"Dissemination of Education for Knowledge, Science and Culture"
-Shikshanmaharshi Dr. Bapuji Salunkhe

Shri Swami Vivekanand Shikshan Sanstha, Kolhapur

Vivekanand College, Kolhapur (Autonomous)

Department of Physics

M.Sc. Part- I

Mathematical Physics

Surprise Test

Date : 05/11/2021 **Total Marks: 20**

Day: - Tuesday **Time :- 3pm to 4pm**

Instructions:-

1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Use of log table and calculator is allowed.

Q. 1 Select most correct alternative **(05)**

i) The product of Eigen values of matrix is equal to it's .....

a) Trace b) Determinant c) Eigen vector d) Characteristics equation

ii) Every matrix satisfies it's ....

a) Characteristic equation b) Eigen values

c) Eigen vector d) All of above

iii) If the determinant of matrix is zero then the matrix is called as ......

a) Singular matrix b) Identity matrix

c) Equal matrix d) Periodic matrix

iv) The sum and product of the eigen values of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are respectively...

(*a*) 7 and 7 (*b*) 7 and 5 (*c*) 7 and 6 (*d*) 7 and 8

v) Multiplication of matrices is ......

a) commutative b) not commutative c) not associative d) determinant

Q.2) Attempt any One **(10)**

i) Find the inverse of following matrix

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

ii) Find the Eigen values of the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Q.3) Attempt any One **(5)**

i) State and prove Cayley-Hamilton theorem

ii) Explain inner product and outer product of matrix with suitable example.

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Vivekanand College, Kolhapur (Autonomous)
Department of Physics
M.Sc. Part- I
Mathematical Physics
Surprise Test
Attendance Sheet

Date : 05/11/2021

Roll. No.	Name of Candidate	Sign
1601	Vishwambar S. Bandar	Bangar
1602	Jayashri Madhukar Chvan	Chavan
1603	Shivani Suryakant Desai	Desai
1604	Ankita Ravindra Digraje	D Ankita
1605	Mahesh M. Gadekar	Gadekar
1606	Aarti kumar Jadhav	Jadhav
1607	Dhanani C. Jadhav	DhaJ
1608	Pooja S. Jadhav	PJ.
1609	Priyanka Jitendra Kadam	Kadam
1610	Sayali Jayasing Patil	Patil
1614	Preeyanka Gundoji Patel	PPatel
1616	Shital Vilas Patil	Spatil
1618	Patil snehal Sanjay	SS patil
1620	Seema Rajesh Tivale	Tivale

Teacher Incharge ShindeAV
(Mr. A.Y. Shinde)

Head of the
Department of Physics
Vivekanand College, Kolhapur

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Vivekanand College, Kolhapur (Autonomous)

Department of Physics

M.Sc. Part- I

Mathematical Physics

Surprise Test

Result

Date : 05/11/2021

Roll. No.	Marks	Roll. No.	Marks
1601	12		
1602	04		
1603	07		
1604	09		
1605	16		
1606	12		
1607	11		
1608	10		
1609	07		
1610	02		
1611	19		
1612	-		
1613	-		
1614	14		
1615	-		
1616	16		
1617	-		
1618	18		
1619	-		
1620	09		

Teacher Incharge......ShindeAV......................
(Mr. A. V. Shinde)

Head of the
Department of Physics
Vivekanand College, Kolhapur

VIVEKANAND COLLEGE KOLHAPUR ESTD JUNE 1964 (Autonomous)

Name - Jarkoli smit kallappa.

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-शिक्षणमहर्षी डॉ. बापूजी साळुंखे

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VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT		Signature of Supervisor	
Suppliment No. :	16/20	Subject	: Mathematical physics
Roll No.	: 1605	Test / Tutorial No. :	internal exam
Class	: M.Sc I (physic)	Div. :	

Q. 1

(I) The product of eigen value of matrix is equal to its ~~determinant~~ Trace

(II) Every matrix satisfies its All of above

(III) if the determinant of matrix is zero then the matrix is called as ~~singular~~ Identify matrix

(IV) the sum & product of the eigen value of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are respectively 7&5

(V) multiplication of matrices is commutative

Q. 2

(1) $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$|A| = 3(-3+4) + 3(2) + 4(-2)$

$= 3(1) + 6 - 8$

$= 3 + 6 - 8$

$= 1$

$|A| = 1$

10

$A11 = (-1)^{1+1}[1]$

$A12 = (-1)^{1+2}[2]$

$A13 = (-1)^{1+3}[-2]$

$B11 = (-1)^{2+1}[-1]$

$B12 = (-1)^{2+2}[3]$

$B13 = (-1)^{2+3}[3]$

$C11 = (-1)^{3+1}[0]$

$C12 = (-1)^{3+2}[-4]$

$C13 = (-1)^{3+3}[-3]$

Cofactors A $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}$

adj A $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A.$

$= \frac{1}{1} \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

$$A^{-1} = 1 \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$A^{-1} \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

Q.3
(ii) Inner product and outer product of matrix :

Inner product :- Inner product is multiplication of one product to another product and thus is called a inner product.

(1) $A \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$ $B \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$

$$AB = \begin{bmatrix} 1\times1+2\times3 & 1\times1+2\times3 \\ 3\times1+4\times3 & 3\times2+4\times4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1+6 & 1+6 \\ 3+12 & 6+8 \end{bmatrix}$$

5

$$AB = \begin{bmatrix} 7 & 7 \\ 15 & 14 \end{bmatrix}_{2\times 2}$$

This is the inner product of matrix A & B.

(2) $A \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}$ $B \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}$

$$AB \begin{bmatrix} 1+8+21 & 2+10+24 & 3+12+27 \\ 4+20+42 & 8+25+48 & 12+30+36 \\ 7+32+63 & 14+40+72 & 21+48+81 \end{bmatrix}$$

$$AB \begin{bmatrix} 30 & 36 & 42 \\ 66 & 81 & 78 \\ 102 & 126 & 150 \end{bmatrix}_{3\times 3}$$

this is inner product examples

Name - Smith Kallappa Jarkoli

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SUPPLIMENT

Signature of Supervisor	
Suppliment No. : 1.	Subject : Mathematical physics
Roll No. : 1605	Test / Tutorial No. : internal exam
Class : M.SC I (physics)	Div. :

outer product of matrix :- one matrix multiply by another matrix B A matrixies inner order product of another matrix B outer multiplication.

example :- outer product of matrix :

$$A \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} & 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \\ 3\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} & 4\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 3 & 4 & 6 & 8 \\ 3 & 6 & 4 & 8 \\ 9 & 12 & 12 & 16 \end{bmatrix}$$

this the example of output product of matrix

Q-2

(ii) $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$|A - \lambda I| =$

$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix}$

1 Extra

$(2-\lambda)[(3-\lambda)(2-\lambda) - 2] - [2(1)(2-\lambda) + 1(1)] + 1[1(2) - 1(3-\lambda)]$

$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$(\lambda-1)(\lambda^2 - 6\lambda + 5) = 0$

