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13 October 2021

onstant

$\therefore$ total linear momentum of system of particles is conserved.

12:11 pm

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12:16 pm

Torque:- time rate of change of ~~linear~~ angular momentum is is called torque and it is denoted by $\bar{N}$

$$\bar{N} = \frac{d\bar{L}}{dt} = \frac{d}{dt}\,\bar{r}\times\bar{P} = \frac{d}{dt}(\bar{r}\times m\dot{\bar{r}})$$

$$= \dot{\bar{r}}\times m\dot{\bar{r}} + \bar{r}\times m\ddot{\bar{r}}$$

$$= 0 + \bar{r}\times m\ddot{\bar{r}}$$

$$= \bar{r}\times m\bar{a}$$

$$\bar{N} = \bar{r}\times\bar{F}$$

$i\times i$

$i\cdot i = 1$

$\bar{a}\cdot\bar{b} = |a|\cdot|b|\cdot\cos\theta$

$\bar{a}\times\bar{b} = |a|\cdot|b|\sin\theta\cdot\hat{n}$

* Angular momentum of system of particles:-

Consider the system of n particles of masses $m_1, m_2, \dots m_n$ and position vector $\bar{r}_1, \bar{r}_2, \dots \bar{r}_n$.

$\therefore$ Angular momentum of i^{th} particle is

$$\bar{L}_i = \bar{r}_i\times\bar{P}_i.$$

Total angular momentum of system is

$$\bar{L} = \sum_{i=1}^{n}\bar{L}_i = \sum_{i=1}^{n}\bar{r}_i\times\bar{P}_i.$$

Now total torque on the system is

$$\bar{N} = \frac{d}{dt}\bar{L} = \frac{d}{dt}\sum_i \bar{r}_i\times\bar{P}_i$$

$$= \sum_i \dot{\bar{r}}_i\times\bar{P}_i + \sum_i \bar{r}_i\times\dot{\bar{P}}_i$$

F_{ij} j

$-F_{ji}$

$r_j\times F_{ji}$

$r_i\times F_{ij}$

$r_j\times(-F_{ji})$

$(r_i - r_j)\times F_{ji}$

Now $\bar{P}_i = m_i\dot{\bar{r}}_i$

$$\therefore \sum_i \dot{\bar{r}}_i\times m_i\dot{\bar{r}}_i = 0$$

$$\therefore \bar{N} = 0 + \sum_i \bar{r}_i\times m_i\ddot{\bar{r}}_i$$

$$\bar{N} = \sum_i \bar{r}_i\times\bar{F}_i \quad \text{---} \; (1)$$

$$\text{Now } \bar{N} = \sum_i r_i\times\left(\bar{F}_i^{(e)} + \sum_{j=1}^{n} F_{ji}^{(int)}\right)$$

$$= \sum_i r_i\times F_i^{(e)} + \sum_i \bar{r}_i\times\sum_j F_{ji}^{(int)} \quad \text{---} \; (2)$$

$$\bar{N} = \sum_i r_i\times F_i^{(e)} + \sum_{i,j}\bar{r}_i\times F_{ji}^{(int)}$$

Consider 2^{nd} term

$$\sum_{i,j}\bar{r}_i\times F_{ji}^{(int)} = (r_2 - r_1)\times\bar{F}_{12}^{(int)} + (r_3 - r_1)\times\bar{F}_{13}^{(int)} + (r_4 - r_1)\times\bar{F}_{14}^{(int)} + \dots$$

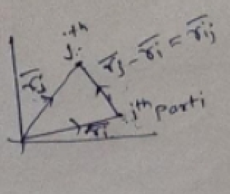
$$= \sum_{i,j}(\bar{r}_i - \bar{r}_j)\times\bar{F}_{ij}^{(int)}$$

$$= \sum_{i,j}\bar{r}_{ij}\times\bar{F}_{ij}^{(int)} \quad \text{---} \; (3)$$

1:07 pm

$$\bar{N} = \sum_i \bar{r}_i\times F_i^{(e)} + \sum_{i,j}\bar{r}_{ij}\times\bar{F}_{ij}^{(int)}.$$

where $\bar{r}_{ij} = \bar{r}_i - \bar{r}_j$ is along line joining i^{th} particle to j^{th} particle



