Santha's

VIVEKANAND COLLEGE , KOLHAPUR.

DEPARTMENT OF MATHEMATICS

B.Sc.III ,2021-2022

Project work on

'TRANSPORATION METHODS'

BY

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Under the guidance of

Prof. S. P. Patankar

Prof. Sanjay Thorat

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I owe a great many thanks to a great many people who helped and supported me during the writing to this project.

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I express my thanks to Principle Dr. R. R. Kumbhar for extending his suneeded. I would also thank my college and faculty members without whom this project would have been a distance reality I also extend my heartful thanks to my family and well wishers.

DECLARATION

I, the undersigned hearby declared that project report entitled "TRANSPORATION METHODS" submitted by me to Vivekanad college, Kolhapur in partical fulfillment of the environment for the award of degree of B.Sc.III in Mathematics Department is record of bonafide project work carried out by me under the guidance of Prof. S. P. Patankar (HOD), Prof. Sanjay Thorat is my original work and interpretation drawn there in are based on material collection by my own self.

DATE

PLACE-KOLHAPUR

Vasundhara Atul Bendake.

VBendake

ROLL NO.-

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"ज्ञान, विज्ञान आणि सुसंस्कार यांच्यासाठी शिक्षणाचा प्रसार"

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श्री स्वामी विवेकानंद शिक्षण संस्थेचे विवेकानंद कॉलेज कोल्हापूर

CERTIFICATE

Department of Mathematics

This is to certify that, Miss Vasundhara Atul Benake. Student of Vivekanand college , Kolhapur has successfully completed a project on 'TRANSPORTATION METHODS' in B.sc.III year at Department of Mathematics in the year 2021-2022.

DATE-

Teacher-in-charge

Examiner

Head of Department

HEAD
Department of Mathematics
Vivekanand College, Kolhapur

THE TRANSPORTATION AND ASSIGNMENT PROBLEMS

we will discuss the transportation and assignment problems which are two special kinds of linear programming. The transportation problem deals with transporting goods from their sources to their destinations. The assignment problem, on the other hand, deals with assigning people or machines to jobs.

❖ The Transportation Problem

Example

Consider the following snow removal problem: there are a number of districts in a city. After a snowfall, the snow in each area must be moved out of the district into a convenient location. In a city, these locations are large grates (leading to the sewer system), a couple large pits, and a couple entry points to the river. Each of these destinations has a capacity. The goal is to minimize the distance traveled to handle all of the snow.

This problem is an example of a transportation problem. In such a problem, there are a set of nodes called sources (4 sources where snow is collected), and a set of nodes called destinations (3 destinations where snow will be transported). All arcs go from a source to a destination. There is a per unit cost on each arc (the cost of transporting one unit from a source to a destination). Each source has a supply of material, and each destination has a demand. We assume that the total supply equals the total demand (possibly adding a fake source or destination as needed). For the snow removal problem, the network might look like that in Fig.4.1

Transportation problems are often used in transportation planning. For instance, in an application where goods are at a warehouse, one problem might be to assign customers to a warehouse so as to meet their demands. In such a case, the warehouses are the sources, the customers are the destinations, and the costs represent the per unit transportation costs.