$(\lambda-1)(\lambda^2 - 3\lambda - 2\lambda + 5) = 0$

$(\lambda-1)(\lambda-2)(\lambda-3)$

$$\begin{array}{r|rrrr} 1 & 1 & -7 & 11 & -5 \\ & & 1 & -6 & 5 \\ \hline & 1 & -6 & 5 & \boxed{0} \end{array}$$

$\boxed{\lambda = 1, 2, 3}$ eigen values of matrix

Shweta Sardar Mithari

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SUPPLIMENT

Signature of Supervisor	
Subject : Mathematical physics	
Test / Tutorial No. :	
Div. :	

18/20

Suppliment No. :

Roll No. : 1608

Class : MSc - I

Q. 1

i) b) Determinant

ii) d) All of above

iii) a) Singular matrix

iv) c) 7 and 6

v) b) Not commutative

Q.2

i) Given that,

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\text{Cofactor of matrix } A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\therefore A_1 = (-1)^{1+1}(-3+4) = +1$$
$$A_2 = (-1)^{1+2}(2-0) = -2$$
$$A_3 = (-1)^{1+3}(-2-0) = -2$$
$$B_1 = (-1)^{2+1}(-3+4) = -1$$
$$B_2 = (-1)^{2+2}(3-0) = 3$$
$$B_3 = (-1)^{2+3}(-3-0) = -3$$
$$C_1 = (-1)^{3+1}(-12+12) = 0$$
$$C_2 = (-1)^{3+2}(12-8) = -4$$
$$C_3 = (-1)^{3+3}(-9+6) = -3$$

$$\therefore \text{Cofactor of matrix } A = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & -3 \\ 0 & -4 & -3 \end{bmatrix}$$

$$\therefore Adj \cdot A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & -3 & -3 \end{bmatrix}$$

but we have,

$$A^{-1} = \frac{1}{|A|} \cdot Adj \cdot A \qquad \text{------ (I)}$$

$$\therefore |A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$$

$$= 3(-3+4)+3(2-0)+4(-2\cancel{\times}0)$$
$$= 3\times1+3\times2+4\times-2$$
$$= 3+6-8$$
$$= 9-8$$
$$= 1$$

$\therefore$ eq^n (I) becomes,

$$A^{-1} = \frac{1}{1} \times \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & -3 & -3 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & -3 & -3 \end{bmatrix}$$

---

ii) Given,

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

We know that, $|A - \lambda I| = 0$

$$\therefore \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} = 0$$

3

Extra

$$\therefore (2-\lambda)\left[(3-\lambda)(2-\lambda) - 2\right] - 2\left[(2-\lambda) - 1\right] + 1\left[2-(3-\lambda)\right] = 0$$

$$(2-\lambda)\left[6-3\lambda-2\lambda+\lambda^2-2\right] - 2\left[1-\lambda\right] + 1\left[(1-\lambda)\right] = 0$$

$$(2-\lambda)\left[\lambda^2-5\lambda-4\right] - 2 + 2\lambda + 1 - \lambda = 0$$

$$(2-\lambda)\left[\lambda^2-5\lambda-4\right] - 1 + \lambda = 0$$

$$2\lambda^2 - 10\lambda - 8 - \lambda^3 - 5\lambda^2 + 4\lambda - 1 + \lambda = 0$$

$$-3\lambda^2 - 5\lambda - 9 - \lambda^3 = 0$$

$$\lambda^3 + 3\lambda^2 + 5\lambda + 9 = 0.$$

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SUPPLIMENT	Signature of Supervisor
Suppliment No. :	Subject :
Roll No. :	Test / Tutorial No. :
Class :	Div. :

Q.3

ii) Inner Product and Outer product of matrix

Let us consider a matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ & $B = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

In inner product matrix there is multiplication of matrix A & B with ~~Ro~~row to column.

Here we have,

$$A \cdot B = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times3+1 & 0+0 \\ 2\times3+3\times1 & 2\times2+3\times1 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 4 & 0 \\ 9 & 7 \end{bmatrix}$$

Here, $A \cdot B = B \cdot A$

In these matrix multiplication the order of matrices are in same order.

Outer product of matrix

In these matrix we can multiply every no. with indivisually.

Let us consider two matrices,

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} & 2 \times \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} \\ 3 \times \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} & 0 \times \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & 2+8 \\ 3+2 & 6+4 \\ 3+12 & 0+0 \\ 9+6 & 0+0 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 10 \\ 5 & 10 \\ 15 & 0 \\ 15 & \end{bmatrix}$$

5

i) Cayley - Hamilton Theorem

For. e.g. $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$\therefore \; |A - \lambda I| = \begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix}$

$= 2-\lambda\left[(3-\lambda)(2-\lambda)-2\right] - 2\left[(2-\lambda)-1\right] + 1\left[2-(3-\lambda)\right] = 0$

$= 2-\lambda\left[6-3\lambda-2\lambda+\lambda^2-2\right] - 2\left[1-\lambda\right] + 1\left[(1-\lambda)\right] = 0$

$= 2-\lambda\left[\lambda^2-5\lambda-4\right] - 2+2\lambda+1-\lambda = 0$

$= 2\lambda^2 - 10\lambda - 8\lambda^3 - 5\lambda^2 + 4\lambda - 1 + \lambda = 0$

$= -3\lambda^2 - 5\lambda - 9 - \lambda^3 = 0$

$= \lambda^3 - 3\lambda^2 + 5\lambda + 9 = 0$

put $\lambda = 0$

$\therefore \quad A^3 - 3A^2 + 5A + 9I = 0 \quad \text{------- eq}^n \text{ (I)}$

We can find,

$A^2 = A \cdot A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2\times2+2\times1+1\times1 & 2\times2+2\times3+2\times2 & 1\times1+1\times1+1\times2 \\ 1\times2+1\times1+1\times1 & 3\times2+3\times3+3\times2 & 1\times1+1\times1+1\times2 \\ 1\times2+1\times1+1\times1 & 2\times2+2\times3+2\times2 & 2\times1+2\times1+2\times2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & 14 & 4 \\ 4 & 20 & 4 \\ 4 & 14 & 8 \end{bmatrix}$

$$\therefore A^2 \cdot A = \begin{bmatrix} 7 & 14 & 4 \\ 4 & 20 & 4 \\ 4 & 14 & 8 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7\times2+7\times1+7\times1 & 14\times2+14\times3+14\times2 & 4\times1+4\times1+4\times2 \\ 4\times2+4\times1+4\times1 & 20\times2+20\times3+20\times2 & 4\times1+4\times1+4\times2 \\ 4\times2+4\times1\times4\times1 & 14\times2+14\times3\times14\times2 & 8\times1+8\times1+8\times2 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 28 & 98 & 16 \\ 16 & 100 & 16 \\ 16 & 98 & 32 \end{bmatrix}$$

$\therefore$ From eq^n (I)

$$A^3 - 3A^2 + 5A + 9I = 0$$

$$\begin{bmatrix} 28 & 98 & 16 \\ 16 & 100 & 16 \\ 16 & 98 & 32 \end{bmatrix} - 3\begin{bmatrix} 7 & 14 & 4 \\ 4 & 20 & 4 \\ 4 & 14 & 8 \end{bmatrix} + 5\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 28 & 98 & 16 \\ 16 & 100 & 16 \\ 16 & 98 & 32 \end{bmatrix} - \begin{bmatrix} 21 & 42 & 12 \\ 12 & 60 & 12 \\ 12 & 42 & 24 \end{bmatrix} + \begin{bmatrix} 10 & 10 & 5 \\ 5 & 15 & 5 \\ 5 & 10 & 10 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = 0$$

$$\begin{bmatrix} 7 & 56 & 4 \\ 4 & 40 & 4 \\ 4 & 56 & 8 \end{bmatrix} + \begin{bmatrix} 19 & 10 & 5 \\ 5 & 24 & 5 \\ 5 & 10 & 19 \end{bmatrix} = 0$$

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VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Subject : Mathematical method in Physics.