Similarly $F_{ji}^{(int)}$ is also lies on the line joining i^{th} particle and j^{th} particle

i.e $\bar{r}_{ij}$ and $F_{ji}^{(int)}$ are collinear vectors.

$$\therefore \bar{r}_{ij}\times F_{ij}^{(int)} = 0$$

$$\therefore \bar{N} = \sum_i \bar{r}_i\times F_i^{(e)}$$

$$\bar{N} = \bar{N}^{(e)} \quad \text{---} \; (4)$$

For the system of particles.

* ~~Law~~ Conservation theorem for angular momentum:-

Statement:- If total external torque applied on system of particle is zero then the angular

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Torque:- time rate of change of ~~linear~~ angular momentum is called torque and it is denoted by $\bar{N}$

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$$= \sum_i\dot{\bar{r}}_i\times\bar{P}_i + \sum_i\bar{r}_i\times\dot{\bar{P}}_i$$

Now $\bar{P}_i = m_i\dot{\bar{r}}_i$

$$\therefore \sum_i\dot{\bar{r}}_i\times m_i\dot{\bar{r}}_i = 0$$

$$\therefore \bar{N} = 0 + \sum_i\bar{r}_i\times m_i\ddot{\bar{r}}_i$$

$$\bar{N} = \sum_i\bar{r}_i\times\bar{F}_i \quad \text{--- (1)}$$

F_{ij} j

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$r_j\times F_{ji}$

$r_i\times F_{ij}$

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$$\text{Now } \bar{N} = \sum_i r_i\times\left(\bar{F}_i^{(e)} + \sum_{j=1}^{n}F_{ji}^{(int)}\right)$$

$$= \sum_i r_i\times F_i^{(e)} + \sum_i\bar{r}_i\times\sum_j F_{ji}^{(int)} \quad \text{--- (2)}$$

$$\bar{N} = \sum_i r_i\times F_i^{(e)} + \sum_{ij}\bar{r}_i\times F_{ji}^{(int)}$$

Consider 2nd term

$$\sum_{ij}\bar{r}_i\times F_{ji}^{(int)} = (r_2 - r_1)\times\bar{F}_{12}^{(int)} + (r_3 - r_1)\times\bar{F}_{13}^{(int)} + (r_4 - r_1)\times\bar{F}_{14}^{(int)} + \ldots$$

$$= \sum_{ij}(\bar{r}_i - \bar{r}_j)\times\bar{F}_{ij}^{(int)}$$

$$= \sum_{ij}\bar{r}_{ij}\times\bar{F}_{ij}^{(int)} \quad \text{--- (3)}$$

1:07 pm

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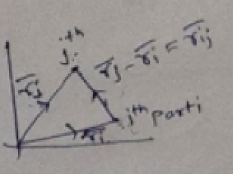
where $\bar{r}_{ij} = \bar{r}_i - \bar{r}_j$ is along line joining ith particle to jth particle



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$$\bar{N} = \bar{N}^{(e)} \quad \text{--- (4)}$$

For the system of particles.

* Conservation theorem for angular momentum:-
Statement:- If total external torque applied on the system of particle is zero then the angular momentum of system of particle is conserved.

$\Rightarrow$ Given $\bar{N}^{(e)} = 0 \Rightarrow \bar{N} = 0$

$$\frac{d}{dt}\bar{L} = 0.$$

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~ Rutu +91 70309 82280
Sir.. When you will provide the notes?
2:35 pm

constant).

* Constraints and constrained motion:–
Sometime the motion of system is not free i.e it is limited by putting some restriction on position co-ordinates of the particle involved in the system then the motion under such restriction is called constrained motion. and the restrictions are called as constraints. and the mathematical relation between position co-ordinates due to this restriction is called constraint relation.

ex. 1] Motion of particle on circle
2] Motion of simple pendulum
3] Motion of any curve on the plane

2:35 pm

* Classification of constraints:–
I] Classification based on nature of constraint relation:–
1] Holonomic constraints:–
If the constraints relation of the system is described by the equation of the form

$$f_j(q_i, t) = 0$$

where q_i is position co-ordinate and t is time then the constraints are holonomic constraints.
ex Simple pendulum, motion of rigit body and so on.

