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Article *in* The European Physical Journal D · March 2019

DOI: 10.1140/epjd/e2019-90369-8

Eur. Phys. J. D (2019) 73: 45
https://doi.org/10.1140/epjd/e2019-90369-8

THE EUROPEAN
PHYSICAL JOURNAL D

Regular Article

Self-focusing of higher-order asymmetric elegant Hermite-cosh-Gaussian laser beams in collisionless magnetized plasma

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Received 25 July 2018 / Received in final form 9 December 2018
Published online 7 March 2019

Abstract. In the present paper, we have explored the complexity in self-focusing of asymmetric elegant Hermite-cosh-Gaussian (EHChG) laser beams in magnetized plasmas. We have studied transverse electromagnetic (TEM) mode indices (m, n) of laser with distinct values $(0, 0)$, $(0, 2)$ and $(0, 4)$. The nonlinearity in dielectric constant considered herein is mainly due to the ponderomotive force. General form of differential equations for beam-width parameters in both transverse dimensions of the beam are established by using WKB and paraxial approximations through parabolic wave equation approach. The behaviour of beam-width parameters with dimensionless distance of propagation is studied at different parameters of plasma and those of the considered beams family.

1 Introduction

The investigation of laser-plasma interaction is motivated by many applications such as laser-based plasma accelerators [1], laser-driven fusion [2,3], harmonic generation [4], new radiation sources [5], ionospheric modification [6,7], etc. The propagation of laser beam over several Rayleigh lengths is an essential requirement in these applications while keeping efficient interaction with plasma. The development of high power laser beams nowadays makes feasibility of investigate various nonlinear optical effects in plasmas. The phenomenon of self-focusing [8,9] is one of the important nonlinear optical effect that spans the gamut to many self-action effects.

The self-focusing of electromagnetic beams, having a non-uniform distribution of irradiance is caused by the corresponding non-uniform distribution of the dielectric function due to inherent nonlinearities. Taking into account the three types of nonlinearities [10] viz. relativistic, collisional and ponderomotive, self-focusing of laser beams in plasmas has received considerable attention. In the case of collisionless plasma, self-focusing is caused by the ponderomotive force which pushes the electrons away from the region of highest laser intensity leaving behind the region of lowered electron density. Consequently, increasing the dielectric function leads to self-focusing of the propagating laser beam. The other important mechanism which is dominant at high intensity is the relativistic nonlinearity. In case of high-intensity short-pulse lasers, the quiver motion of electrons leads to increasing the mass of electrons. Consequently, it causes increase in the dielectric constant of the plasma which results in increase in self-focusing of the laser beam. This is generally known as relativistic self-focusing. Hora [11] has presented numerous theoretical discussion on the concept of relativistic self-focusing of high-power laser beams in plasmas. The interaction of intense laser beams with magnetized plasma is a relatively new field of study in which magnetic field significantly affects the propagation of laser beams in plasma. Several effects are observable in laser-magnetized plasma interaction, viz. second harmonic generation [12], laser wakefield acceleration [13], longitudinal pulse compression and amplification [14], etc. Review of literature highlights the fact that most of the research work on self-focusing of laser beams in magnetized plasma has been confined to cylindrically symmetric Gaussian beam. But the beam shaping technology has made several beams with variety of intensity profiles practically available. Only few studies have been reported on self-focusing of elliptic Gaussian beams [15,16], hollow Gaussian beams [17,18], super-Gaussian beams [19,20], Hermite-Gaussian beams [21,22], cosh-Gaussian beams [23,24], q-Gaussian beams [25,26], etc.