Suppliment No. : —

16/20

Test / Tutorial No. : —

Roll No. : 1612

Div. : —

Class : M.Sc - I

Q.1

1) — A] Trace

2) — D] All of the above

3) — B] Identity matrix

4) — A] 7 and 7.

5) — B] not commutative

Q. 2.

2) — $\cancel{A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}}$ $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$\therefore \quad A - \lambda I = \begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix}$

Now, $|A - \lambda I| = \begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix}$

$= (2-\lambda)\left[(3-\lambda)(2-\lambda) - 2\right] - 2\left[(2-\lambda) - 1\right] + 1\left[2 - (3-\lambda\right.$

$= (2-\lambda)\left[6 - 2\lambda - 3\lambda + \lambda^2 - 2\right] - 2\left[1 - \lambda\right] + 1\left[\lambda - 1\right]$

$= (2-\lambda)(\lambda^2 - 5\lambda + 4) - 2 + 2\lambda + \lambda - 1$

$= 2\lambda^2 - 10\lambda + 8 - \lambda^3 + 5\lambda^2 - 4\lambda + 3\lambda - 3$

10

$= -\lambda^3 + 7\lambda^2 - 11\lambda + 5$

$\therefore \quad |A - \lambda I| = \lambda^3 - 7\lambda^2 + 11\lambda - 5$

We know,

$|A - \lambda I| = 0$

$\therefore \quad \lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$$\begin{array}{c|cccc} 1 & 1 & -7 & 11 & -5 \\ & & 1 & -6 & 5 \\ \hline & 1 & -6 & 5 & \boxed{0} \end{array}$$

$\therefore (\lambda - 1)(\lambda^2 - 6\lambda + 5) = 0$

$\therefore \quad \lambda - 1 = 0 \quad$ or $\quad \lambda^2 - 6\lambda + 5 = 0$

$\therefore \quad \lambda = 1 \quad\quad \therefore (\lambda - 5)(\lambda - 1) = 0$

$\therefore \quad \lambda = 5, 1$

$\therefore \lambda = 1, 5, 1$

i.e. the Eigen values of the matrix A.

Q. 3.

2) ① Inner product : If two square matrices $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 6 & 5 \end{bmatrix}$ are multiplied, their inner product is given by,

$$A \cdot B = \begin{bmatrix} 2\times3 + 3\times6 & 2\times4 + 3\times5 \\ 1\times3 + 4\times6 & 1\times4 + 4\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 6+18 & 8+15 \\ 3+10 & 4+20 \end{bmatrix}$$

$$\therefore A \cdot B = \begin{bmatrix} 24 & 23 \\ 13 & 24 \end{bmatrix}$$

5

② Outer product / Direct product : Two square matrices given as,

$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 3 & 4 \\ 6 & 5 \end{bmatrix}$ ~~are~~

multiplied, their direct product is given by,

$$A \cdot B = \begin{bmatrix} 2\begin{bmatrix} 3 & 4 \\ 6 & 5 \end{bmatrix} & 3\begin{bmatrix} 3 & 4 \\ 6 & 5 \end{bmatrix} \\ 1\begin{bmatrix} 3 & 4 \\ 6 & 5 \end{bmatrix} & 4\begin{bmatrix} 3 & 4 \\ 6 & 5 \end{bmatrix} \end{bmatrix}$$

$$\therefore A \cdot B = \begin{bmatrix} 6 & 8 & 9 & 12 \\ 12 & 10 & 18 & 15 \\ 3 & 4 & 12 & 16 \\ 6 & 5 & 24 & 20 \end{bmatrix}$$

$$\hat{a} = \left(\frac{m\omega}{2\hbar}\right)^{1/2} x + i\left(\frac{1}{2m\hbar\omega}\right)^{1/2} p \quad \text{---} (3)$$

$$\hat{a}^{+} = \left(\frac{m\omega}{2\hbar}\right)^{1/2} x - i\left(\frac{1}{2m\hbar\omega}\right) p \quad \text{---} (4)$$

Hamiltonion adjoint is x, so eq^n (3) + (4) ~~p~~ becomes

$$\hat{a}\hat{a}^{+} = \left(\frac{m\omega}{2\hbar}\right) x^2 + \left(\frac{1}{2m\hbar\omega}\right) p^2 + \frac{i}{2\hbar}(px - xp)$$

$$= \frac{m\omega x^2}{2\hbar} + \frac{p^2}{2m\hbar\omega} + \frac{i}{2\hbar}(px - xp)$$

$$= \cancel{m\omega x}^2$$

$$= \frac{p^2}{2m\hbar\omega} + \frac{m\omega x^2}{2\hbar} + \frac{i}{2\hbar}(px - xp)$$

$$\hat{a}\hat{a}^{+} = \frac{1}{2m\hbar\omega}\left(p^2 + (m\omega x)^2\right) + \frac{i}{2\hbar}[p, x] \quad \text{---} (5)$$

We know,

$$\frac{1}{2m}\left(p^2 + (m\omega x)^2\right) = H \quad \text{from eq}^n \ (2)$$

so eq^n (5) becomes

$$\hat{a}\hat{a}^{+} = \frac{H}{\hbar\omega} + \frac{i}{2\cancel{\hbar}} - i\cancel{\hbar}$$

$$\hat{a}\hat{a}^{+} = \frac{H}{\hbar\omega} + \frac{1}{2} \quad \text{---} (6)$$

$$\hat{a}^{+}\hat{a} = \frac{H}{\hbar\omega} - \frac{1}{2} \quad \text{—} (7)$$

We have to fine $[a, a^{+}]$

$$[a, a^{+}] = (a, a^{+}) - (a^{+}, a)$$

$$= \frac{\cancel{H}}{\cancel{\hbar}\omega} + \frac{1}{2} - \frac{\cancel{H}}{\cancel{\hbar}\omega} + \frac{1}{2}$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

$$\therefore \left.\begin{aligned} [a, a^{+}] &= 1 \\ [a^{+}, a] &= -1 \end{aligned}\right\} \text{—} (8)$$

Ground state energy we can find from eq^n (7)

$$\frac{H}{\hbar\omega} - \frac{1}{2} = 0$$

Ground state energy $H \Rightarrow E_0$

$$\frac{E_0}{\hbar\omega} - \frac{1}{2} = 0$$

$$\frac{E_0}{\hbar\omega} = \frac{1}{2} \qquad E_0 = \frac{1}{2}\hbar\omega \quad \text{—} (9)$$

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VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor	
Suppliment No. :	Subject : Quantum Mechanics
Roll No. : 1601	Test / Tutorial No. :
Class : MSc I	Div. :

Total energy of n^{th} state is given by

$$E_n = E_0 + n\hbar\omega$$

$$E_n = \frac{1}{2}\hbar\omega + n\hbar\omega$$

$$E_n = \left(\frac{1}{2} + n\right)\hbar\omega \quad \text{--- (10)}$$

$\therefore n = 0, 1, 2, 3, \ldots$

08

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VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor :

13/20

Suppliment No. :

Subject : Quantum Mechanics.