2] Non-holonomic constraints:–
The constraints which are not holonomic constraints

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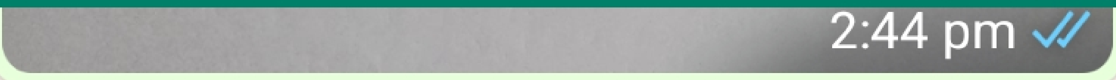

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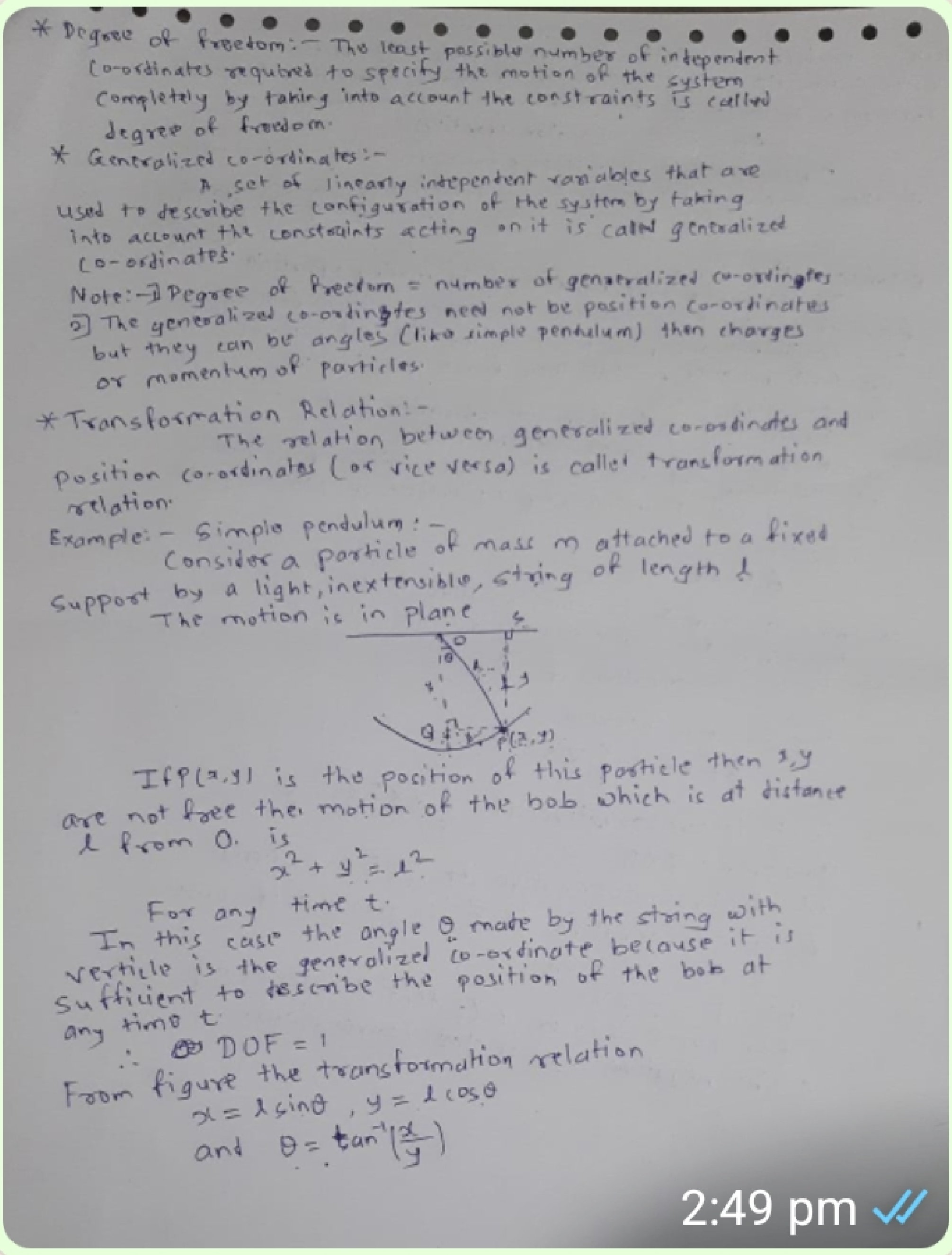


* Degree of freedom:– The least possible number of independent co-ordinates required to specify the motion of the system completely by taking into account the constraints is called degree of freedom.