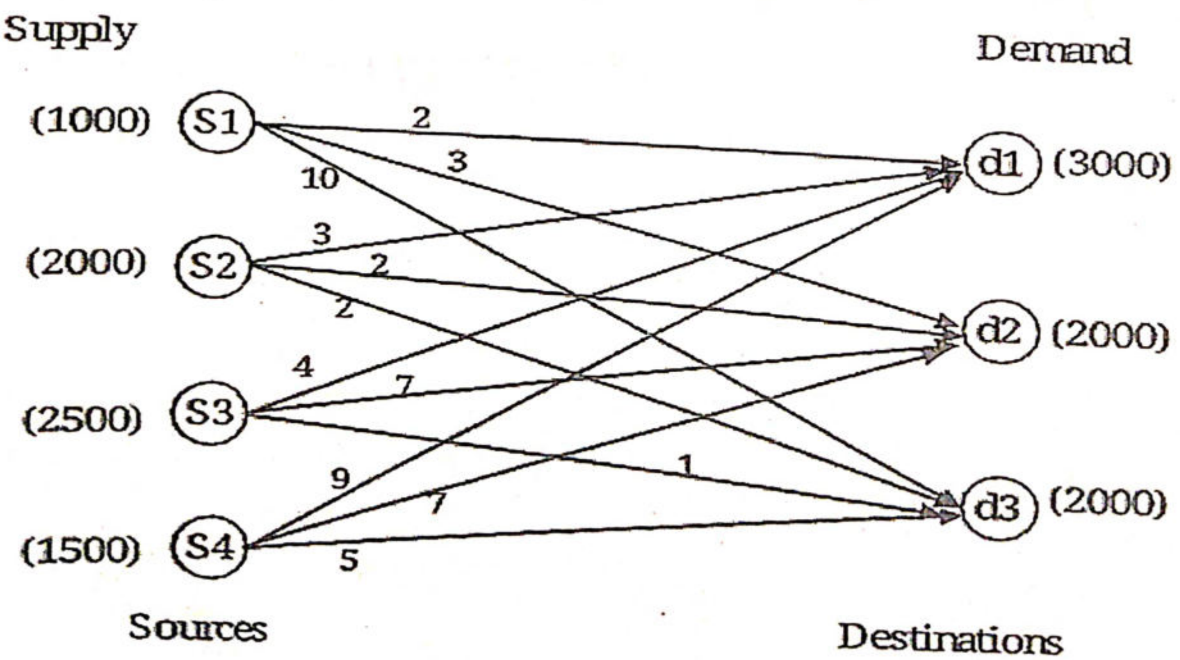

Figure 4.1: Snow transportation network

Example

One of the main products of Qaha Company is canned peas. The peas are prepared at three canneries (Qaha, Benha, and Tanta) and are then shipped by trucks to four distributing warehouses in Cairo, Alexandria, Portsaid, and Aswan. Because shipping costs are a major expense, management has begun a study to reduce them. For the upcoming season, an estimate has been made of what the output will be from each cannery, and how much each warehouse will require to satisfy its customers. The shipping costs from each cannery to each warehouse have also been determined. This is summarized in Table 4.1 and Fig. 4.2. Formulate this transportation problem as a linear programming problem by defining the variables, the objective and the constraints.

Table 4.1: Shipping data for example 4.2

		Warehouse Shipping cost per truck load				Supply
		1	2	3	4	
Cannery	1	464	513	654	867	75
	2	352	416	690	791	125
	3	995	682	388	685	100
Demand		80	65	70	85	

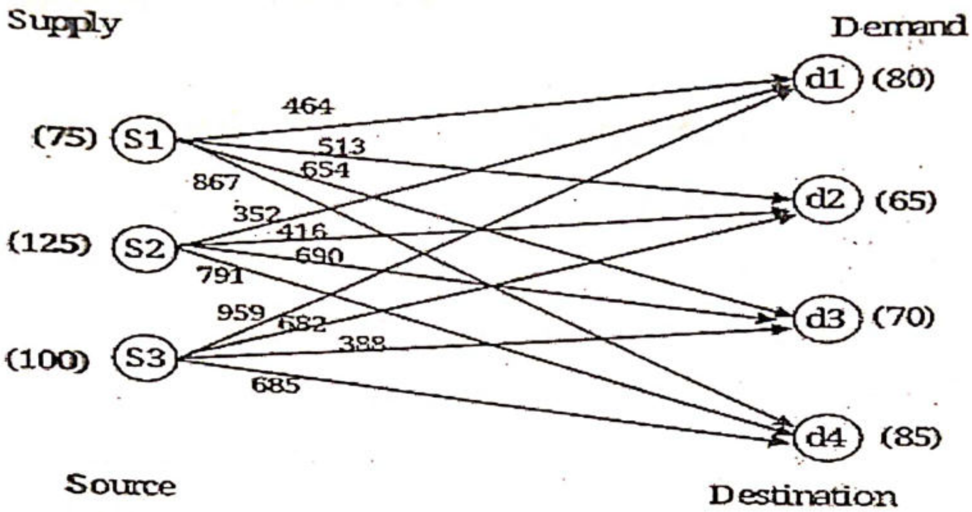

Figure 4.2: Network representation of example 4.2

You should find it an easy exercise to model this as a linear program. If we let *xij* be the number of truckloads shipped from cannery *i* to warehouse *j*, the problem is to:

$$minimize\ Z = 464x11 + 513x12 + 654x13 + 867x14 + 352x21$$

$$+416x22+690x23+791x24+995x31+682x32+388x33+685x34$$

Subject to

$$x11 + x12 + x13 + x14 = 75$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 125$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 100$$
$$x_{11} + x_{21} + x_{31} = 80$$
$$x_{12} + x_{22} + x_{32} = 65$$
$$x_{13} + x_{23} + x_{33} = 70$$
$$x_{14} + x_{24} + x_{34} = 85$$
$x_{ij} \geq 0$ for all i and j.

This is an example of the ***transportation model***. As has been pointed out, this problem has a specific structure. All the coefficients of the variables in the constraint equations are 1 and every variable appears in exactly two constraints. All constraint equations are formed using the equal sign not the $\geq$ or the $\leq$ signs. This is the special structure that distinguishes this problem as a transportation problem.

❖ *The Transportation Problem Model*

In general, the transportation model is concerned with distributing (literally or figuratively) a commodity from a group of supply centers, called sources to a group of receiving centers, called destinations to minimize total cost. In general, source i ($i = 1, 2, 3, \ldots, m$) has a supply of s_i units, and destination j ($j = 1, 2, 3, \ldots, n$) has a demand for d_j units. The cost of distributing items from a source to a destination is proportional to the number of units. This data can be conveniently represented in a table like that for the sample problem of example 4.2. Each source has a fixed supply of units, s_i, where the entire supply must be distributed to the destinations. Each destination has a fixed demand for units, d_j, and the entire demand must be received from the sources. The cost of distributing units from any particular source i to any particular destination j, c_{ij}, is directly proportional to the number of units distributed, x_{ij}, $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ from source i to destination j.

Any transportation model can be described completely by a parameter table as given in Table 4.2. In transportation problems every s_i and d_j has integer values and also all other variables.

Table 4.2: Transportation problem representation

		Cost per distributed unit				Supply
		Destination				
		1	2	...	n	
Source	1	c_{11}	c_{12}	...	c_{1n}	s_1
	2	c_{21}	c_{22}	...	c_{2n}	s_2
	:	:	:		:	:
	m	c_{m1}	c_{m2}	...	c_{mn}	s_m
Demand		d_1	d_2	...	d_n	

A typical transportation problem is shown in Table 4.2, using a matrix with the rows representing sources and columns representing destinations. The algorithms for solving the problem are based on this matrix representation. The costs of shipping from sources to destinations are indicated by the entries in the matrix. If shipment is impossible between a given source and destination, a large cost of M is entered. This discourages the solution from using such cells. Supplies and demands are shown along the margins of the matrix.

In general, Let Z represents the total distribution cost and *xij* (i = 1, 2,....., m; j = 1, 2, ..., n) be the number of units to be distributed from source i to destination j.

Minimize $$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to $$\sum_{j=1}^{n} X_{ij} = S_i \qquad i = 1, 2, \ldots\ldots, m$$

$$\sum_{i=1}^{m} X_{ij} = d_j \qquad j = 1, 2, \ldots\ldots, n$$

$$X_{ij} \geq 0 \qquad \text{for all } i \text{ and } j$$

For a transportation problem to posses a feasible solution, the following property indicates when this will occur. A necessary and sufficient condition for a transportation problem to have any feasible solution is that:

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} d_j$$

This property may be verified by observing that the constraints require that both

$$\sum_{i=1}^{m} S_i \quad \text{and} \sum_{j=1}^{n} d_j \quad \text{equal} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}$$