Roll No. : 1602

Test / Tutorial No. :

Class : MSc - I Physics

Div. :

.1

1) The first excited state energy of LHO is $\frac{3}{2}\hbar\omega$.

2) Quantum computers are used in All of the above

3) The commutation relation between $[a^{+}, a]$ is -1

4) Entanglement is the phenomenon of quantum mechanics to study of movement of two particles.

5) The key of experimental invalidation of a result called Bell inequalities.

05

Q. 2.

1) Creation and anhiliation operator :-

i) Creation and annihiliation operator are mathematical operator and it is applicatio in quantum mechanics.

ii) <u>Creation operator</u> :- It is called as raising operator denoted by $a^{\dagger}$. In H operator particle increases. (+)

iii) <u>Annihiliation operator</u> :- It is called as Lowering operator denoted by a. In this particle decreases (-)

Schrodengers eq^n is written as,

$$H\psi = E\psi$$

where H = Hamiltonian operator
E = Eigen value
ψ = wave function.

We know the Hamiltonion LHO is given as,

$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2 \quad \text{--- (1)}$$

we know that,

$$\omega^2 = \frac{k}{m}.$$

$$\therefore k = \omega^2 m.$$

Hence eq^n (1) becomes,

$$H = \frac{p^2}{2m} + \frac{1}{2}\omega^2 m x^2$$

$$H = \frac{1}{2m}\left[p^2 + (\omega m x)^2\right] \quad \text{--- (2)}$$

We have new operator a and $a^\dagger$

$$\therefore \quad a = \left(\frac{\omega m}{2\hbar}\right)^{1/2} x + i\left(\frac{1}{2\hbar\omega m}\right)^{1/2} p \quad \text{——} \ (3)$$

$$a^\dagger = \left(\frac{\omega m}{2\hbar}\right)^{1/2} x - i\left(\frac{1}{2\hbar\omega m}\right)^{1/2} p \quad \text{——} \ (4)$$

$a^\dagger$ is hermition adjoint of x.

Now we have to find from eqⁿ (3) & (4)

$$aa^\dagger = \left(\frac{\omega m}{2\hbar}\right) x^2 + \left(\frac{1}{2\hbar\omega m}\right) p^2 + \frac{i}{2\hbar}(px - xp)$$

$$= \frac{m\omega x^2}{2\hbar} + \frac{p^2}{2\hbar m\omega} + \frac{i}{2\hbar}(px - xp)$$

$$= \frac{p^2}{2\hbar m\omega} + \frac{x^2 m\omega}{2\hbar} + \frac{i}{2\hbar}(px - xp)$$

$$= \frac{1}{2\hbar m\omega}\left(p^2 + (xm\omega)^2\right) + \frac{i}{2\hbar}(px - xp) \quad \text{——} \ (5)$$

We know that

$$H = \frac{1}{2m}\left[p^2 + (xm\omega)^2\right] \quad \text{——} \ \text{from eq}^n \ (2)$$

$\therefore$ Eqⁿ (5) becomes

$$aa^\dagger = \frac{H}{\hbar\omega} + \frac{i}{2\cancel{\hbar}}(-i\cancel{\hbar})$$

$$aa^\dagger = \frac{H}{\hbar\omega} + \frac{1}{2} \quad \text{——} \ (6)$$

Similarly,

$$a^\dagger a = \frac{H}{\hbar\omega} - \frac{1}{2} \quad \text{——} \ (7)$$

Now we have to find

$$[a a^\dagger] = aa^\dagger - a^\dagger a.$$

from eqⁿ (6) & (7)

$$[a\,a^\dagger] = \frac{H}{\hbar\omega} + \frac{1}{2} - \frac{H}{\hbar\omega} + \frac{1}{2}$$

$$[a\,a^\dagger] = 1$$

Similarly,

$$[a^\dagger a] = -1$$

Ground state energy is given as

$$\frac{H}{\hbar\omega} + \frac{1}{2} = 0$$

Hamiltonian is ground state energy

$$H \longrightarrow E_0.$$

$$\therefore \frac{E_0}{\hbar\omega} + \frac{1}{2} = 0$$

$$\therefore E_0 = \frac{1}{2}\hbar\omega$$ — ground state energy.

Energy at n^{th} level is given as

$$E_n = E_0 + n\hbar\omega$$

$$= \frac{1}{2}\hbar\omega + n\hbar\omega$$

$$= \left(\frac{1}{2}+n\right)\hbar\omega$$ — Enery for n^{th} level of L.H.O.

08

Q. 3.

1) Bell inequalities :

It is given as, the key of experimental invalidation of a result. called as Bell inequalities.

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VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor :

Suppliment No. :

14/20

Roll No. : 1604

Class : M.Sc. I[st] [Sem II[nd]]

Subject : Quantum Mechanics.

Test / Tutorial No. :

Div. :

Que-1

1.) c.) $3/2\ \hbar\omega$

2.) d.) all of the above.

3.) d.) -1

4.) d.) movement

5.) b.) Bell inequalities.

05

Que-2
Ans 1.) Creation and annihilation operator are mathmatical operator

Creation $\longrightarrow$ $\hat{a}$ $\rightarrow$ for increasing
Annihilation $\longrightarrow$ a^+ $\rightarrow$ for decreasing.

for the Schrodingers equation.
$H\Psi$

Hamiltonion eqn for L.H.O. Energy -

$$H = \frac{P^2}{2m} + \frac{1}{2}kx^2 \quad \text{--- (1)}$$

$\because \omega^2 = k/m$
$k = \omega^2 . m$ ~~$k = \omega^2 / m$~~

$$H = \frac{P^2}{2m} + \frac{1}{2}\omega^2 m x^2$$

Now multipling and deviding of 'm' in second term.

$$H = \frac{1}{2m}\left[P^2 + (\omega m x)^2\right] \quad \text{--- (2)}$$

Here, we are taking another operator.

$$\hat{a} = \left(\frac{m\omega}{2\hbar}\right)^{1/2} x + i\left(\frac{1}{2\hbar m\omega}\right)^{1/2} . P \quad \text{--- (3)}$$

$$a^t = \left(\frac{m\omega}{2\hbar}\right)^{1/2} x - i\left(\frac{1}{2\hbar m\omega}\right)^{1/2} . P \quad \text{--- (4)}$$

