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Analytical investigation on domain of decentered parameter for self-focusing of Hermite-cosh-Gaussian laser beam in collisional plasma

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In the present paper, an analytically investigated domain of decentered parameter and its effect on the self-focusing of Hermit-cosh-Gaussian (HChG) laser beams in a collisional plasma have been studied theoretically. The nonlinearity in the dielectric constant of plasma arising due to the non-uniform heating of carriers along the wavefront of the laser beam has been employed in the present investigation. The nonlinear differential equation of beam width parameter for various laser modes of HChG beam is obtained by following the standard Akhamanov's parabolic equation approach under Wentzel-Kramers-Brillouin and paraxial approximations. The analytical treatment has enabled us to define three distinct regions: self-focusing, self-trapping and defocusing, which are presented graphically. *Published by AIP Publishing.* https://doi.org/10.1063/1.5016938

I. INTRODUCTION

Interaction of light with matter can be classified as a nonlinear phenomenon, especially for an intense source of light. The invention of high power laser beams has made it possible to study such nonlinear interaction of strong electromagnetic waves with plasmas. High energy laser beam and plasma interaction gives a variety of phenomena which are important in many applications, such as high harmonic generation,[1–3] x-ray lasers,[4] laser-driven fusion, laser based plasma accelerators,[5–8] etc. These applications require the laser beam to pass over several Rayleigh lengths in a plasma with sustained energy exchange between the laser beam and plasma media. The attention of researchers has been attracted by different kinds of laser beams such as Gaussian beams,[9–13] Cosh-Gaussian,[14–17] Hermit-cosh-Gaussian (HChG) beams,[18,19] Elliptical-Gaussian beams,[20,21] Bessel-Gaussian beams,[22] Leguerre-Gaussian beams,[23–25] etc. High energy laser beams induce intensity dependent nonlinear changes in the refractive index which results in a self-focusing phenomenon. There is experimental evidence of the phenomenon of self-focusing observed in a plasma channel created by the instantaneous response of the nonlinear refractive index to the light beam.[26] Self-focusing is responsible for the optical damage that occurred in a solid by a high-power laser beam.[27] Thus, the study of the self-focusing phenomenon is important due to the above stated applications and its relation to other nonlinear optical effects mentioned above.

When a laser beam, having a non-uniform intensity distribution along its wavefront, propagates through the plasma, electrons get heated and the temperature gradient is setup. In a collisional plasma, the main source of field dependence of effective dielectric constant is not ponderomotive force, but the nonuniform redistribution of carriers on account of inhomogeneous heating of carriers arising from transverse variation of the electric field along the wavefront. In the steady state, this mechanism is seen to be more effective than the ponderomotive force mechanism in characterizing the field dependence of the effective dielectric constant. Thus, the effective dielectric constant gets modified significantly, due to self-induced nonlinearity dependence on the intensity of the laser beam. Basically, self-induced nonlinearity causes the self-focusing and defocusing phenomena in the plasma medium.[27,28]

In recent years, considerable interest has been evinced towards the study of self-focusing of various modes of HChG laser beam passing through different plasma media. It has been found that HChG beams can be generated in a laboratory by the superposition of two decentered Hermite-Gaussian beams as the Cosh-Gaussian one.[29] The present theoretical analysis employs a parabolic equation approach under Wentzel-Kramers-Brillouin (WKB) and paraxial approximations. The decentered parameter plays an important role in the self-focusing phenomenon. A literature survey shows that the decentered parameter has been explored in different numerical intervals. However, in the present paper, we have emphasized analytically to set the numerical domain of decentered parameter for various nonlinear phenomena such as self-focusing, self-trapping, and defocusing.

The systematic organization of this paper is as follows: Sec. II describes the evolution of the beam width parameter of the HChG laser beam propagating through a collisional plasma for mode index values $m=0$, 2, and 4. In Sec. III, a detailed discussion of analytical investigation is presented with graphs. Finally, a brief conclusion is added in Sec. IV.