As shown, the number of functional constraints equals the number of sources (m) and destinations (n). However, number of basic variables $= m + n - 1$. The reason behind that, the functional constraints are equality constraints, and this set of $m + n$ equations has one extra (redundant) equation that can be deleted without changing the feasible region (i.e., any one of the constraints is automatically satisfied whenever the other $m + n - 1$ constraints are satisfied. This can be verified as any supply constraint exactly equals the sum of the demand constraints minus the sum of other supply constraints. ***Therefore, any basic feasible (BF) solution will have only m + n – 1 basic variables (non-zero variables) while all other variables will have zero value. Also, the sum of allocations for each row or each column equals its supply or demand.***

We will generally assume that the total supply equals the total demand. If this is not true for a particular problem, ***dummy*** sources or destinations can be added to make it true. The text refers to such a problem as a ***balanced transportation problem***. These dummy centers may have zero distribution costs, or costs may be assigned to represent unmet supply or demand. For example, suppose that cannery 3, in Example 4.2, makes only 75 truckloads. The total supply is now 25 units little. ***A dummy supply node can be added with supply 25 to balance the problem*** and the cost from the dummy to each warehouse can be added to represent the cost of not meeting the warehouse's demand.

In the pea shipping example, a basic solution might be to ship 20 truckloads from cannery 1 to warehouse 2 and the remaining 55 to warehouse 4, 80 from cannery 2 to warehouse 1 and 45 to warehouse 2 and, finally, 70 truckloads from cannery 3 to warehouse 3 and 30 to warehouse 4. Even though the linear programming formulation of the pea shipping example has seven constraints other than non-negativity, a basic solution has only six basic variables! This is because the constraints are linearly dependent: the sum of the first three is identical to the sum of the last four. As a consequence, the feasible region defined by the constraints would remain the same if we only kept six of them. In general, a basic solution to the transportation model will have a number of basic variables equal to the number of sources plus the number of destinations minus one.

Note that the resulting table of constraint coefficients has a special structure as shown below in Table 4.3. Any linear programming problem that fits this special formulation is classified as a transportation problem; this is why the transportation problem is generally considered a special type of linear programming problem.

Table 4.3: Constraint coefficients of the transportation problem

Table 4.3: Constraint coefficients of the transportation problem

A =

x_{11}	x_{12}	....	x_{1n}	x_{21}	x_{22}	....	x_{2n}	....	x_{m1}	x_{m2}	....	x_{mn}	
1	1	....	1										Supply constraints
				1	1	....	1						
									1	1	....	1	
1				1					1				Demand constraints
	1				1					1			
								....					
			1				1					1	

For many applications as the supply and demand quantities have integer values, all the basic variables in every basic feasible solution also have integer values.

Example

Assume three sources and three destinations with the cost of supply as given in Fig. 4.3. No transport between source S3 and destination D1, and Source S1 and destination D3. The matrix representation of this example is sown in Table 4.4.

Table 4.4: Matrix representation of the transportation problem

Table 4.4: Matrix representation of the transportation problem

		Destinations			Supply
		1	2	3	
	1	3	1	M	5
Sources	2	4	2	4	7
	3	M	3	3	3
Demand		7	3	5	

The network model of the transportation problem is shown in Fig. 4.3. Sources are identified as the nodes on the left and destinations on the right. Allowable shipping links are shown as arcs, while disallowed links are not included.