In recent years, a considerable interest has been evinced for new class of laser beams which is a more generalized case for an elegant Hermite–Gaussian beam and cosh-Gaussian beam, i.e. elegant Hermite-cosh-Gaussian beam (EHChG) was studied for its propagation properties [27]. As such, the nonlinear effects arrived by propagation of such laser beams through plasmas are highly sensitive to

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laser-plasma coupling parameters. Thus, such beams can be utilized to achieve efficient interaction with the plasmas. The literature has shown that a lot of studies have been carried out on propagation of this type of beams such as the work of Patil et al. [28–31], Nanda et al. [32,33], Kaur et al. [34] and Valkunde et al. [35]. Recently, we have studied the self-focusing of cosh-Gaussian laser beams in collisionless magnetized plasma [36–38].

In the present paper, we have studied the influence of decentred parameter on self-focusing in the interaction of asymmetric EHChG laser beams for transverse electromagnetic (TEM) mode indices (m, n) of distinct values $(0, 0)$, $(0, 2)$ and $(0, 4)$ with collisionless magnetized plasma. In the present study, Cartesian coordinate system has been employed which enables us to study evolution of two transverse beam-width parameters simultaneously. As usual, present analysis employs parabolic wave equation approach under WKB and paraxial approximations. The systematic organization of this paper is as follows: Section 2 describes the evolution of general beam width parameter differential equations for EHChG laser beams propagating through collisionless magnetized plasma. In Section 3, detailed discussion of the results has been presented. Finally, a brief conclusion is added in Section 4.

2 Basic formulation

Consider the propagation of EHChG laser beams through collisionless magnetized plasma along the z-direction, which is also the direction of the static magnetic field. The effective dielectric constant for extraordinary (right circularly polarised) and ordinary (left circularly polarised) modes of laser in collisionless magnetized plasma can be expressed in the following form [16],

$$\varepsilon_\pm = \varepsilon_{0\pm} + \phi(E_\pm E_\pm^*), \tag{1}$$

where $\varepsilon_{0\pm}$ and $\phi(E_\pm E_\pm^*)$ are the linear and nonlinear part of $\varepsilon_\pm$, respectively. Here,

$$\varepsilon_{0\pm} = 1 - \frac{\Omega_p^2}{1 \mp \Omega_c}, \tag{2}$$

$$\phi_\pm = \frac{\Omega_p^2}{2(1 \mp \Omega_c)}[1 - \exp(-(\gamma_\pm \alpha E_{0\pm}^2))], \tag{3}$$

where $\gamma_\pm = [1 \mp (\Omega_c/2)]/(1 \mp \Omega_c)^2$, $\alpha = 3m\alpha_0/4M$, $\alpha_0 = e^2/6m\omega^2 K_{\mathrm{B}} T_0$, $\Omega_p = \omega_p/\omega$, $\Omega_c = \omega_c/\omega$, $\omega_p = (4\pi N_0 e^2/m)^{1/2}$ is the plasma frequency, $\omega_c = eB_0/mc$ is the electron cyclotron frequency, e and m are the electronic charge and its rest mass, respectively. N_0 be the unperturbed density of plasma electrons, M is the mass of ion, ω is the angular frequency of laser used, K_{B} is the Boltzmann constant, B_0 is external magnetic field and T_0 is an equilibrium plasma temperature.

The wave equation is considered to comprise two field configurations (viz., extraordinary and ordinary modes). The evolution of electric field can thus be written as,

$$\frac{\partial^2 E_\pm}{\partial z^2} - 2ik_\pm \frac{\partial E_\pm}{\partial z} + \delta_\pm \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) E_\pm + \frac{\omega^2}{c^2}(\epsilon_\pm - \epsilon_{0\pm})E_\pm = 0, \tag{4}$$

where $\delta_\pm = [1 + (\epsilon_{0\pm}/\epsilon_{0zz})]/2$ with $\epsilon_{0zz} = 1 - \Omega_p^2$.