II. FORMULATION

A. Field distribution of HChG laser beam in collisional plasma

Consider the HChG laser beam propagating into the plasma along the z direction, starting at $z=0$ and having a field distribution of the form

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$$A(r,z) = \frac{E_0}{2f} H_m\left(\frac{\sqrt{2}r}{r_0 f}\right)\exp\left(\frac{b^2}{4}\right) \times \left\{\exp\left[-\left(\frac{r}{r_0 f}+\frac{b}{2}\right)^2\right] + \exp\left[-\left(\frac{r}{r_0 f}-\frac{b}{2}\right)^2\right]\right\}, \tag{1}$$

where m is the mode index of a laser beam, b is the decentered parameter, r is the radial distance from the center of the beam, r_0 is the waist width of Gaussian amplitude distribution, E_0 is the amplitude of Gaussian beam for the central position at $r = z = 0$, and f is the dimensionless beam width parameter. The effective dielectric constant of plasma can also be written as[30]

$$\varepsilon = \varepsilon_0 + \phi(EE^*), \tag{2}$$

where $\varepsilon_0 = 1 - (\omega_p^2/\omega^2)$ is the linear part of dielectric constant with $\omega_p = (4\pi n e^2/m)^{1/2}$ as the plasma frequency. Here, e and m are the electronic charge and its rest mass, respectively.

The second term in Eq. (2), the intensity dependent part of dielectric constant for a collisional plasma, is given by

$$\phi(EE^*) = \frac{\omega_p^2}{\omega^2}\left(\frac{\alpha EE^*}{2+\alpha EE^*}\right), \tag{3}$$

with

$$\alpha = \left(\frac{e^2 M}{6 k_B T_0 \omega^2 m^2}\right),$$

where M, k_B, and T_0 are the mass of the ion, the Boltzmann constant, and the equilibrium temperature of plasma, respectively.

B. Beam width parameter differential equations

The general wave equation of propagation of the laser beam inside the plasma medium with effective dielectric constant ε can be written as

$$\nabla^2 \vec{E} + \frac{\omega^2}{c^2}\varepsilon \vec{E} + \vec{\nabla}\left(\frac{\vec{E}.\vec{\nabla}\varepsilon}{\varepsilon}\right) = 0. \tag{4}$$

The last term on the left-hand side of Eq. (4) can be neglected, provided that $k^{-2}\nabla^2(ln\varepsilon) \ll 1$, where $k = \frac{\omega}{c}\sqrt{\varepsilon}$ represents the wave number

$$\nabla^2 \vec{E} + \frac{\omega^2}{c^2}\varepsilon \vec{E} = 0. \tag{5}$$

In a cylindrical coordinate system, Eq. (5) can be written as

$$\frac{\partial^2 \vec{E}}{\partial z^2} + \frac{\partial^2 \vec{E}}{\partial r^2} + \frac{1}{r}\frac{\partial \vec{E}}{\partial r} + \frac{\omega^2}{c^2}\varepsilon \vec{E} = 0. \tag{6}$$

We employ the WKB approximation to solve this equation. Solution of Eq. (6) for cylindrically symmetric beams can be expressed as

$$E = A(r,z)\exp[i(\omega t - k_0 z)], \tag{7}$$

where ω is the frequency of laser used. Neglecting $\partial^2 A/\partial z^2$ implies that the characteristic distance (in the z-direction) of intensity variation is much greater than the wavelength. Equation (6) reduces to

$$2ik\frac{\partial A}{\partial z} = \frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} + \frac{k^2 A}{\varepsilon_0}\phi(AA^*). \tag{8}$$

To solve this equation, we now introduce an eikonal S as

$$A = A_0(r,z)\exp[-ikS(r,z)], \tag{9}$$

where $A_0(r, z)$ and $S(r, z)$ are the real function of r and z with $S(r, z)$ as the eikonal of the beam which determines the convergence/divergence of the beam.

Substituting Eq. (9) into Eq. (8) and equating the real and imaginary parts, we get

$$2\left(\frac{\partial S}{\partial z}\right) + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{\varepsilon_0}\phi\left(\frac{1}{2}AA^*\right) + \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) \tag{10}$$

and

$$\frac{\partial A_0^2}{\partial z} + \frac{\partial S}{\partial r}\frac{\partial A_0^2}{\partial r} + A_0^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) = 0. \tag{11}$$

The solution of Eqs. (10) and (11) for the cylindrical HChG laser beam can be written as

$$S = \frac{r^2}{2}\beta(z) + \phi(z), \tag{12}$$

$$A_0^2 = \frac{E_0^2}{4f^2} H_m^2\left(\frac{\sqrt{2}r}{r_0 f}\right)\exp\left(\frac{b^2}{2}\right) \times \left\{\exp\left[-2\left(\frac{r}{r_0 f}+\frac{b}{2}\right)^2\right] + \exp\left[-2\left(\frac{r}{r_0 f}-\frac{b}{2}\right)^2\right] + 2\exp\left[-\left(\frac{2r^2}{r_0^2 f^2}+\frac{b^2}{2}\right)^2\right]\right\}, \tag{13}$$

where $\beta(z)$ can be expressed as $(1/f)(\partial f/\partial z)$, which represents the reciprocal of the curvature of the wavefront, $\phi(z)$ is the phase shift, and f is the beam width parameter which is a measure of both axial intensity and width of the beam.