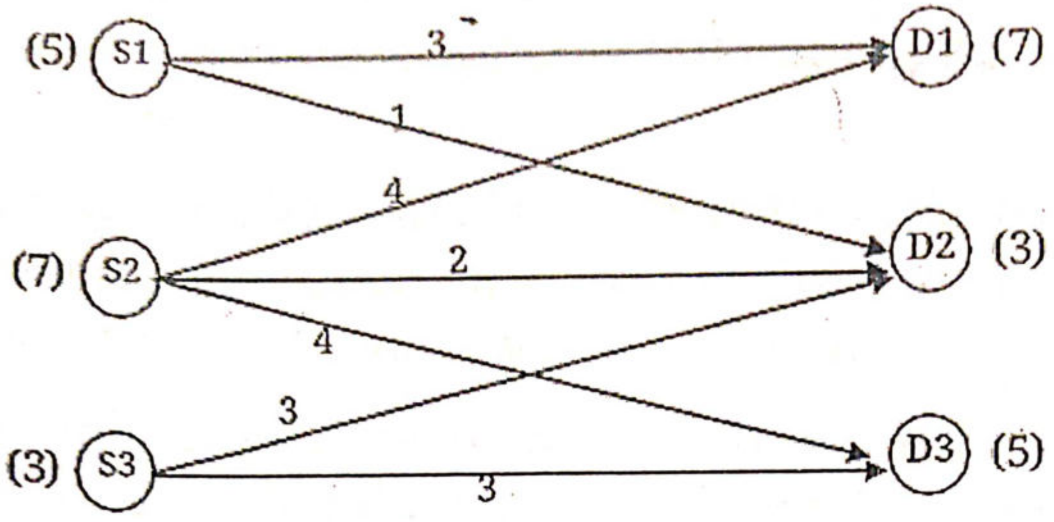

Figure 4.3: Network flow model of the transportation problem

On each supply node the positive external flow indicates supply flow entering the network. On each destination node a demand is a negative fixed external flow indicating that this amount must leave the network.

Solving the Transportation Problem Using Vogel's Approximation Method

General procedure for constructing an initial basic feasible solution (BFS):

To begin: all source rows and destination columns of the transportation simplex tableau are initially under consideration for providing a basic variable.

1. From the rows and columns still under consideration, select the next basic variable (allocation) according to some criterion.

2. Make the allocation large enough to exactly use up the remaining supply in its row or the remaining demand in its column (whichever is smaller).

3. Eliminate that row or column from further consideration. (If the row and column have the same remaining supply and demand, then arbitrarily select the row as the one to be eliminated).

4. If only one row or one column remains under consideration, then the procedure is completed by selecting every remaining variable associated with that row or column to be basic with the only feasible allocation. Otherwise, return to the first step.

Vogel's approximation method: For each row and column remaining under consideration, calculate its difference, the arithmetic difference between the smallest and next-to-the-smallest unit cost c_{ij} still remaining in that row or column. In that row or column having the largest difference, select the variable having the smallest remaining unit cost. Ties for the largest difference, or for the smallest remaining unit cost, broken arbitrarily.

Example

Find an initial basic feasible solution for the problem presented in Example 4.4 using *Vogel's* approximation method

Solution

(1)		Destination				Supply	Row diff.
		1	2	3	4		
Source	1	40	20 (20)	20	50	30	0
	2	20	50	10	60	50	10
Demand		20	20	30	10	$X_{12} = 20$	
Col. Diff.		20	30	10	10	Eliminate col. 2	

First step: The highest difference is in column 2. The least value in this column is 20. Accordingly, the first basic variable to be assigned a value is *x12*. Then *x12* = 20, the value is shown in the demand row. So, column 2 will be eliminated as its all demand is satisfied. Also, the supply value of row number 1 will be equal to 10 as a 20 of the original value has been assigned to variable *x12*

(2)		Destination			Supply	Row diff.
		1	3	4		
Source	1	40	20	50	10	20
	2	20 (20)	10	60	50	10
Demand		20	30	10	$X_{21} = 20$	
Col. Diff.		20	10	10	Eliminate col. 1	

Second step: The highest difference is in column 1 and row 1; this tie may be broken arbitrary by either choosing row1 or column 1. Choosing column 1, the least value in this column is 20. Accordingly, the second basic variable to be assigned a value is x_{21}. Then $x_{21} = 20$, the value is shown in the demand row. So, column 1 will be eliminated as its all demand is satisfied. Also, the supply value of row number 2 will be equal to 30 as a 20 of the original value has been assigned to variable x_{21}.