Introducing the concept of eikonal, one can express $E_\pm = A_{0\pm}(x, y, z)\exp[-ik_\pm S_\pm(x, y, z)]$, where $S_\pm$ is the eikonal of beam and is related to the curvature of wavefront of the beam. Within the framework of WKB and paraxial approximations, for initially EHChG laser beams one finds,

$$A_{0\pm}^2 = \frac{E_{0\pm}^2}{f_{1\pm} f_{2\pm}} \left[H_m\left(\frac{x}{r_{0\pm} f_{1\pm}}\right) H_n\left(\frac{y}{r_{0\pm} f_{2\pm}}\right)\right]^2 \times \left[\cosh\left(\frac{b_1 x}{r_{0\pm} f_{1\pm}}\right)\cosh\left(\frac{b_2 y}{r_{0\pm} f_{2\pm}}\right)\right]^2 \times \exp\left[-2\left(\frac{x^2}{r_{0\pm}^2} + \frac{y^2}{r_{0\pm}^2}\right)\right], \tag{5}$$

and

$$S_\pm = \frac{1}{\delta_\pm}\left(\frac{x^2}{2f_{1\pm}}\frac{df_{1\pm}}{dz} + \frac{y^2}{2f_{2\pm}}\frac{df_{2\pm}}{dz}\right) + \psi_\pm(z), \tag{6}$$

where $E_{\pm 0}$ is the initial amplitude of laser with initial beam-width $r_{0\pm}$, H_m and H_n are the Hermite polynomials of order m and n, respectively, b_1 and b_2 are the decentred parameters of EHChG beams with $f_{1\pm}$ and $f_{2\pm}$ as the corresponding beam-width parameters in x and y dimensions, respectively.

Following approach given by Akhmanov et al. [8] and its extension by Sodha et al. [9], we have obtained general expressions for beam-width parameters $f_{1\pm}$ and $f_{2\pm}$ for mode indices (m, n) of distinct values $(0, 0)$, $(0, 2)$ and $(0, 4)$ as,

$$\frac{d^2 f_{1\pm}}{d\zeta_\pm^2} = \frac{A_1}{2}\frac{\delta_\pm^2}{f_{1\pm}^3} + \mathrm{Y}_1 \frac{B_1\delta_\pm}{(1\mp\Omega_c)}\frac{\rho_{0\pm}^2\Omega_p^2\gamma_\pm P_{0\pm}}{f_{1\pm}^3 f_{2\pm}^2} \times \exp\left(-Y_2\frac{\gamma_\pm P_{0\pm}}{f_{1\pm} f_{2\pm}}\right), \tag{7}$$

and

$$\frac{d^2 f_{2\pm}}{d\zeta_\pm^2} = \frac{A_2}{2}\frac{\delta_\pm^2}{f_{2\pm}^3} + \mathrm{Y}_1 \frac{B_2\delta_\pm}{(1\mp\Omega_c)}\frac{\rho_{0\pm}^2\Omega_p^2\gamma_\pm P_{0\pm}}{f_{2\pm}^3 f_{1\pm}^2} \times \exp\left(-Y_2\frac{\gamma_\pm P_{0\pm}}{f_{2\pm} f_{1\pm}}\right), \tag{8}$$

where $A_1 = 4(1 - b_1^2)$, $A_2 = 4(1 - b_2^2)(1 + n)$, $B_1 = b_1^2 - 2$, $B_2 = b_2^2 - [2(n + 1)]$, $P_{0\pm} = \alpha E_{0\pm}^2$, $\rho_{0\pm} = r_{0\pm}\omega/c$ is the equilibrium beam radius, $\zeta_\pm = z/k_\pm r_{0\pm}^2$ is the dimensionless distance of propagation. The general relation between numerical coefficients $Y_{1,2}$ in the second term on right-hand side of above equations and the varied mode index n