* Generalized co-ordinates:–
A set of linearly independent variables that are used to describe the configuration of the system by taking into account the constraints acting on it is called generalized co-ordinates.

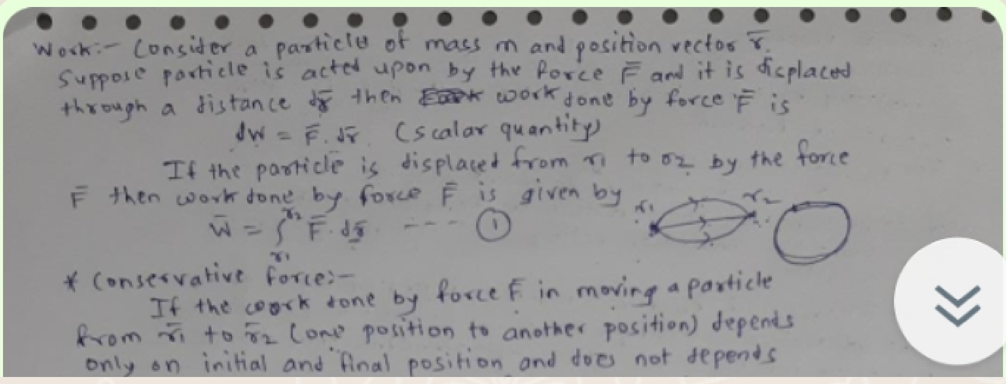
Note:– 1] Degree of freedom = number of generalized co-ordinates
2] The generalized co-ordinates need not be position co-ordinates but they can be angles (like simple pendulum) then charges or momentum of particles.

* Transformation Relation:–
The relation between generalized co-ordinates and position co-ordinates (or vice versa) is called transformation relation.

Example:– Simple pendulum:–
Consider a particle of mass m attached to a fixed support by a light, inextensible, string of length l
The motion is in plane

If $P(x,y)$ is the position of this particle then x, y are not free the motion of the bob which is at distance l from O. is
$$x^2 + y^2 = l^2$$
For any time t.
In this case the angle θ made by the string with verticle is the generalized co-ordinate because it is sufficient to describe the position of the bob at any time t
$\therefore$ DOF = 1
From figure the transformation relation
$$x = l\sin\theta, \quad y = l\cos\theta$$
and $\theta = \tan^{-1}\left(\frac{x}{y}\right)$

2:49 pm



Work:– Consider a particle of mass m and position vector $\bar{r}$.
Suppose particle is acted upon by the force $\bar{F}$ and it is displaced through a distance $d\bar{r}$ then ~~Each~~ work done by force $\bar{F}$ is
$$dW = \bar{F}\cdot d\bar{r} \quad \text{(scalar quantity)}$$
If the particle is displaced from r_1 to r_2 by the force $\bar{F}$ then work done by force $\bar{F}$ is given by
$$\bar{W} = \int_{r_1}^{r_2} \bar{F}\cdot d\bar{r} \quad \text{--- (1)}$$
r_1 r_2

* Conservative force:–
If the work done by force $\bar{F}$ in moving a particle from $\bar{r}_1$ to $\bar{r}_2$ (one position to another position) depends only on initial and final position and does not depends

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1:28 pm

⇒ gravitational force is conse...

* Virtual Displacement:-
A virtual displacement of a system refers to a change in the configuration of the system as a result of any arbitrary infinitesimal change in the co-ordinate (say δr_i) consistant with force F and constraints imposed on the system at given time instant t.