(3)		Destination		Supply	Row diff.
		3	4		
Source	1	20	50	10	30
	2	10	60	30	50
Demand		30	10	$X_{23} = 30$	
Col. Diff.		10	10	Eliminate row 2	

Third step: The highest difference is in row 2. The least value in this row is 10. Accordingly, the third basic variable to be assigned a value is x_{23}. Then $x_{23} = 30$, the value is shown in the demand row. So, row 2 will be eliminated as its all supply is used (i.e, equal zero). Also, all the demand of column 3 is satisfied. Accordingly, the last basic variable, x_{14}, equals 10. Hence, the basic variables are:

$x_{12} = 20$, $x_{21} = 20$, $x_{23} = 30$, $x_{13} = 0$, and $x_{14} = 10$

The sum of all basic variables = 20 + 20 + 30 + 10 = 80
$= \Sigma\, si = \Sigma\, dj$
Z = 20 x 20 + 20 x 20 + 30 x 10 + 10 x 50 = 1600 minute.

MODI METHOD

The MODI (*modified distribution*) method allows us to compute improvement indices quickly for each unused square without drawing all of the closed paths. Because of this, it can often provide considerable time savings over other methods for solving transportation problems.
MODI provides a new means of finding the unused route with the largest negative improvement index. Once the largest index is identified, we are required to trace only one closed path. This path helps determine the maximum number of units that can be shipped via the best unused route.

How to Use the MODI Method

In applying the MODI method, we begin with an initial solution obtained by using the northwest corner rule or any other rule. But now we must compute a value for each row (call the values $R1$, $R2$, $R3$ if there are three rows) and for each column ($K1$, $K2$, $K3$) in the transportation table. In general, we let
The MODI method then requires five steps:
1. To compute the values for each row and column, set
$Ri + Kj = Cij$

but *only for those squares that are currently used or occupied*. For example, if the square at the intersection of row 2 and column 1 is occupied, we set $R2 + K1 = C21$.

2. After all equations have been written, set $R1 = 0$.
3. Solve the system of equations for all R and K values.
4. Compute the improvement index for each unused square by the formula improvement index $(Iij) = Cij \_ Ri \_ Kj$.
5. Select the largest negative index and proceed to solve the problem as you did using the stepping-stone method.

Solving the Arizona Plumbing Problem with MODI

Let us try out these rules on the Arizona Plumbing problem. The initial northwest corner solution is shown in Table T4.1. MODI will be used to compute an improvement index for each unused square.

Note that the only change in the transportation table is the border labeling the *Ri s* (rows) and *Kj s* (columns).

We first set up an equation for each occupied square:

1. $R1 + K1 = 5$
2. $R2 + K1 = 8$
3. $R2 + K2 = 4$
4. $R3 + K2 = 7$
5. $R3 + K3 = 5$

Letting $R1 = 0$, we can easily solve, step by step, for $K1$, $R2$, $K2$, $R3$, and $K3$.

1. $R1 + K1 = 5$

$0 + K1 = 5$ $K1 = 5$

2. $R2 + K1 = 8$

$R2 + 5 = 8$ $R2 = 3$

3. $R2 + K2 = 4$

$3 + K2 = 4$ $K2 = 1$

4. $R3 + K2 = 7$

$R2 + 1 = 7$ $R3 = 6$

5. $R3 + K3 = 5$

$6 + K3 = 5$ $K3 = -1$

R_i	K_j — FROM \ TO	K_1 ALBUQUERQUE	K_2 BOSTON	K_3 CLEVELAND	FACTORY CAPACITY
R_1	DES MOINES	[5] 100	[4]	[3]	100
R_2	EVANSVILLE	[8] 200	[4] 100	[3]	300
R_3	FORT LAUDERDALE	[9]	[7] 100	[5] 200	300
	WAREHOUSE REQUIREMENTS	300	200	200	700

values to occur as well. After solving for the *R*s and *K*s in a few practice problems, you may become so proficient that the calculations can be done in your head instead of by writing the equations out.
The next You can observe that these *R* and *K* values will not always be positive; it is common for zero and negativestep is to compute the improvement index for each unused cell. That formula is
improvement index $= Iij = Cij$ _ Ri _ K We have:

We have:

$$\text{Des Moines–Boston index} = I_{DB} \text{ (or } I_{12}) = C_{12} - R_1 - K_2 = 4 - 0 - 1$$
$$= +\$3$$
$$\text{Des Moines–Cleveland index} = I_{DC} \text{ (or } I_{13}) = C_{13} - R_1 - K_3 = 3 - 0 - (-1)$$
$$= +\$4$$
$$\text{Evansville–Cleveland index} = I_{EC} \text{ (or } I_{23}) = C_{23} - R_2 - K_3 = 3 - 3 - (-1)$$
$$= +\$1$$
$$\text{Fort Lauderdale–Albuquerque index} = I_{FA} \text{ (or } I_{31}) = C_{31} - R_3 - K_1 = 9 - 6 - 5$$
$$= -\$2$$

Because one of the indices is negative, the current solution is not optimal. Now it is necessary to trace only the one closed path, for Fort Lauderdale–Albuquerque, in order to proceed with the solution procedures.

The steps we follow to develop an improved solution after the improvement indices have been computed are outlined briefly:

1. Beginning at the square with the best improvement index (Fort Lauderdale–Albuquerque), trace a closed path back to the original square via squares that are currently being used.
2. Beginning with a plus (+) sign at the unused square, place alternate minus (_) signs and plus signs on each corner square of the closed path just traced.
3. Select the smallest quantity found in those squares containing minus signs. *Add* that number

to all squares on the closed path with plus signs; *subtract* the number from all squares assigned minus signs.

4. Compute new improvement indices for this new solution using the MODI method.

FROM \ TO	A	B	C	FACTORY
D	$5; 100	$4	$3	100
E	$8; 100	$4; 200	$3	300
F	$9; 100	$7	$5; 200	300
WAREHOUSE	300	200	200	700

FROM \ TO	A	B	C	FACTORY
D	$5; 100	$4	$3	100
E	$8	$4; 200	$3; 100	300
F	$9; 200	$7	$5; 100	300
WAREHOUSE	300	200	200	700

Following this procedure, the second and third solutions to the Arizona Plumbing Corporation problem can be found. See Tables T4.2 and T4.3. With each new MODI solution, we must recalculate the *R* and *K* values. These values then are used to compute new improvement indices in order to determine whether further shipping cost reduction is possible

VOGEL'S APPROXIMATION METHOD: ANOTHER WAY TO FIND AN INITIAL SOLUTION

In addition to the northwest corner and intuitive lowest-cost methods of setting an initial solution to transportation problems, we introduce one other important technique—*Vogel's approximation method* (VAM). VAM is not quite as simple as the northwest corner approach, but it facilitates a very good initial solution—as a matter of fact, one that is often the *optimal* solution.

Vogel's approximation method tackles the problem of finding a good initial solution by taking into account the costs associated with each route alternative. This is something that the northwest corner rule did not do. To apply the VAM, we first compute for each row and column the penalty faced if we should ship over the *second best* route instead of the *least-cost* route

FROM \ TO	Warehouse at Albuquerque	Warehouse at Boston	Warehouse at Cleveland	Factory Capacity
Des Moines factory	$5	$4	$3	100
Evansville factory	$8	$4	$3	300
Fort Lauderdale factory	$9	$7	$5	300
Warehouse requirements	300	200	200	700

Des Moines capacity constraint

Cell representing a source-to-destination (Evansville to Cleveland) shipping assignment that could be made

Cleveland warehouse demand

Total demand and total supply

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

Transportation Table for Arizona Plumbing Corporation

The six steps involved in determining an initial VAM solution are illustrated on the Arizona Plumbing Corporation data. We begin with Table T4.4.

VAM Step 1: For each row and column of the transportation table, find the difference between the two lowest unit shipping costs. These numbers represent the difference between the distribution cost on the *best* route in the row or column and the *second best* route in the row or column. (This is the *opportunity cost* of not using the best route.)

