A Project Submitted to

Vivekanand College, Kolhapur(Autonomous)

(स्वायत्त) कोल्हापूर

Affiliated to
Shivaji University, Kolhapur
For the Degree of Bachelor of Science
In Mathematics
By

ASHAM IMAM PANHALKAR

Roll No :- 8321

Exam Seat No :

B.Sc III(Mathematics)
2022-23

Project name:- APPLICATIONS OF ORDINARY DIFFERENTIAL EQUATIONS

Under the Guidance of

Mr. Thorat S.P.

Head Of The Department Of Mathematics

"Education For Knowledge ,Science & culture"
-Shikshanmaharshi Dr. Bapuji Salunkhe.

Shri Swami Vivekanand Shikshan Sanstha's,

Vivekanand College, Kolhapur (Autonomous)

DEPARTMENT OF MATHEMATICS

CERTIFICATE

This is to certify that Mr./Ms./Mrs. **Asham Imam Panhalkar** has successfully completed the project work on topic "Applications of Oridinary Differential Equations" towards the partial fulfilment for the course of Bachelor of Science (Mathematics) work of Vivekanand College , Kolhapur(Autonomous) during the academic year 2022-2023. This report represents the bonafide work of student.

Place : Kolhapur

Date :

SThorat

Mr. S. P. Thorat

Head Of Dept. Of Mathematics

HEAD
Department of Mathematics
Vivekanand College, Kolhapur

DECLERATION

I undersigned hereby declare that project entitled "Applications of Ordinary Differential Equations". Completed under the guidance of Mr.S.P. Thorat sir. (Department of Mathematics Vivekanand College (Autonomous), Kolhapur). Based on the experiment results and cited data. I declare that this is my original work which is submitted to Vivekanand College, Kolhapur in this academic year.

Mr./Ms: Asham Imam Panhalkar

ACKNOWLEDGEMENT

On the day of completion of this project, the numerous memories agreeing rushed in my mind with full of gratitude to this encouraged and helped me a lot at various stages of this work.

I offer sincere gratitude to all of them. I have great pleasure to express my deep sense of indebtedness and heart of full gratitude to my project guide Mr. S.P. Thorat sir. For his expert and valuable guidance and continuous encouragement given to me during the course of project work.

I am thankful to Dr. R.R. Kumbhar sir (Principal, Vivekanand College, Kolhapur) and Mr. S.P. Thorat sir (Head of Dept. of Mathematics) for allowing me to carry out our project work and extending me all the possible infra-structural facilities of department.

I would like to thank all my teachers Ms. S.M. Malavi, Ms. P.P. Kulkarni, Mr. A.A. Patil, Ms. M.G. Goliwadekar and Mr. G.B. Kolhe for co-operation, help and maintaining cheerful environment during my project.

I would also like to thanks non-teaching staff Mr. D.J. Birnale.

I would like to thanks my entire dear friends for their constant encouragement and co-operation .1 am indebted to my parents who shaped me to this status with their blunt less vision and selfness agenda.

Place :- Kolhapur

Date :-

Mr./Ms. :- Asham Imam Panhalkar

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ABSTRACT

In this project we have discussed few methods to solve first order differential equations. We solve in this chapter first-order differential equations modelling phenomena of cooling, population growth, radioactive decay, mixture of salt solutions, series circuits, survivability with AIDS, draining a tank, economics and finance, drug distribution, pursuit problem and harvesting of renewable natural res

INTRODUCTION

A first order differential equation is an equation

$$dy/dx = f(x,y) \qquad (1)$$

in which f(x,y) is a function of two variables defined on a region in the xy-plane. The equation is of first order because it involves only the first derivative dy/dx (and not higher-order derivatives). we point out that the equations

$$Y' = f(x,y) \text{ and } dy/dx = f(x,y)$$

are equivalent to Equation (1) and all three forms will be used interchangeably in the text.

A solution of Equation (1) is a differentiable function y = y(x) defined on an interval I of x-values (perhaps infinite) such that

$$d/dx[y(x)] = f(x,y(x))$$

On that interval. That is when y(x) and its derivative y'(x) are substituted into Equation (1), the resulting equation is true for all x over the interval I. The general solution to a first-order differential equation is a solution that contains all possible solutions. The general solution always contains an arbitrary constant, but having this property doesn't mean a solution is the general solution. That is, a solution may contain an arbitrary constant without being the general solution Establishing that a solution is the general solution may require deeper results from the theory of differential equations and is the best studied in more advanced course.

POPULATION GROWTH AND DECAY

The differential equation,

$$dN(t)/dt=kN(t)$$

Where N(t) denotes population at time t and k is a constant of proportionality, serves as a model for population growth and decay of insects, animals and human population at certain places and duration. Solution of this equation is

$N(t)=Ce^{kt}$, where C is the constant of integration.