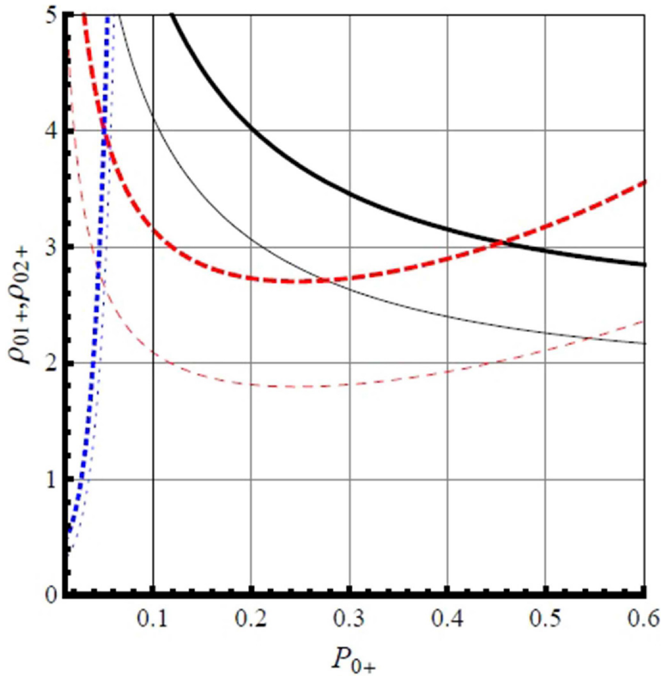

Fig. 1. Critical curves for E-mode of EHChG laser beams in collisionless magnetized plasma. The mode indices (m, n) for solid curves correspond to $(0, 0)$, dashed curves correspond to $(0, 2)$ and dotted curves correspond to $(0, 4)$. Thick curves are for ρ_{01+} and thin curves are for ρ_{02+}. The other parameters are $N_0 = 10^{19}\,\mathrm{cm}^{-3}$, $\omega = 1.778 \times 10^{14}\,\mathrm{rad/s}$, $B_0 = 0.10\,\mathrm{MG}$, $b_1 = 0.4$ and $b_2 = 0.8$.

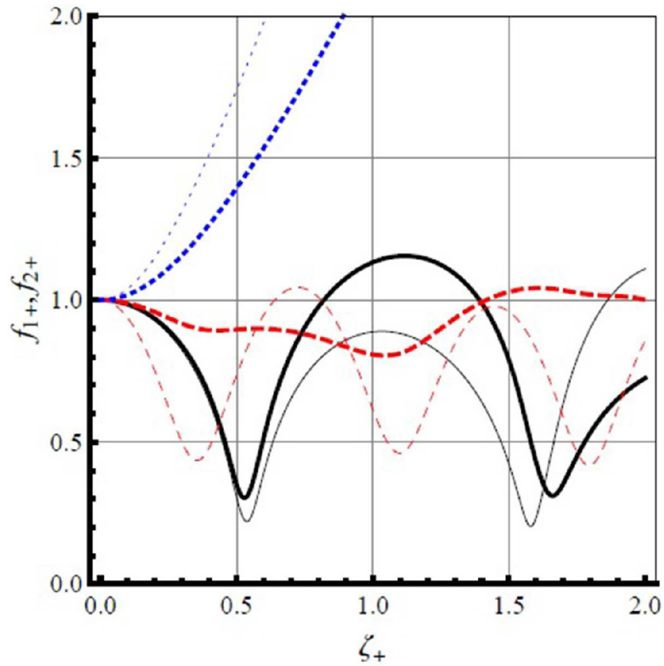

Fig. 2. Variation of beam-width parameters f_{1+}, f_{2+} with dimensionless distance of propagation ζ_+ for E-mode of EHChG laser beams with $\rho_{01+,02+} = 4$ and $P_{0+} = 0.5$ in collisionless magnetized plasma. The mode indices (m, n) are $(0, 0)$ for solid curves, $(0, 2)$ for dashed curves and $(0, 4)$ for dotted curves. Thick curves are for f_{1+} and thin curves are for f_{2+}. The other parameters are same as in Figure 1.

is $Y_{1,2} = u_{1,2} + v_{1,2}n + w_{1,2}n^2$. Here. $u_{1,2}$, $v_{1,2}$ and $w_{1,2}$ are constants for relevant quadratic fit. We strictly prohibit this generalization for $n > 4$ as it has not been verified by detailed calculations.