$\hat{a}a^+$ for the hamltonion adjoint -

$$(\hat{a}a^t) = \left(\frac{m\omega}{2\hbar}\right)x^2 + \left(\frac{1}{2\hbar m\omega}\right)P^2 + \frac{i}{2\hbar}[Px - x$$

$$= \frac{m\omega x^2}{2\hbar} + \frac{P^2}{2\hbar m\omega} + \frac{i}{2\hbar}[Px - xP]$$

$$= \frac{P^2}{2\hbar m\omega} + \frac{m\omega x^2}{2\hbar} + \frac{i}{2\hbar}[Px - xP]$$

$$= \frac{1}{2\hbar m\omega}\left[p^2 + (m\omega x)^2\right] + \frac{i}{2\hbar}[-i\hbar] \quad -(5)$$

from eq^n (2)

$$(\hat{a}\,a^+) = \frac{H}{2\hbar m\omega} + \frac{i(-i)}{2}$$

$$= \frac{H}{2\hbar m\omega} - \frac{1}{2} \quad -(6)$$

similarly

$$(a^+\hat{a}) = \frac{H}{2\hbar m\omega} + \frac{1}{2} \quad -(7)$$

Now from eq^n (6) + (7)

$$[a^+, a] = \hat{a}a^+ - a^+\hat{a}$$

$$= \cancel{\frac{H}{2\hbar m\omega}} - \frac{1}{2} - \cancel{\frac{H}{2\hbar m\omega}} - \frac{1}{2}$$

$$[a^+, a] = -1$$

$$[a, a^+] = 1$$

Ground state Energy for a particle.

from eq^n (6)-

$$= \frac{H}{2\hbar m\omega} - \frac{1}{2}$$

$$= \frac{E_0}{2\hbar m\omega} - \frac{1}{2} = 0$$

$$\boxed{E_0 = \frac{1}{2} m\omega}$$

for 'nth number state

$$E_n = E_0 + n\hbar$$

$$= \frac{1}{2} m\omega$$

$$\boxed{E_n = \left(n + \frac{1}{2}\right)\hbar m\omega}$$

Que-3 1.) Bell Inequlities :- The key of experimental invalidation of a result called Bell Inequlities.

It's also a work like EPR Paradox bell finds the result.

Here, we are considering the two particle δ_1 & δ_2 having vector measurement $\bar{a}$ & $\bar{b}$ so,

the resultant could be $A = \bar{a}\delta_1$ & $B = \bar{b}\delta_2$.

Let take a parameter 'λ'.

and these two particle having spin $\pm 1/2$.

Now,

$$[A(a) \cdot B(b)]\lambda = A(a,\lambda) \cdot B(b,\lambda) \quad \text{--- (1)}$$

Here,

$\rho(\lambda)$ be the density distribution Parameter. so the normalization condition would be -

$$\int \rho(\lambda)\, d\lambda = 1 \quad \text{--- (2)}$$

describe the energy state could be -

$$\langle \psi | \delta_1 a \cdot \delta_2 b | \varphi \rangle = 1 \quad \text{--- (3)}$$

03

above eqn show that the singlet and the total resultant of the measurement is ± 1.

$$\boxed{E = (\psi, \lambda) \cdot (a\,\delta_1)\ (b\,\delta_2)}$$

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VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

09/20

Signature of Supervisor	
Suppliment No. :	Subject : quantum mechanics II
Roll No. : 1605	Test / Tutorial No. : internal exam 2022-023
Class : mSC I	Div. :

Q. 1

(1) the first excited state energy of L.H.O is $3/2 \hbar\omega$

(2) quantum computer are used in all of the above

(3) the communication relation betn $[a^+, a]$ is 1

(4) entanglement is the phenomenon of quantum mechanic to study of movement of two particles.

(5) the key of experimental invalidation of a result called Bell inequalitie

04

Q.3

(1) Bell inequalities :-

the key of experimental invalidation of a result is called Bell inequali

$A(a)B(b)\lambda = A(a\lambda)\, Bb(\lambda)$

$A(\bar{a}\,\lambda) = \pm 1$

$B = (\bar{b}\lambda) = \pm 1$

$\therefore \int \rho(\lambda)\, d\lambda = 1$

$E_{Qm} = \langle \psi | \delta_1 a \, \delta_2 b | \psi \rangle = +1$

$E = \int \rho(\lambda) A(a\lambda)\, B(b\lambda)\, d\lambda$

this is the Bell inequalities,

01

Q.2

(1) entanglement and EPR paradox :-

entanglement is the phenomenon of quantum mechanics to study of movement of two particles
the particle in a group for a small or large distance.

EPR paradox :-
E = Eistein, P = Podolsky, R = Rosein.
EPR paradox is a incomple description of physical reality. EPR paradox is the reality of element.

$|01\rangle, |10\rangle$

$|\psi\rangle = \frac{1}{\sqrt{2}}$

$|01\rangle - |10\rangle$

$= |01\rangle - |10\rangle = \sqrt{2}$

$= \frac{|ab\rangle - |ba\rangle}{\sqrt{2}}$

$\cancel{= (\alpha \beta - \beta \nu) = 1}$

$= |a\rangle |b\rangle$

$\alpha, \beta, \nu, \delta$

04

$|a\rangle = \alpha |a\rangle + \beta |b\rangle.$

$$(\langle \delta - \beta v) = 1$$

$$= \boxed{\frac{|ab\rangle - |ba\rangle}{\sqrt{2}}}$$

this is the entagkment and EPR paradox

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VIVEKANAND COLLEGE (Autonomous), KOLHAPUR

Class MSc I Div. Roll No. 1335

Suppliment No. Subject

Test / Tutorial No. 2

full dependance density $D(P, q, t)$

$$\frac{dD}{dt} = \frac{d}{dt}\left[\frac{d\dot{p}_i}{\partial p_i} p_i + \frac{\partial \dot{q}_i}{\partial q_i} \partial p\right] - \frac{d p_i}{\partial \partial p} \dot{p}_i + \frac{\partial q_i}{\partial q_i} \dot{q}_i \quad \text{—(B)}$$

than Adding eq^n (A) + (B)

$$\boxed{\frac{dD}{dt} = 0}$$

The liouville's theorm hus the value's o become the density of phase space value remains zero then the unltravalent hus can be affect in zero space phase

03

Q3

1) Macrostate :-

a, b, c, d are the 4 particle in microsstate which probality 1/2

the componets wise distrabution

Sr No	compounts	arrangement of particle				
		I	II	III	IV	V
1	1	0	1	2	3	4
2	2	4	3	2	1	0

their are compoundewise distribution has 5 arrangement particle

(0, 4) (1, 3) (2, 2), (3, 1) (4, 0).

each compsitionwise distribation of particle is known as microstate.