$dN(t)/N(t)=kdt$

Integrating both sides we get

$In(t)=kt+In\ C$

Or $In(N(t)/C)=kt$

Or $N(t)=Ce^{kt}$

C can be determined if N(t) is given at certain time.

Example:

The population of a community is known to increase at a rate proportional to the number of people present at a time t. If the population has doubled in 6 years, how long it will take to triple?

Solution:

Let N(t) denote the population at time t. Let N(0) denote the initial population (population at t=0).

$\frac{dN}{dt} \propto N(t)$

$t=6$

$N(6)=2N(0)$

dN/dt=kN(t)

Solution is N(t)=Ae^kt, where A=N(0)

Ae^6k=N(6)=2N(0)=2A

Or e^6k=2 or k=(1/6)ln2

Find t when N(t)=3A=3N(0)

or $N(0)\, e^{kt}=3N(0)$

or $3 = e^{(1/6)(\ln 2)t}$

or in $3 =((\ln 2)t)/6$

or $t=(6\ln 3)/\ln 2=9.6$ years (approximately 9 years 6 months)

Example:

Let population of country be decreasing at the rate proportional to Its population. If the population has decreased to 25% in 10 years, how long will it take to be half?,

Solution:

This phenomenon can be modeled by $dN/dt=kN(t)$

Its solution is

$N(t)=N(O)\, e^{kt}$, where

$N(0)$ in the initial population

For $t=10$, $N(10)=(1/4)\,N(0)$

$(1/4)N(0) = N(0)\,e^{10k}$

or $e^{10k}= 1/4$

or $k=(1/10)\ln(1/4)$

Set $N(t)=(1/4)\,N(0)$

$N(0)e^{kt}=(1/2)N(0)$

Or $t= 8.3$ years approximately.

CARBON DATING

Carbon Dating:-

The key to the carbon dating of paintings and other materials such as fossils and rocks lies in the phenomenon of radioactivity discovered at the turn of the century. The physicist Rutherford and his colleagues showed that the atoms of certain radioactive elements are unstable and that within a given time period a fixed portion of the atoms spontaneously disintegrate to form atoms of a new element. Because radioactivity is a property of the atom, Rutherford showed that the radioactivity of a substance is directly proportional to the number of atoms of the substance present. Thus, if $N(t)$ denotes the number of atoms present at time t, then dN/dt , the number of atoms that disintegrate per unit time, is proportional to N; that is,

$$dN/dt = -\lambda N \qquad (1)$$

The constant λ, which is positive, is known as the decay constant of the substance. The larger λ is, the faster the substance decays.

To compute the half life of substance in terms of A, assume that at time t=to, N(to)=No. The solution of the initial value problem

dN/dt= - λN

N(to)= No (2)

is

N(t)=Noe^- λ (t-to)

Or (N/ No)e^ -λ (t-to)

Taking logarithms of both sides we obtain

-λ (t-to)=ln(N/ No) (3)

If N/No =1/2 , then – λ(t-to)=ln(1/2) , so that

t-to= (ln2)/ λ =0.6931/ λ

Thus the half life of a substance is ln2 divided by the decay constant λ.

The half-life of many substances have been determined and are well published. For example, half-life of carbon-14 is 5568 years, and the half-life of uranium 238 is 4.5 billion years.

Remark :-

a) in (1) λ is positive and is decay constant. We may write equation (1) in the form

dN/dt= λN , where is λ negative constant, that is, λ <0.

b) The dimension of X is reciprocal time. It t is measured in years, then λ has the dimension of reciprocal years, and if t is measured in minutes, then X has the dimension of reciprocal minutes.

c) From (3) we can solve for

$$t-t_o =(1/ \lambda)\ln(N/N_o) \quad (4)$$

If to is the time the substance was initially formed or manufactured, then the age of the substance is (t-to). The decay constant λ is known or can be computed in most cases. N can be computed quite usually. Computation or pre-knowledge of No will yield the age of the substance.

By the Libby's discovery discussed in Section 1.4.2. the present rate R(t) of disintegration of the C-14 in the sample is given by

$$R(t)= \lambda N(t)= \lambda N_o e^{-\lambda t}$$

and the original rate of disintegration is

$$R(o)= \lambda N_o.$$

Thus

$R(t)/ R(0)= e^{-\lambda t}$ so that

$$T= (1/ \lambda)\ln(R(o)/R(t)) \qquad (5)$$

d) If we measure R(t), that present rate of disintegration of the C-14 in the charcoal and observe that R(o) must equal the rate of disintegration of the C-14 in the comparable amount of living wood then we can compute the age t of the charcoal.

e) The process of estimating the age of an artifact is called **carbon dating.**

Example:

Suppose that we have an artifact, say a piece of fossilized wood, and measurements show that the ratio of C-14 to carbon in the sample is 37% of the current ratio. Let us assume that the wood died at time 0, then compute the time T it would take for one gram of the radio active carbon to decay this amount.