Note:- Actual displacement occures in a time interval dt but virtual displacement occures at a fixed time instant t. This change in denoted by δ and $\delta t = 0$ because time is fixed.

* Virtual Work:- The work done by the force F in causing the virtual displacement δr_i is called virtual work.

2:29 pm

* Principle of virtual work:-
If the system is in equillibrium (i.e. total force on each particle is zero (vanishesh) i.e. $\bar{F}_i = 0$) then virtual work of the force $\bar{F}_i$ during virtual displacement δr_i also vanishesh (zero)
i.e. $\bar{F}_i \, \delta \bar{r}_i = 0$

$$\sum_{i=1}^{n} \bar{F}_i \cdot \delta \bar{r}_i = 0 \quad \text{--- (1)}$$

Note:- The principle of virtual work is applicable only in statics i.e for system in equillibraium. The analogus principle in dynamics is proposed by D'-Alembert's

* D'-Alembert's principle:-
Equation of motion of a particle is
$\bar{F}_i = \dot{\bar{P}}_i \rightarrow (\bar{F}_i - \dot{\bar{P}}_i) = 0$
which can state that the system will be in equillibrium under ~~action~~ the force equal to actual force and reverse effective force $(-\dot{\bar{P}}_i)$
$\therefore$ By principle of virtual work

$$\sum_{i=1}^{n} (\bar{F}_i - \dot{\bar{P}}_i) \cdot \delta \bar{r}_i = 0 \quad \text{--- (2)}$$

This is mathematical form of D'-Alembert's principle statement:-
"A system of particle moves in a such a way that the total virtual work done by the applied force and reverse effective force is zero."

Note:- All the Laws in mechanics can be derived from D'-Alembert's principle hence it is called fundamental principle of mechanics.

Q. Use D'-Alembert's principle to find equation of motion of simple pendulum.
⇒ Consider a particle of mass m and position vector $\bar{r} = (x, y)$ attached to a fixed support by a rigid string of length l.
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1:32 pm

* Generalized velocity :-

From the transformation relation we have

$$\bar{r}_i = \bar{r}_i(q_1, q_2, \cdots q_n, t) = \bar{r}_i(q_j, t)$$

where $q_1, q_2, \cdots q_n$ are generalized co-ordinates

differentiating we get

$$\dot{\bar{r}}_i = \sum_j \frac{\partial r_i}{\partial q_j}\dot{q}_j + \frac{\partial r_i}{\partial t} \quad \text{---} (1)$$

This is expression for velocity of i^{th} particle and the terms $\dot{q}_j$ is called generalized velocity

* δ-variation of $\dot{\bar{r}}_i$:-

From eq^n (1)

$$\delta\bar{r}_i = \sum_j \frac{\partial r_i}{\partial q_j}\delta q_j + \frac{\partial r_i}{\partial t}\delta t$$

$$\delta r_i = \sum_j \frac{\partial r_i}{\partial q_j}\delta q_j \quad \cdots [\delta t = 0 \text{ for virtual displacement})$$

* Generalized force :-

If $\bar{F}_i$ is force acting on ith particle whose position vector is $\bar{r}_i$ then the virtual work done by all this $\bar{F}_i$ is

$$\delta W = \sum_i \bar{F}_i \, \delta r_i$$

$$= \sum_i F_i \sum_j \frac{\partial r_i}{\partial q_j}\delta q_j$$

$$= \sum_j \left(\sum_i \bar{F}_i \frac{\partial r_i}{\partial q_j}\right)\delta q_j$$

$$= \sum_j Q_j \, \delta q_j$$

where $Q_j = \sum_i \bar{F}_i \frac{\partial r_i}{\partial q_j}$

is the component of ~~gen~~ generalized force

* Lagrange's equation of motion :-

Q. Derive Lagrange's equation of motion from D-Alembert's principle.

Ans :- Consider a system of particles of masses m_i and position vector $\bar{r}_i$. If $q_1, q_2, \cdots q_n$ are generalized co-ordinates then the position vector $\bar{r}_i$ are given by