In the general form is

(1, n+1) (2, n+2) - - - (n n+1)

& total number has (n n-1)

2) microstate

macrostate	Arrangement wise distribution		microstate
	I	II	
(0,4)	abcd	0	1
(1,3)	a	bcd	4
	b	acd	
	c	abd	
	d	abc	
(2,2)	ab	cd	6
	ac	bd	
	ad	cb	
	bc	ad	
	bd	ac	
	ba	cd	
(3,1)	abc	d	4
	abd	c	
	acd	b	
	bcd	a	
(4,0)	abcd	0	1

(0,4) has only one microstate.
(1,3) has 4 microstate
(2,2) has 6 microstate

Q3

1) Macrostate :-

a, b, c, d are the 4 particle in microsstate which probality 1/2

the componets wise distrabution

Sr No	Compounts	arrangement of particle				
		I	II	III	IV	V
1	1	0	1	2	3	4
2	2	4	3	2	1	0

their are compoundewise distribution has 5 arrangement particle

(0, 4) (1, 3) (2, 2), (3, 1) (4, 0).

each compsitionwise distribution of particle is known as microstate.

In the general form is

(1, n+1) (2, n+2) - - - (n n+1)

& total number has (n n-1)

2) microstate

macrostate	Arrangement wise distribution I	II	microstate
(0, 4)	abcd	0	1
(1, 3)	a	bcd	4
	b	acd	
	c	abd	
	d	abc	
(2, 2)	ab	cd	6
	ac	bd	
	ad	cb	
	bc	ad	
	bd	ac	
	ba	cd	
(3, 1)	abc	d	4
	abd	c	
	acd	b	
	bcd	a	
(4, 0)	abcd	0	1

(0, 4) has only one microstate.
(1, 3) has 4 microstate
(2, 2) has 6 microstate

each particle compoundwise distribution of macro state is known as microstate.

for exampl 16 microstate on

$16 = 2^4$.

then 2^n

general form of microstate is 2^n.

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VIVEKANAND COLLEGE (Autonomous), KOLHAPUR

12/20

Class M.Sc-I Div. ______ Roll No. 1336

Suppliment No. ______ Subject physics

Test / Tutorial No. M.Sc-I, Sem-II Internal Examination

Q1. 1. a. microstate

2. b. accessible microstate

3. c. $\delta(\ln W) = 0$

4. d. an excited

5. b. number of particles in the system

3

Q2. 1. Liouvill'es Theorem

$\leftarrow$ f degree of freedom

Configuration of the system
position co-ordinate $\Rightarrow \partial q_1 \; \partial q_2 \cdots \partial q_f$
momentum co-ordinate $\Rightarrow \partial p_1 \; \partial p_2 \cdots \partial p_f$

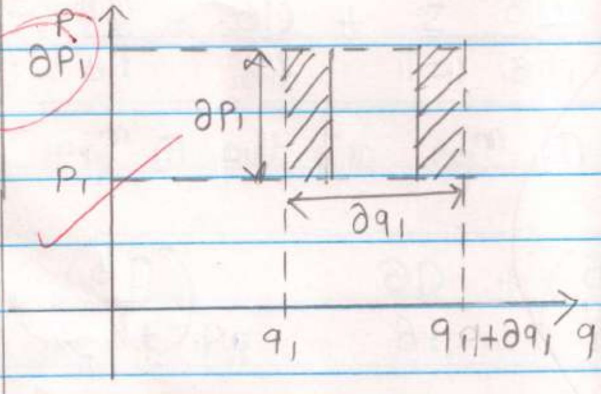
$\therefore$ consider the infinitesimal hyper volume the no. of phase point density doesn't change with time

$$\frac{dD}{dt} = 0$$

The volume $d\tau = \partial q_1 \, \partial q_2 \cdots \partial q_f \, \partial p_1 \, \partial p_2 \cdots \partial p_f$

$\therefore$ density distribution function is $D = D(q, p, t)$

$\therefore$ No of phase points
$\partial m = D(q\, p\, t)\, \partial q_1 \cdots \partial q_f \, \partial p_1 \cdots \partial p_f \; d\tau$



$p + \delta p$

the no. of phase point entering in the 1^{st} face in time dt

$\therefore$ D ∂q_1 dt $\partial q \cdots \partial q_f \; \partial p \cdots \partial p_f$

Q2.

1. the no. of phase point leaving

$$\therefore D\dot{q}_1\, dt\, \partial q_1 \cdots \partial p_f + \frac{\partial}{\partial q_1}\left(D\dot{q}_1\, dt\, \partial q_1 \cdots \partial p_f\right)$$

$$\therefore D\dot{q}_1\, dt\, \partial q_1 \cdots \partial p_f + D\frac{\partial \dot{q}_1}{\partial q_1} dt + \frac{\partial D}{\partial q_1}\dot{q}_1\, dt\, \partial q_1 \cdots \partial p_f$$

$\therefore$ the net no. of phase points entering
= enter - leave

$$\therefore D\dot{q}_1 dt\, \partial q_1 \cdots \partial p_f - D\dot{q}_1 dt\, \partial q_1 \cdots \partial p_f - D\frac{\partial \dot{q}_1}{\partial q_1} dt + \frac{\partial D}{\partial q_1}\dot{q}_1$$

$$= -\left[+D\frac{\partial \dot{q}_1}{\partial q_1} dt + \frac{\partial D}{\partial q_1}\dot{q}_1\right]\delta q_1$$

$$= -\left(+D\frac{\partial \dot{q}_1}{\partial q_1} + \frac{\partial D}{\partial q_1}\dot{q}_1\right) d\Gamma$$

For p- co-ordinate

$$= -\left[D\frac{\partial \dot{p}_1}{\partial p_1} + \frac{\partial D}{\partial p_1}\dot{p}_1\right] d\Gamma$$

$\therefore$ the total rate of change of no. of phase point in time dt

$$\therefore \frac{\partial}{\partial t}(dm)\, dt = -\left[D\left(\frac{\partial \dot{q}_1}{\partial q_1} + \frac{\partial \dot{p}_1}{\partial p_1}\right) + \left(\frac{\partial D}{\partial q_1}\dot{q}_1 + \frac{\partial D}{\partial p_1}\dot{p}_1\right)\right] dt$$

Q2. 1. Acc. to Hamiltonian

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \quad \dot{q}_i = \frac{\partial H}{\partial p_i}$$

$$\therefore \frac{\partial \dot{q}_i}{\partial q_i} + \frac{\partial \dot{p}_i}{\partial p_i} = \frac{\partial^2 H}{\partial p_i \partial q_i} - \frac{\partial^2 H}{\partial p_i \partial q_i} = 0$$

consider

$$\frac{\partial}{\partial t}(dm)\,dt = -\left[\sum_{i=1}^{f} \frac{\partial D}{\partial q_i}\dot{q}_i + \frac{\partial D}{\partial p_i}\dot{p}_i\right] dt\, d\tau$$

$$\therefore D^{(pqt)} = (dm)\,dt\,d\tau$$

$$\therefore \frac{\partial D}{\partial t}\,dt\,d\tau = -\left[\sum_{i=1}^{f} \frac{\partial D}{\partial q_i}\dot{q}_i + \frac{\partial D}{\partial p_i}\dot{p}_i\right] dt\,d\tau$$

$$\therefore \frac{\partial D}{\partial t} = -\sum_{i=1}^{f} \frac{\partial D}{\partial q_i}\dot{q}_i + \frac{\partial D}{\partial p_i}\dot{p}_i \quad \text{--- (1)}$$