Solution:

We know that,

$dm/dt = km$

This is a separable differential equation. Write it in the form

$(1/m)dm = kdt$

Integrate it to obtain

$\ln|m| = kt + c$

Since mass is positive, $|m| = m$ and

$\ln(m)=kt+c$.

Then

$m(t) = e^{kt}=c=Ae^{kt}$, where $A = e$ is positive constant. Let at some time, designated at time zero, there are M grams present. This is called the initial mass. Then

$m(o) = A = M$, so

$m(t) = Me^{kt}$

If at some later time T we find that there are MT grams, then

$m(T) = MT = Me\ T$

Then

$Ln(MT/M)=kT$

Hence

$K= (1/T)\ln(MT/M)$

This gives us k and determines the mass at any time:

$m(t) = Me^k$

Let $T=\lambda$ be the time at which half of the mass has radiated away, that is, half-life. At this time, half of the mass remains, so $MT=M/2$ and $MT/M =1/2$

Now the expression for mass becomes,

$m(t) = Me^{(t/\lambda)\ln(1/2)}$

or $m(t) = Me^{(t/\lambda)\ln 2}$

Half-life of C-14 is 5600 years approximately, that is,

$\lambda =5600$

$\ln 2/5600 =-0.00012378$

≈ means approximately equal (all decimal places are not listed).

Therefore,

m(t)=Me^-0.00012378t

or m(t)/M =0.37=e^-000012378t

by the given condition that M(t)/M is .37 during t.

T=- In(0.37)/ 0.00012378 = 8031 years approximately..

DRUG CONCENTRATION IN

HUMAN BODY

To combat the infection to human a body appropriate dose of medicine is essential. Because the amount of the drug in the human body decreases with time medicine must be given in multiple doses. The rate at which the level y of the drug in a patient's blood decays can be modeled by the decay equation.

$dy/dt=-ky$

where, k is a constant to be experimentally determined for each drug. If initially, that is, at t=0 a patient is given an initial dose Yp. then the drug level y at any time t is the solution of the above differential equations, that is,

$Y(t)=Ype^{-kt}$

Example:

A representative of a pharmaceutical company recommends that a new drug of his company be given every T hours in doses of quantity y_o, for an extended period of time. Find the steady state drug in the patient's body.

Solution:

Since the initial dose is y_o, the drug concentration at any time

$t \geq 0$ is found by the equation $y=y_o e^{-kt}$, the solution of the equation

$dy/dt= -ky$

At $t=T$ the second dose of y_o is taken, which increases the drug

level to

$y(T)=y_o+y_o e^{-kt}=y_o(1+e^{-kt})$

The drug level immediately begins to decay. To find its mathematical expression we solve the initial-value problem:

$dy/dt = -ky$

$y(T)=y_o(1+e^{-kT})$

Solving this initial value problem we get

$y=y_o(1+e^{-kT})e^{-k(t-T)}$

This equation gives the drug level for $t>T$. The third dose of yo is to be taken at $t=2T$ and the drug just before this dose is taken is given by

$Y-Y_o=(1+e^{-kT})e^{-k(2T-T)} = y_o(1+e^{-KT})e^{-KT}$

The dosage yo taken at $t=2T$ raises the drug level to

$y(2T) = y_o+ y_o(1+e^{-kT})e^{-kT} = y_o(1+e^{-kT} + e^{-2kt})$

Continuing in this way, we find after (n+1)th dose is taken that

the drug level is,

$y(nT)=y_o(1+e^{-kT}+e^{-2kT}+.....+e^{-nkT})$

We notice that the drug level after (n+1)th dose is the sum of the first n terms of a geometric series, with first term as yo and the common ratio ex. This sum can be written as

$$Y(nT) = Yo(1-e^{-(n+1)kT})/1-e^{-KT}$$

As n becomes large, the drug level approaches a steady state value,

say Ys given by,

$$Ys = \lim y(nT)$$

n trends to infinity

$$Ys=Yo/ 1-e^{-kT}$$

The steady state value Ys, is called the saturation level of the drug.

CONCLUSION

- In the above project that is "Applications of Ordinary Differential Equation" we conclude after its study that, it is a gearing application in many aspects and sections related to human development.

- Some of the vital applications discussed above are population growth, radio-active decay and carbon dating, drug concentration in human body. we can cure the particular person from the side effects of drugs like this, it plays an important role in medical field also.

- problems like population growth can be minimized if we have a previous idea about its increment which is possible by differential equation.

- Along with it, the concentration of drugs can be successfully estimated in human body and before any drastic problem.

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