$$\bar{r}_i = \bar{r}_i(q_1, q_2, \cdots q_n, t)$$

$$\bar{r}_i = \bar{r}_i(q_j, t)$$

$$\delta r_i = \sum_j \frac{\partial r_i}{\partial q_j}\delta q_j$$

From D'-Alembert's principle

2:49 pm

$$\sum_i (F_i - \dot{P}_i) \cdot \delta r_i = 0$$

$$\sum_i F_i \, \delta r_i = \sum_i \dot{P}_i \, \delta r_i$$

$$\sum_i F_i \sum_j \frac{\partial r_i}{\partial q_j}\delta q_j = \sum_i m_i \ddot{\bar{r}}_i \, \delta r_i$$

$$\sum_j \left(\sum_i F_i \frac{\partial r_i}{\partial q_j}\right)\delta q_j = \sum_i m_i \ddot{\bar{r}}_i \, \delta r_i$$

$$\sum_j Q_j \, \delta q_j = \sum_i m_i \ddot{\bar{r}}_i \, \delta r_i$$

$$\sum_j Q_j \, \delta q_j = \sum_{i,j} m_i \left[\ddot{\bar{r}}_i \frac{\partial r_i}{\partial q_j}\right]\delta q_j \quad \text{---} (1)$$

where $Q_j = \sum_i F_i \frac{\partial r_i}{\partial q_j}$ is component of generalized force.

Consider $\cancel{\frac{d}{dt}\left(\dot{\bar{r}}_i \cdot \frac{\partial r_i}{\partial q_j}\right)}$

$$\frac{d}{dt}\left(\dot{\bar{r}}_i \frac{\partial r_i}{\partial q_j}\right) = \ddot{\bar{r}}_i \frac{\partial r_i}{\partial q_j} + \dot{\bar{r}}_i \cdot \frac{d}{dt}\left(\frac{\partial r_i}{\partial q_j}\right) \quad \text{----} (2)$$

From eq^n (2), eq^n (1) becomes

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Now we consider different cas[illegible]

Case I :- System is conservative

In this case $\overline{F_i} = -\nabla_i V = -\frac{\partial}{\partial r_i} V = -\frac{\partial V}{\partial r_i}$

where V is potential energy of the system

$\therefore Q_j = \sum_i F_i \frac{\partial r_i}{\partial q_j} = \sum_i -\frac{\partial V}{\partial r_i} \cdot \frac{\partial r_i}{\partial q_j} = \frac{-\partial V}{\partial q_j}$

eqn (10) becomes

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = \frac{-\partial V}{\partial q_j}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial}{\partial q_j}(T-V) = 0 \quad \text{--- (11)}$$

1:11 pm

For conservative system the potential energy V will not depends on the generalized velocity $\dot{q}_j$

$$V = V(q_j)$$

i.e. $\frac{\partial V}{\partial \dot{q}_j} = 0.$

$\therefore$ eqn (11) becomes

$$\frac{d}{dt}\left[\frac{\partial}{\partial \dot{q}_j}(T-V)\right] - \frac{\partial}{\partial q_j}(T-V) = 0$$

Define $L = T - V$. a Lagrangian of given conservative system $L = L(q_j, \dot{q}_j, t)$

$$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0 \quad \text{--- (12)}$$

This is Lagrange's equation of motion for conservative holonomic system.

Case II :- For non-conservative system of particle:-

For non-conservative system of particle Lagrange's equation of motion is given by eqn (10)

In this case the potential energy will depend on generalized velocity

i.e $V = V(q_j, \dot{q}_j, t)$

In some cases [eg a charged particle in electromagnetic field] the component of generalized force can be expressed as

$$Q_j = -\frac{\partial V}{\partial q_j} + \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{q}_j}\right) \quad \text{--- (*)}$$

$\therefore$ eqn (10) becomes

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = -\frac{\partial V}{\partial q_j} + \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{q}_j}\right)$$

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{q}_j}(T-V)\right) - \frac{\partial}{\partial q_j}(T-V) = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0 \quad \text{--- (13)}$$

Note:- For a non-conservative system the Lagrange's equation of motion is given by eqn (10) not by [illegible] conservative