$\therefore$ The full of probability density is given by

$$\therefore \frac{dD}{dt} = \frac{\partial D}{\partial t} + \sum_{i=1}^{f} \frac{\partial D}{\partial q_i}\frac{\partial q_i}{\partial t} + \sum_{i=1}^{f} \frac{\partial D}{\partial p_i}\frac{\partial p_i}{\partial t} \quad \text{--- (2)}$$

eq^n (1) put in eq^n (2) we get

$$\therefore \left(\frac{dD}{dt}\right)_{pq} = \frac{\partial D}{\partial t} + \left(\frac{\partial D}{\partial q}\right)_{p,t} + \left(\frac{\partial D}{\partial p}\right)_{q,t}$$

5

$$\therefore \boxed{\frac{dD}{dt} = 0}$$

the Theorem States that the rate of cha

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VIVEKANAND COLLEGE (Autonomous), KOLHAPUR

Class M.Sc-I Div. Roll No. 1336

Suppliment No. 1 Subject physics

Test / Tutorial No. M.Sc-I, Sem-II Internal Examination

Q3. 2. Concept of ensemble with its types

Ensemble -

It is the collection of large number of macroscopically identical and essentially independent system.

macroscopically identical means it have same values of tempreture, pressure, volume, number of particles.

microscopically differ in parity, Symmetry and quantum states.

Types of ensemble

i Micro-canonical ensemble

ii. Canonical ensemble

iii Grand-canonical ensemble

i Micro-canonical ensemble -

It is the collection of large no. of essentially independent system having same Energy, volume, and no. of particle

In this ensemble the system is seperated

Q3. 2.

Energy Volume no.of pag	Energy Volume no.of parti
Energy Volume no. of par	Energy volume no. of parti

outer walls rigid impermeable and inducting

ii Canonical ensemble -

It is the collection of large number of essentially independent system having Same Tempreture, volume, No. of particle

In this ensemble the System is Separated by rigid impearmeable and conducting walls

Tempreture volume no of parti	T V N
T V N	T V N

outer walls rigid impermeable inducting

Inner walls rigid impermeable conducting

iii Grand- canonical ensemble

It is the collection of large number of essentially independent system having Same Tempreture, volume and chemical potential

In this ensemble system is Separated by the permeable and conducting walls.

Tempreture volume	T V

outer walls rigid

" ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार "
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

Signature of Supervisor

Shri Swami Vivekanand Shikshan Sanstha's

VIVEKANAND COLLEGE (Autonomous), KOLHAPUR

$\frac{10}{20}$ July

Class M.Sc - I Div. Roll No. 1333

Suppliment No. Subject Physics

Test / Tutorial No.

Q. 1

1) a) microstates

2) b) accessible microstate.

3) c) $S(\ln W) = 0$

4) a) An equillibrium

5) c) thermal equillibrium system.

5

Q.3

→2] Ensembles can be defined as It is the collection of no. of macroscopically identical but esentially independent system.

Here the term macroscopically identical means constituent system of ensembles having same microscopic condition like pressure, volume, temperature, no. of particles etc. Here again the term esentially independent means constituent system of ensembles having different microscoopic condition like quantum stat etc.

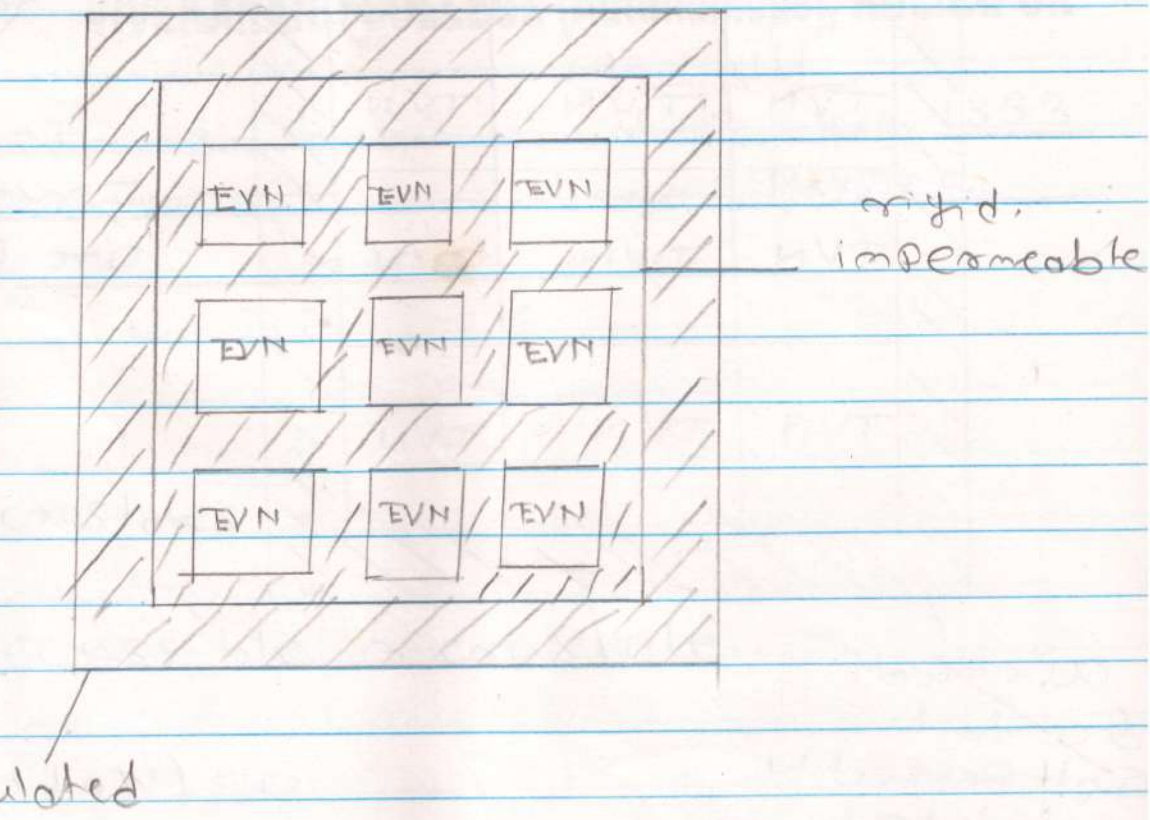
There are 3 type of ensembles.

1) Microcanonical ensemble.
2) Canonical ensemble.
3) Grand canonical ensemble.

1) Microcanonical ensemble.