For an initially plane wavefront $df_{1\pm,2\pm}/d\zeta_\pm = 0$ and $f_{1\pm,2\pm} = 1$ and $\zeta_\pm = 0$, the condition $d^2 f_{1\pm,2\pm}/d\zeta_\pm^2 = 0$ leads to the propagation of EHChG laser beams without convergence or divergence i.e., in self-trapped mode. These conditions are known as critical conditions. Thus, by putting $d^2 f_{1\pm,2\pm}/d\zeta_\pm^2 = 0$ in equations (7) and (8), we obtain a relation between $\rho_{01\pm}$ and $\rho_{02\pm}$ in both x and y dimensions as,

$$\rho_{01\pm}^2 = -\frac{A_1\delta_\pm(1\mp\Omega_c)}{2Y_1B_1\Omega_p^2\gamma_\pm P_{0\pm}}\exp(Y_2\gamma_\pm P_{0\pm}), \tag{9}$$

$$\rho_{02\pm}^2 = -\frac{A_2\delta_\pm(1\mp\Omega_c)}{2Y_1B_2\Omega_p^2\gamma_\pm P_{0\pm}}\exp(Y_2\gamma_\pm P_{0\pm}). \tag{10}$$

3 Results and discussion

Equations (7) and (8) are the second-order, nonlinear, coupled, differential equations which govern self-focusing and defocusing of EHChG laser beam in collisionless magnetized plasma for both the modes of polarization of laser. The first term on right-hand sides of the equations (7) and (8) shows diffraction effect which is responsible for divergence of laser beams (i.e., defocusing effect), and the second term is due to the ponderomotive nonlinearity in dielectric constant which is responsible for convergence of the laser beams (i.e., focusing effect). It is interesting to note that for mode indices $(m, n) = (0, 0)$, coefficients $Y_{1,2}$ in equations (7) and (8) become unity and these equations reduce to equations (7) and (8) of our earlier work [36] for self-focusing of asymmetric cosh-Gaussian laser beams in collisionless magnetized plasma. Equations (7) and (8) can be solved numerically by using fourth order Runge–Kutta method for following laser-plasma parameters: $N_0 = 10^{19}\,\mathrm{cm}^{-3}$, $\omega = 1.778 \times 10^{14}\,\mathrm{rad/s}$, $B_0 = 0.10\,\mathrm{MG}$, $\rho_{01+,02+} = 4$, $b_{1,2} = 0.0$–0.8 and $P_{0+} = 0.5$.

Figure 1 represents the critical curves for E-mode with decentred parameters $b_1 = 0.4$ and $b_2 = 0.8$ in collisionless magnetized plasma. It is observed from this figure that for unlike decentred parameter values in both transverse dimensions of the beam, the nature of equilibrium beam radius ρ_{0+} with initial intensity parameter P_{0+} is different for different mode indices. We have highlighted this nature for mode indices $(m, n) = (0, 0), (0, 2), (0, 4)$. It is to be noted that for $n = 0, 2$, ρ_{0+} decreases initially, slowly attains a minimum $\rho_{0+\min}$ value and again increases with increase in P_{0+}. It is also observed that $\rho_{0+\min}$ is lower for $n = 2$ than that of $n = 0$. However, for $n = 4$, ρ_{0+} increases sharply with P_{0+}. As such, focusing region decreases with increase in mode index n.