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Ex. Show that the Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot q_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$$

can also be written as

$$\frac{\partial \dot T}{\partial \dot q_j} - 2\frac{\partial T}{\partial q_j} = Q_j$$

⇒ The kinetic energy of system is

$$T = T(q_j, \dot q_j, t)$$

$$\dot T = \sum_k \frac{\partial T}{\partial q_k}\dot q_k + \sum_k \frac{\partial T}{\partial \dot q_k}\ddot q_k + \frac{\partial T}{\partial t}$$

$$\frac{\partial \dot T}{\partial \dot q_j} = \frac{\partial T}{\partial q_j} + \sum_k \frac{\partial^2 T}{\partial \dot q_j \partial q_k}\dot q_k + \sum_k \frac{\partial^2 T}{\partial \dot q_j \partial \dot q_k}\ddot q_k + \frac{\partial^2 T}{\partial \dot q_j \partial t}$$

$$\frac{\partial \dot T}{\partial \dot q_j} = \frac{\partial T}{\partial q_j} + \sum_k \left[\frac{\partial^2 T}{\partial \dot q_j \partial q_k}\dot q_k + \frac{\partial^2 T}{\partial \dot q_j \partial \dot q_k}\ddot q_k\right] + \frac{\partial^2 T}{\partial \dot q_j \partial t} \quad \text{--- (1)}$$

Now we find

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot q_j}\right) = \sum_k \frac{\partial^2 T}{\partial q_k \partial \dot q_j}\dot q_k + \sum_k \frac{\partial^2 T}{\partial \dot q_k \partial \dot q_j}\ddot q_k + \frac{\partial^2 T}{\partial t\, \partial \dot q_j} \quad \text{--- (2)}$$

Substracting (2) from (1)

$$\frac{\partial \dot T}{\partial \dot q_j} - \frac{d}{dt}\left(\frac{\partial T}{\partial \dot q_j}\right) = \frac{\partial T}{\partial q_j} \quad \text{--- (3)}$$

Now Lagrange's eqⁿ is

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot q_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$$

∴ eqⁿ (3) becomes

$$\frac{\partial \dot T}{\partial \dot q_j} - \left[\frac{\partial T}{\partial q_j} + Q_j\right] = \frac{\partial T}{\partial q_j}$$

$$\frac{\partial \dot T}{\partial \dot q_j} - 2\frac{\partial T}{\partial q_j} = Q_j$$

which is required equation.

Q. Show that if force acting on the particle is conservative then the total energy of the particle is conserved.

⇒ Consider a particle of mass m. Let $\bar F$ be a conservative force acting on particle.
Suppose that particle is displaced from position P_1 to P_2 under action of force $\bar F$
then the workdone is

$$W = \int_{P_1}^{P_2} \bar F \cdot d\bar r \quad \text{--- (1)}$$

1:07 pm

By Newton's 2nd law of motion.

$$\bar F = m\bar a = \dot{\bar p} = m\ddot{\bar r}$$

$$W = \int_{P_1}^{P_2} m\ddot{\bar r}\, d\bar r = \int_{P_1}^{P_2} m\ddot{\bar r}\frac{d\bar r}{dt}\, dt$$

$$= \int_{P_1}^{P_2} m\ddot{\bar r}\dot{\bar r}\, dt$$

$$= \int_{P_1}^{P_2} m\frac{1}{2}\frac{d}{dt}(\dot{\bar r}^2)\, dt$$

$$= \int_{P_1}^{P_2} \frac{d}{dt}\left(\frac{1}{2}m\dot r^2\right) dt = \int_{P_1}^{P_2} d\left(\frac{1}{2}m\dot r^2\right)$$

$$= \left[\frac{1}{2}m\dot r^2\right]_{P_1}^{P_2} = \left[\frac{1}{2}mv^2\right]_{P_1}^{P_2} = [T]_{P_1}^{P_2}$$

$$W = T_2 - T_1 \quad \text{--- (2)}$$

where T_i is kinetic energy at point P_i

also $\bar F$ is conservative force

$$\bar F = -\nabla V = -\frac{\partial V}{\partial r}$$

∴ From eqⁿ (1)

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