Following the approach given by Akhamanov *et al.*[31] and developed by Sodha *et al.*,[30] we have obtained a nonlinear differential equation for the mode index of the HChG laser beam. For m = 0:

$$\frac{d^2 f}{d\xi^2} = \frac{12 - 12b^2 - b^4}{3f^3} - \frac{2(2-b^2)\alpha E_0^2 \omega_p^2 f r_0^2}{(\alpha E_0^2 + 2f^2)^2 c^2}. \tag{14}$$

For m = 2:

$$\frac{d^2f}{d\xi^2} = -\frac{84+60b^2+b^4}{3f^3} + \frac{2(b^2-10)\alpha E_0^2\omega_p^2 f r_0^2}{(2\alpha E_0^2+f^2)^2 c^2}. \quad (15)$$

For m = 4:

$$\frac{d^2f}{d\xi^2} = -\frac{308+108b^2+b^4}{3f^3} + \frac{72(b^2-18)\alpha E_0^2\omega_p^2 f r_0^2}{(72\alpha E_0^2+f^2)^2 c^2}. \quad (16)$$

III. RESULTS AND DISCUSSION

Equations (14)–(16) are second order nonlinear differential equations and represent the variation of the beam width parameter f with the normalized distance of propagation ξ for m = 0, 2, and 4, respectively. The first term on the right-hand side of these equations corresponds to the diffraction divergence of the beam and the second term corresponds to the convergence resulting from the nonlinearity. It is important to note that for $b = 0$, we obtain a similar equation obtained earlier by Sodha *et al.*[30] for collisional plasma.

Analytical investigation

The nonlinear equation contains a self-focusing term and a divergence term. We have made an analytical investigation to sustain the competition between these two terms. Consider Eq. (14) of m = 0 mode for further investigation. For critical conditions, taking $d^2f/d\xi^2 = 0, f = 1, \alpha E_0^2 = p$ and $\omega_p r_0/c = \rho$, we obtain

$$\frac{6(2-b^2)p}{(12-12b^2-b^4)(p+2)^2} = \frac{1}{\rho^2}, \quad (17)$$

where ρ^2 will be minimum, when the LHS of Eq. (17) is maximum. For the maximum value of LHS, we can write

$$\frac{d}{dp}\left\{\frac{6(2-b^2)p}{(12-12b^2-b^4)(p+2)^2}\right\} = 0. \quad (18)$$

Solving this equation, we obtain p = 2. Substituting this value into Eq. (17), we get

$$\rho_{0min} = \sqrt{\frac{4(12-12b^2-b^4)}{3(2-b^2)}}. \quad (19)$$

Equation (19) shows that ρ_{0min} is purely b dependent. Figure 1(a) is plotted from this equation, which shows that the value of ρ_{0min} decreases as b increases and reaches a minimum and again increases for a further increase of b values. To support this result, we have plotted the ρ Vs p graph from Eq. (17) for various b values, shown in Fig. 1(b). It shows an initial shift towards the downward direction with an increase in the value of b and reaches a minimum of ρ and starts to shift in the upward direction with a further increase in the value of b. The value of b for a minimum of ρ can be calculated as:for a minimum of ρ_{0min}, we can write $d\rho_{0min}/db = 0$, and solving this condition for Eq. (19), we get $b = \sqrt{6} = 2.44949$.