Figure 2 describes the variation of beam-width parameters f_{1+} and f_{2+} as a function of dimensionless distance of propagation ζ_+ for different mode indices

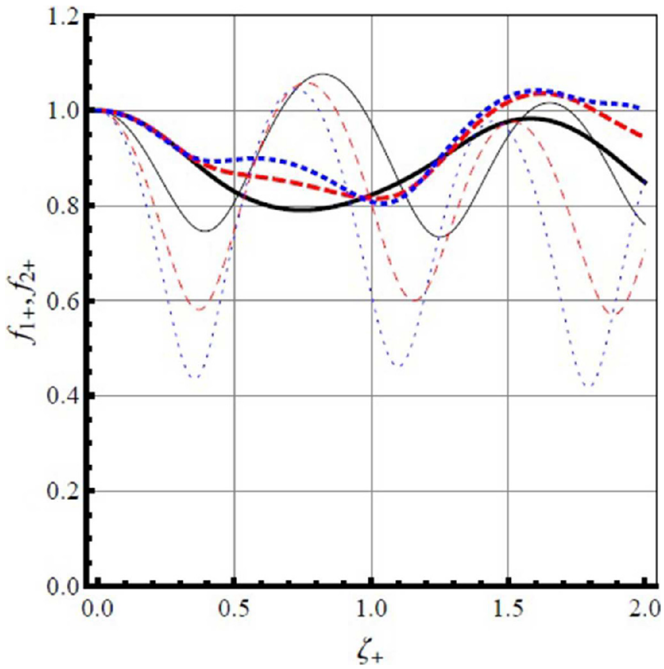

Fig. 3. Variation of beam-width parameters f_{1+}, f_{2+} with dimensionless distance of propagation ζ_+ for E-mode of EHChG laser beams with mode indices (m, n) corresponding to $(0, 2)$ and $\rho_{01+,02+} = 4$, $P_{0+} = 0.5$ in collisionless magnetized plasma. Solid curves correspond to $(b_1 = b_2 = 0.0)$, dashed curves correspond to $(b_1 = 0.2, b_2 = 0.6)$ and dotted curves correspond to $(b_1 = 0.4, b_2 = 0.8)$. Thick curves are for f_{1+} and thin curves are for f_{2+}. The other parameters are the same as in Figure 1.

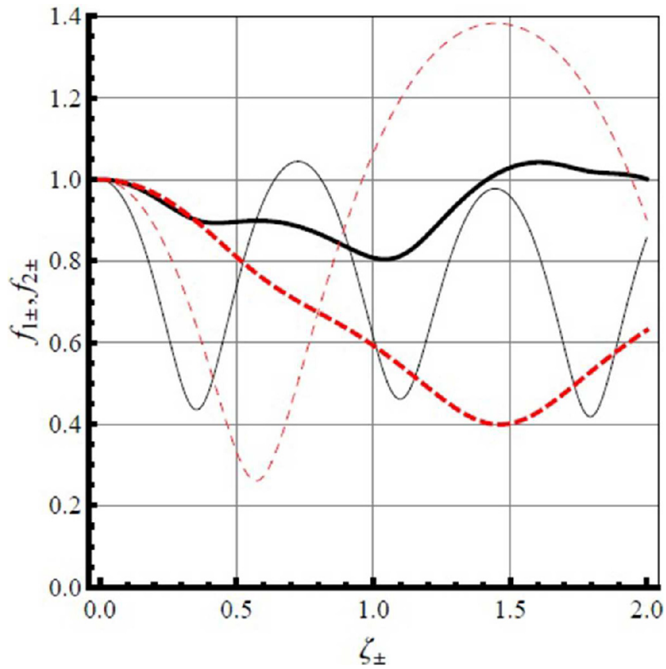

Fig. 4. Variation of beam-width parameters $f_{1\pm}$, $f_{2\pm}$ with dimensionless distance of propagation ζ_+ for EHChG laser beams with mode indices (m, n) corresponding to $(0, 2)$ and $\rho_{01+,02+} = 4$, $P_{0+} = 0.5$ in collisionless magnetized plasma. Solid curves correspond to E-mode, while dashed curve to O-mode. Thick curves are for f_1 and thin curves are for f_2. The other parameters are the same as in Figure 1.