Step 1 has been done in Table T4.5. The numbers at the heads of the columns and to the right of the rows represent these differences. For example, in row *E* the three transportation costs are $8, $4, and $3. The two lowest costs are $4 and $3; their difference is $1.

VAM Step 2: Identify the row or column with the greatest opportunity cost, or difference. In the case of Table T4.5, the row or column selected is column *A*, with a difference of 3.

	3	0	0		
FROM \ TO	ALBUQUERQUE *A*	BOSTON *B*	CLEVELAND *C*	TOTAL AVAILABLE	
DES MOINES *D*	5	4	3	100	1
EVANSVILLE *E*	8	4	3	300	1
FORT LAUDERDALE *F*	9	7	5	300	2
TOTAL REQUIRED	300	200	200	700	

FROM \ TO	A	B	C	TOTAL AVAILABLE	
	1	3	2		
D	5 100	4 X	3 X	100	X
E	8	4	3	300	1
F	9	7	5	300	2
TOTAL REQUIRED	300	200	200	700	

VAM Step 3: Assign as many units as possible to the lowest cost square in the row or column selected.

Step 3 has been done in Table T4.6. Under Column *A*, the lowest-cost route is *D–A* (with a cost of $5), and 100 units have been assigned to that square. No more were placed in the square because doing so would exceed *D*'s availability.

VAM Step 4: Eliminate any row or column that has just been completely satisfied by the assignment just made. This can be done by placing *X*s in each appropriate square.

Step 4 has been done in Table T4.6 *D* row. No future assignments will be made to the *D–B* or *D–C* routes.

VAM Step 5: Recompute the cost differences for the transportation table, omitting rows or columns crossed out in the preceding step.

This is also shown in Table T4.6. *A*'s, *B*'s, and *C*'s differences each change. *D*'s row is eliminated, and *E*'s and *F*'s differences remain the same as in Table T4.5.

VAM Step 6: Return to step 2 and repeat the steps until an initial feasible solution has been Obtained

	~~3~~ 1	~~0~~ 3	~~0~~ 2		
FROM \ TO	A	B	C	TOTAL AVAILABLE	
D	5 100	4 X	3 X	100	~~1~~
E	8	4 200	3	300	~~1~~ 5
F	9	7 X	5	300	~~2~~ 4
TOTAL REQUIRED	300	200	200	700	

In our case, column *B* now has the greatest difference, which is 3. We assign 200 units to the lowest-cost square in column *B* that has not been crossed out. This is seen to be *E–B*. Since *B*'s requirements have now been met, we place an X in the *F–B* square to eliminate it. Differences are once again recomputed. This process is summarized in Table T4.7.

The greatest difference is now in row *E*. Hence, we shall assign as many units as possible to the lowest-cost square in row *E*, that is, *E–C* with a cost of \$3. The maximum assignment of 100 units depletes the remaining availability at *E*. The square *E–A* may therefore be crossed out. This is illustrated in Table T4.8.

The final two allocations, at *F–A* and *F–C*, may be made by inspecting supply restrictions (in the rows) and demand requirements (in the columns). We see that an assignment of 200 units to *F–A* and 100 units to *F–C* completes the table (see Table T4.9).

The cost of this VAM assignment is = (100 units · \$5) + (200 units · \$4) + (100 units · \$3) + (200 units · \$9) + (100 units · \$5) = \$3,900.

It is worth noting that the use of Vogel's approximation method on the Arizona Plumbing Corporation data produces the optimal solution to this problem. Even though VAM takes many more calculations to find an initial solution than does the northwest corner rule, it almost always produces a much better initial solution. Hence VAM tends to minimize the total number of computations needed to reach an optimal solution.

FROM \ TO	A	B	C	TOTAL AVAILABLE
D	5 100	4 X	3 X	100
E	8 X	4 200	3 100	300
F	9 200	7 X	5 100	300
TOTAL REQUIRED	300	200	200	700

References :

1) Dr. Emad elbeltagi : google scholar citations

2) www.google.com