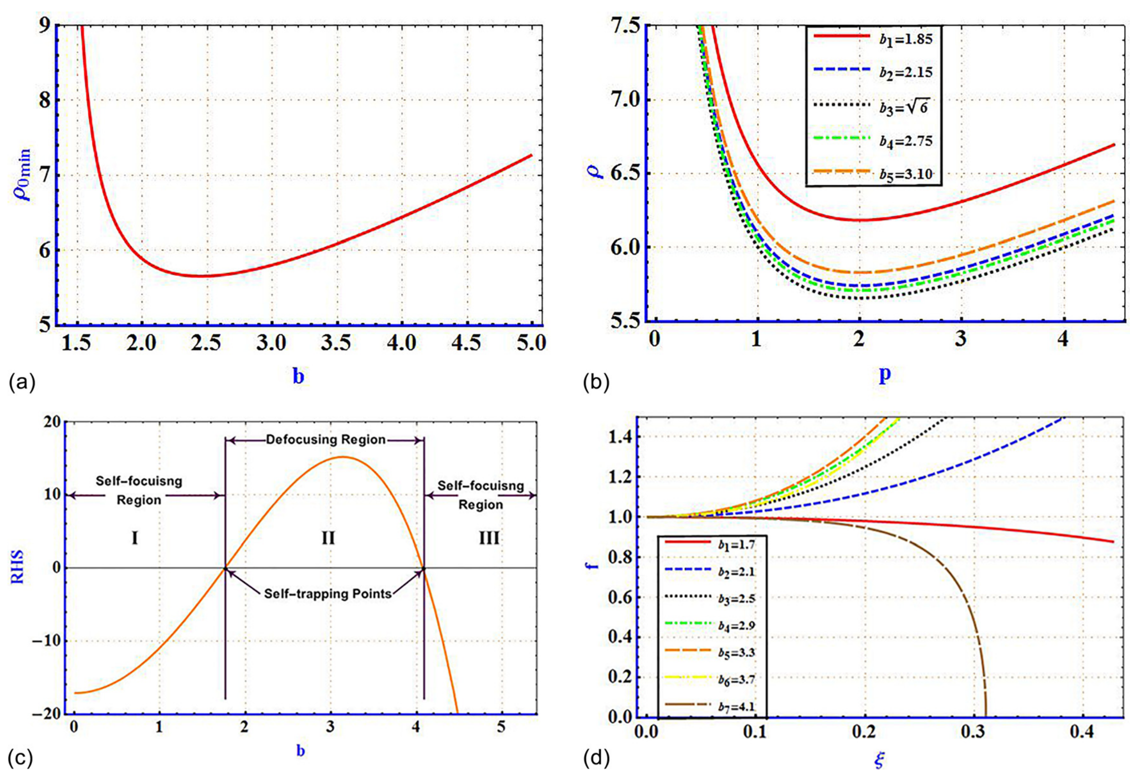

FIG. 1. Set of graphs for mode m = 0. (a) Variation of ρ_{0min} with decentered parameter b. (b) Critical curves for various values of decentered parameter b. (c) Graphical domain of decentered parameter b for different nonlinear effects. (d) Variation of beam width parameter f with a normalised propagation distance ξ for different values of decentered parameter b.