$(m, n) = (0, 0), (0, 2), (0, 4)$ with $\rho_{01+,02+} = 4$, $P_{0+} = 0.5$, $B_0 = 0.1$ MG and the same values of decentred parameters as that in Figure 1. It is obvious from this figure that exact synchronized periodic oscillations of f_{1+} and f_{2+} are not possible for all mode indices $(m, n) = (0, 0), (0, 2), (0, 4)$ due to different decentred parameters $b_1 = 0.4$ and $b_2 = 0.8$ in both transverse dimensions of the beam. The complexity in oscillations of f_{1+} and f_{2+} is clear for both indices $n = 0, 2$. It is to be noted that self-focusing of the beam is weaker for $n = 2$ than that of $n = 0$. However for $n = 4$, sharp defocusing of the beam in both dimensions is observed. Thus, higher order modes with $n = 2, 4$ are dominated by diffraction effect, wherein f_{1+} and f_{2+} values exceed than that of $n = 0$ case. This is due to the asymmetric intensity distribution in the beams through mode indices. Consequently, asymmetric intensity distribution through mode indices results in explicit diffraction leading to increase in f_{1+} and f_{2+} than initial beam-width parameter value. Also, unlike decentring the beam profile in both transverse dimensions of the beam causes complexity in f_{1+} and f_{2+} during propagation through plasmas.

Figure 3 illustrates the variation of f_{1+} and f_{2+} for $n = 2$ as a function of ζ_+ for different values of decentred parameters $(b_1, b_2) = (0.0, 0.0), (0.2, 0.6), (0.4, 0.8)$. It is obvious from this figure that complexity in self-focusing character of f_{1+} and f_{2+} increases with increase in decentred parameters b_1 and b_2. It is apparent from Figure 3 that focusing and defocusing trend of f_{1+} and f_{2+} with ζ_+ is clear for $n = 2$. This means that with increase in decentred parameters in both the dimensions, f_{1+} exhibits little oscillatory behaviour while f_2+ changes with higher amplitude but still maintaining oscillatory behaviour.

The effect of polarization modes (E-mode and O-mode) on the behaviour of $f_{1\pm}$ and $f_{2\pm}$ with $\zeta_\pm$ for $n = 2$, $b_1 = 0.4$ and $b_2 = 0.8$ has been presented in Figure 4. From this figure it is observed that both the modes show the clear propagation behaviour of $f_{1\pm}$ and $f_{2\pm}$. It is evident that focusing of $f_{2\pm}$ is more than that of $f_{1\pm}$. It is also found that initial focusing of O-mode is more than E-mode.

4 Conclusions

In conclusion, influence of decentred parameter on self-focusing of asymmetric EHChG laser beams with TEM mode indices (m, n) equal to $(0, 0)$, $(0, 2)$ and $(0, 4)$ is investigated by taking into account ponderomotive nonlinearity in collisionless magnetized plasma. Following important conclusions are drawn from present analysis:

– As obvious, self-focusing of EHChG laser beams increases with increase in decentred parameters in transverse dimensions of the beam.
– Intensity distribution of asymmetric EHChG laser beams through mode indices results in explicit diffraction in transverse dimensions of the beam.

– Complexity in self-focusing character for TEM_{02} mode is higher than that of TEM_{00} mode of EHChG laser beams. However, for TEM_{04} mode, beam defocusses sharply.

The present results are useful in understanding the TEM mode-driven direct laser acceleration of electron under the influence of magnetic field, where mode index and decentred parameter play crucial role.

Author contribution statement

The beam-width parameter differential equations and numerical computation were performed by B.D. Vhanmore under the supervision of M.V. Takale. The manuscript was mainly drafted by S.D. Patil, following discussions with B.D. Vhanmore. All authors contributed to the conceptual design of the work, actively participated in the discussion of the results, provided valuable comments and insight, and contributed to the revision of the manuscript.

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