It is the large no. of microscopic condition but esentially independent syst having same energy E, volume V, No of particle N

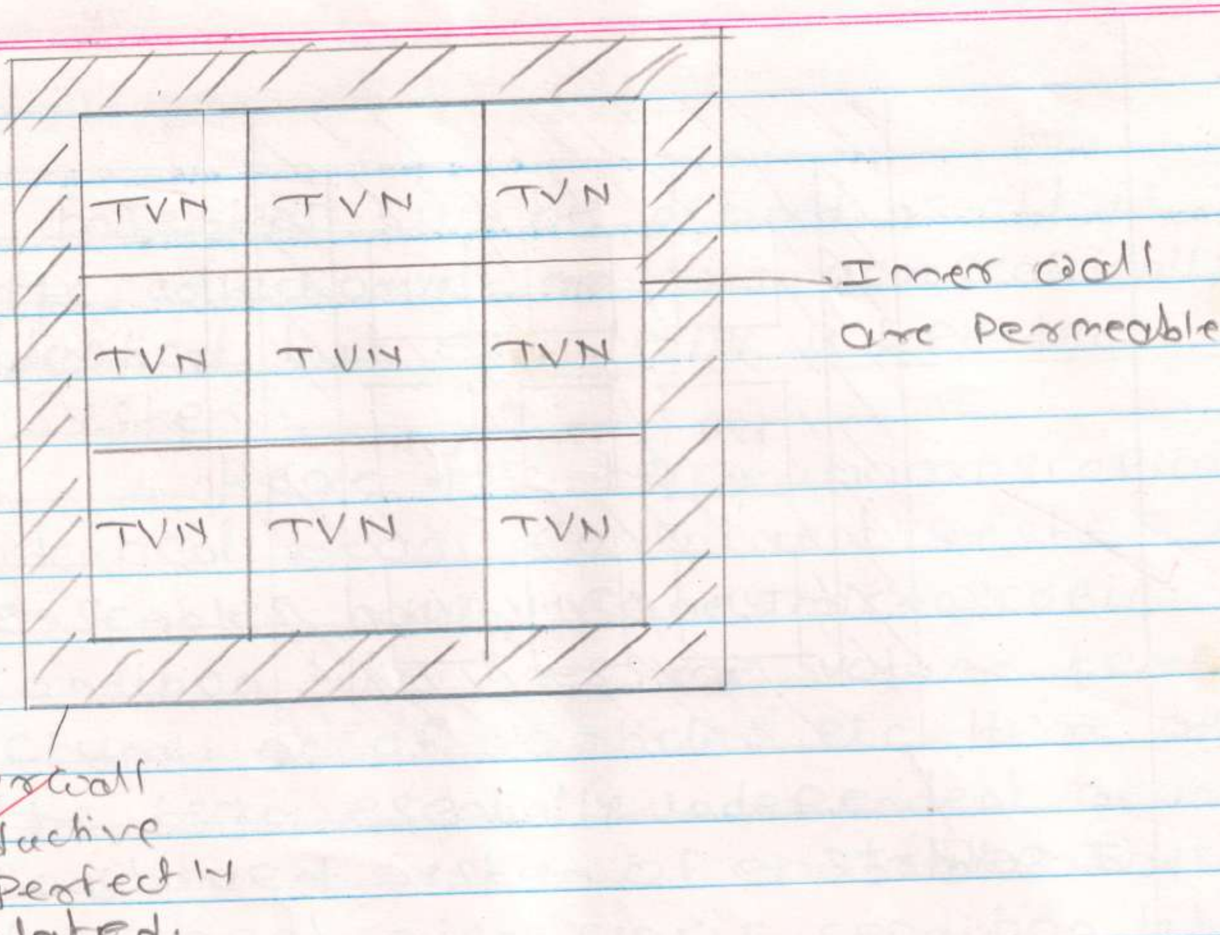
Microcanonical ensemble are ... impermeable &



2) canonical ensemble

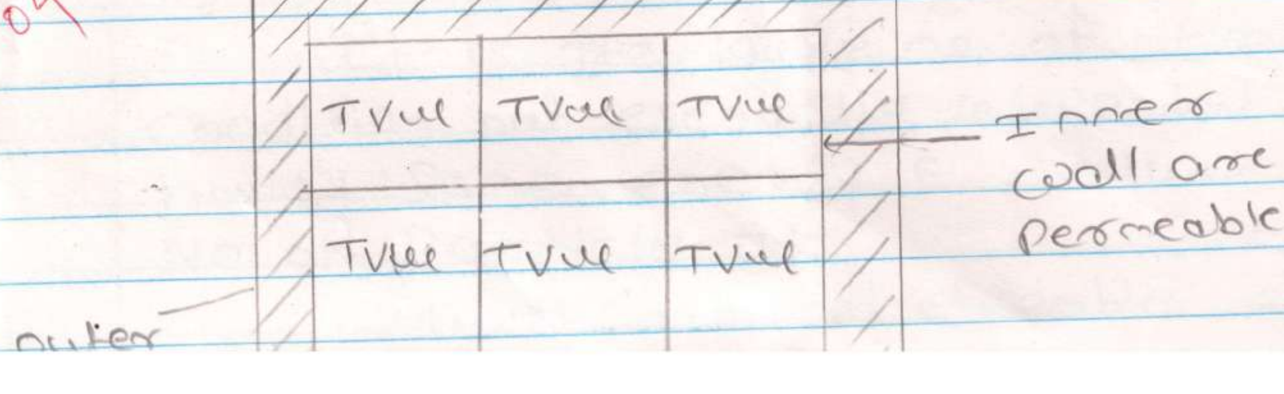
It is the large no. of microscopical condition but esentially independent system having same ~~energy~~ temprature T volume v & no of particle N.

canonical ensemble are separated by rigid, impermeable & ~~perfectly insulated~~ but conductive walls & outer wall are perfectly insulated.



3) Grand canonical system:-
It is the large no. of microscopical condition but esentially independent system having same temprerature T volume v & chemical potential μ.
Grand canonical ensembles are separated by rigid permeable & insulated walls.

04



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In this grand canonical system inner wall are rigid & permeable. but outer wall as-well as inner wall are insulated.

Canoni
Ensemble avarage:-

$$\vec{R_i} = \frac{\int_{-\infty}^{\infty} R(x)\, N(x)\, dx}{\int_{-\infty}^{\infty} N(x)\, dx}.$$

Q.2

1) Liouville's Therem.

Infinitimal hyper volume no. of Phase Posint density does not change with time

$$\frac{dD}{dt} = 0$$

where volume

$$dq_1, \cancel{q} dq_2 \ldots dq_f \quad dp_1, dp_2, \ldots dp$$

$$dT = dq \cdot dp$$

density distribution hunction $D = D(P, q, t)$

No of Phase Point $dy = D(D, q, T)$

For instant phase space two domential the no. of phase point entering the ~~Phase~~ ~~Space~~ 1st face in time dt.

$$= D \frac{dq_1}{dq_t} dt \, q_2 \ldots \cancel{q} \, dq_3 \ldots dp_f \ldots dp_I \ldots dp_f$$

0

$$= D \, q_1 dt \ldots q_2 dt \ldots dp_f$$

~~The no. of Phase Point~~