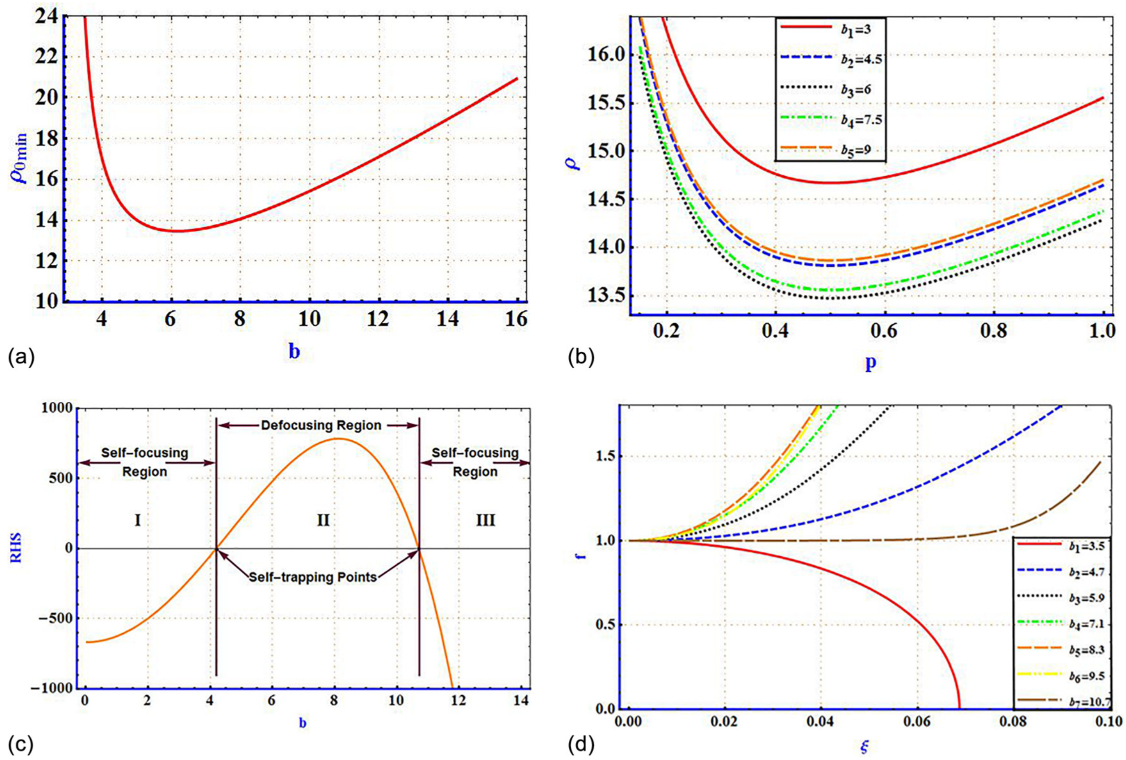

FIG. 2. Set of graphs for mode m = 2. (a) Variation of ρ_{0min} with decentered parameter b. (b) Critical curves for various values of decentered parameter b. (c) Graphical domain of decentered parameter b for different nonlinear effects. (d) Variation of beam width parameter *f* with a normalised propagation distance ξ for different values of decentered parameter b.

So, we have to choose the value of b around $\sqrt{6}$ for further study.

Substituting $\alpha E_0^2 = p$ and $\omega_p r_0/c = \rho$ into Eq. (14), we obtain

$$\frac{d^2f}{d\xi^2} = \frac{12 - 12b^2 - b^4}{3f^3} - \frac{2(2-b^2)pf\rho^2}{(p+2f^2)^2}. \qquad (20)$$

Now, choose the value of ρ well above the critical curve, that is $\rho = 6.5$ and the value of $p = 2$ that we have previously calculated. For critical conditions, taking $f = 1$ and $\frac{d^2f}{d\xi^2} = 0$, the Right Hand Side (RHS) of Eq. (20) becomes

$$RHS = \frac{1}{3}(12 - 12b^2 - b^4) - 10.5625(2 - b^2). \qquad (21)$$

Equation (21) is a pure b dependent equation; from this equation, we can find a self-focusing, self-trapping and defocusing condition, and this equation is plotted in Fig. 1(c). This figure shows different values of the domain of decentered parameter in which a different nonlinear phenomenon occurs.

In Fig. 1(d), we have displayed the variation of beam width parameter *f* with a normalized propagation distance ξ for different values of decentered parameter b, chosen from each domain shown in Fig. 1(c).

From the figure, it is clear that

$$\partial^2 f/\partial\xi^2 < 0 \quad for: \quad 0 < b < 1.75971$$
$$\text{and} \quad for: \quad b > 4.07320.$$

So, in this range of b value, self-focusing is observed

$$\partial^2 f/\partial\xi^2 = 0 \quad for: \quad b = 1.75971$$
$$\text{and} \quad for: \quad b = 4.07320.$$

These are the self-trapping conditions, so the beam passes through the collisional plasma medium without any deviation from these initial b values

$$\partial^2 f/\partial\xi^2 > 0 \quad for: \quad 1.75971 < b < 4.07320.$$

In this domain of b values, the beam gets defocused. From Figs. 1(c) and 1(d), it is clear that, as the b value increases above b = 1.75971, the rate of defocusing increases, reaches a maximum and again the rate starts to decrease for a further increase in the value of b and finally gets self-trapped at $b = 4.07320$.

A similar process is followed for a higher mode index m = 2 and m = 4. Graphical results are shown in the set of Figs. 2 and 3 for the mode index m = 2 and m = 4, respectively, while the numerical results and the domain are tabulated in Table I.

IV. CONCLUSIONS

We have investigated the domain of value of decentered parameter b for a different nonlinear phenomenon, such as self-focusing, self-trapping and defocusing by using simple calculus. The following important conclusions are drawn from the present analysis:

- Nonlinear phenomena are sensitive to the decentered parameter b of the HChG beam, while passing through a collisional plasma.

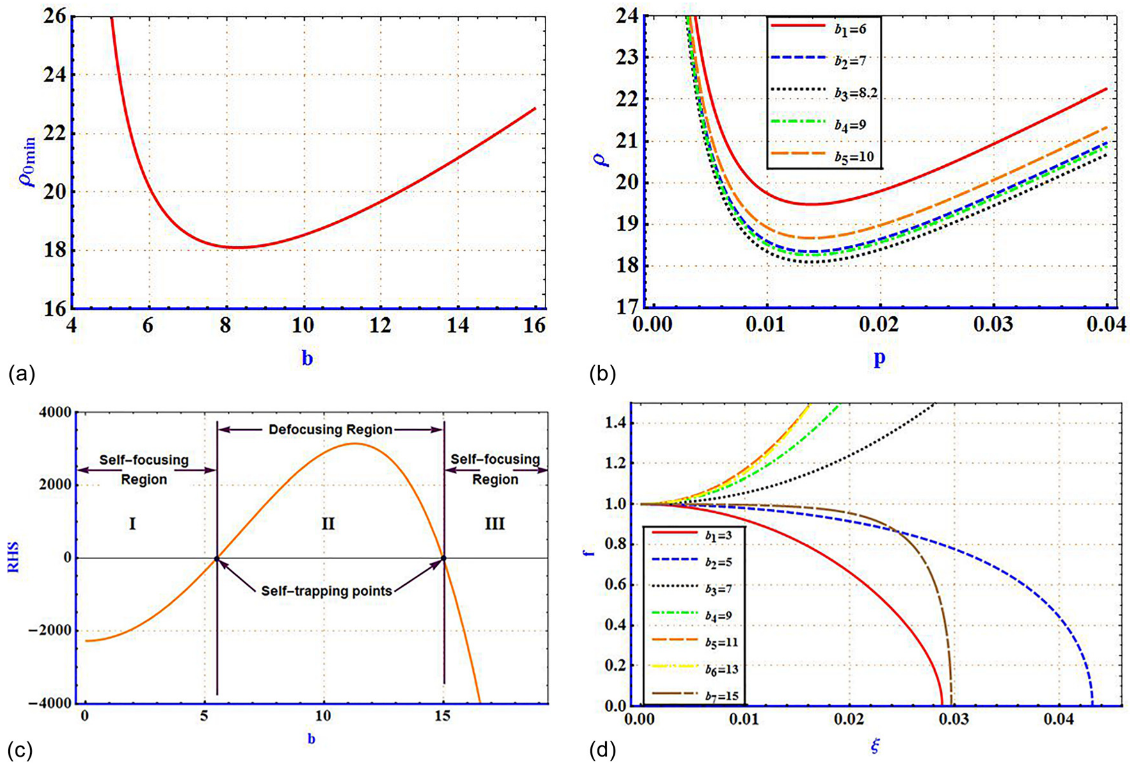

FIG. 3. Set of graphs for mode m = 4. (a) Variation of ρ_{0min} with decentered parameter b. (b) Critical curves for various values of decentered parameter b. (c) Graphical domain of decentered parameter b for different nonlinear effects. (d) Variation of beam width parameter f with a normalised propagation distance ξ for different values of decentered parameter b.

TABLE I. Analytical investigation of m = 2 and m = 4 modes.

Analytical investigation						
Mode (m)	b for ρ_{0min}	Calculated p	Chosen ρ	Self-focusing region: $\frac{d^2f}{d\xi^2} < 0$	Self-trapping points: $\frac{d^2f}{d\xi^2} = 0$	Defocusing region: $\frac{d^2f}{d\xi^2} > 0$
m = 2	6.16441	0.50000	16.00000	$b < 4.18361$ and $b > 10.70030$	$b = 4.18361$ and $b = 10.70030$	$b > 4.18361$ and $b < 10.70030$
m = 4	8.29182	0.01389	22.00000	$b < 5.52023$ and $b > 14.98420$	$b = 5.52023$ and $b = 14.98420$	$b > 5.52023$ and $b < 14.98420$

- With the increase in the mode index of the laser beam, the values of decentered parameter b for ρ_{0min} increase.
- The dimensionless critical beam power $p = \alpha E_o^2$ decreases with an increase in the mode index of the HChG laser beam.
- For higher laser modes of the HChG beam, i.e., for m = 2 and m = 4, the domain of b values for different nonlinear phenomena shifted towards higher values of b.

Thus, the present analytical investigations of decentered parameter offer a choice to govern the propagation characteristics of the HChG laser beam through collisional plasma.

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