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Influence Of Critical Beam Power On Propagation Dynamics Of *q*-Gaussian Laser Beams In Isotropic Collisional Plasma

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Abstract: In current work, the beam-width parameter (BWP) equation is reduced to two variables such as q and critical beam power (p_0) which govern the propagation dynamics and intervals of critical beam power p_0 of a q-Gaussian laser beam have been investigated. The non-linear dependence of dielectric function in collisional plasma used herein is primarily because of heterogeneous heating of carriers along the wavefront of the laser beam. The influence of intervals of p_0 is further used to explore propagation dynamics of q-Gaussian laser beam in isotropic, homogeneous and collisional plasma has been studied. The Akhmanov's parabolic equation approach under WKB (Wentzel-Kramers-Brillouin) and paraxial approximations are used in current work. The results have been found reasonably interesting which are revealed graphically and discussed.

Keywords: Critical beam radius, q-Gaussian, Isotropic Collisional Plasma, Self focusing/defocusing.

1. Introduction:

Since the announcement of the first laser [1,2], the area of laser physics has developed at a rapid rate and its wide variety of applications such as high harmonic generation [3], inertial confinement fusion [4], laser ablation of materials [5], laser coupling to graphene plasmonics [6], X-ray lasers [7], etc. The 3 major mechanisms that contribute to modifications in the dielectric function of plasma in the investigations of laser and plasma interactions are: (1) collisional, (2) relativistic and (3) ponderomotive force. Self-focusing is nonlinear phenomena in which an intense beam of laser incident on a material medium changes the refractive index (R.I.) of the medium; as a consequence, the beam comes to focus within the medium. As laser beam incidents on the plasma, it modifies dielectric function of plasma results in modification of R.I. of the plasma; hence laser beam self-focuses/defocuses [8, 9].

The collisional plasma dynamics is basically dominated by local collisional forces rather than the collective actions in it. The prime source of field dependent dielectric function in the collisional plasma lies in non-uniform rearrangement of carriers because of heterogeneous heating of carriers along wavefront [10]. Most of the studies on the interaction of laser-plasma have been conducted by assuming Gaussian distribution of laser beam [11-13]. But in recent years, there has been considerable interest shown by many researchers towards the q-Gaussian laser beam [14-20]. P.K. Patel *et al.* [21] have reported in their investigation that in the case of Vulcan Peta watt laser the deviated Gaussian distribution called q-Gaussian. Furthermore, M. Nakatsutsumi *et al.* [22] have recommended beam intensity distribution can be rigorously featured by a q-Gaussian of the type $f(r) = f(0)\left(1 + \frac{r^2}{q\, r_0{}^2}\right)^{-q}$. B.D. Vhanmore *et al.* [17] have shown that a q-Gaussian laser beam profile is slowly converted into the Gaussian profile when $q \rightarrow \infty$. Recently, H.A. Salih *et al.* [20] have studied a basic heuristic method that is used to investigate the trouble in the travelling of a q-Gaussian laser beam through unmagnetized plasma for relativistic self-focusing.

The current paper explores the influence of intervals of the critical beam power p_0 on the propagation dynamics of q-Gaussian laser beams for distinct values of q in isotropic, homogeneous and collisional plasma. The present theoretical investigation is depends on the parabolic wave equation approach under WKB as well as paraxial approximations [9, 23].

2. Theoretical Formulation

Consider the q-Gaussian laser beam propagating through isotropic, homogeneous and collisional plasma along z-axis, the initial electric field distribution of laser beam at $z = 0$, is

$$E = A(r,z)\exp[i(\omega t - k_0 z)] \quad (1)$$

where, $k_0 = (\omega/c)\sqrt{\varepsilon_0}$, k_0 is the wave number, c is speed of light, and ω is frequency of laser beam. In the cylindrical co-ordinate system, wave equation describing electric field E of laser beam in the plasmas along with effective dielectric constant ε [9], is stated as,

$$\frac{\partial^2 \bar{E}}{\partial z^2} + \frac{\partial^2 \bar{E}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{E}}{\partial r} + \frac{\omega^2}{c^2}\varepsilon\, \bar{E} = 0 \quad (2)$$

and

$$\varepsilon = \varepsilon_0 + \phi\,(EE^*) - i\varepsilon_i \quad (3)$$

where, $\varepsilon_0 = 1 - \left(\frac{\omega_p{}^2}{\omega^2}\right)$ is linear part of dielectric constant, ω_p is frequency of plasma $\omega_p = \sqrt{4\pi\, n_e\, e^2/\, m_0}$, where m_0, e, and n_e are rest mass of electron, the electronic charge, and plasma electron's density without laser beam, respectively. We restrict for the case when ε_i is field independent and $\varepsilon_i \ll \varepsilon_0$. The term $\phi\,(EE^*)$ represents a field dependent dielectric constant, for collisional plasma [9],

$$\phi\,(EE^*) = \frac{\omega_p{}^2}{\omega^2}\left[1 - \left(1 + \frac{\alpha}{2}EE^*\right)^{\frac{s}{2}-1}\right], \quad (4)$$

with

$$\alpha = \left(\frac{e^2 M}{6\, k_B T_0 \omega^2 m^2}\right),$$

where, k_B, M, m, T_0 are Boltzmann constant, ion mass, electron mass, temperature of the plasma, respectively. The collision nature is described by parameter s. When electrons collide with neutral particles, then s taken as 1, and when electrons collide with ions, then s may be taken as - 3. In the current work, for s = 1 case has been considered for the collisions of neutral particles with electrons. Substituting E and ε from equations (1) and (3) in equation (2), one can obtain parabolic wave equation,

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} + \frac{\omega^2}{c^2}\phi(EE^*)\,A = 2ik_0\,\frac{\partial A}{\partial z} \qquad (5)$$

where

$$A(r,z) = A_0(r,z)\exp\,[-ikS(r,z)] \qquad (6)$$

where, S is the eikonal and $A_0(r,z)$ and $S(r,z)$ are the real functions of r and z. From equations (5) and (6) we get

$$2\left(\frac{\partial S}{\partial z}\right) + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) + \frac{\phi}{\varepsilon_0}\,(A_0 {A_0}^*) \qquad (7)$$

$$\frac{\partial A_0^2}{\partial z} + \left(\frac{\partial S}{\partial r}\right)\left(\frac{\partial A_0^2}{\partial r}\right) + \left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right)A_0^2 = 0 \qquad (8)$$

By following Akhmanov *et al.* [23] and Sodha *et al.* [9], the solutions of equations (7) and (8) are given by

$$S = \frac{r^2}{2}\beta(z) + \phi(z) \qquad (9)$$

and

$$A_0^2(r,z) = \frac{{E_0}^2}{f^2(z)}\left(1 + \frac{r^2}{f^2(z)\,q\,r_0^2}\right)^{-q}, \qquad (10)$$

where, $\phi(z)$ is the phase shift, $\beta(z)$ can written as $\left(\frac{1}{f}\right)\left(\frac{\partial f}{\partial z}\right)$ and it is the inverse of radius of curvature, r_0 is the initial radius of the laser beam, f is BWP and it measures width of the beam as well as axial intensity. By following paraxial approach specified by Akhmanov *et al.* [23] as well as subsequently extended by Sodha *et al.* [9], we get the following second order differential equation,

$$\frac{d^2 f}{d\xi^2} = \left[\frac{(4+q)}{q\,f^3}\right] - \left[\frac{2\,\alpha {E_0}^2 f\,\omega_p^2\,r_0^2}{\left(\alpha {E_0}^2 + 2\,f^2\right)^2 c^2}\right] \qquad (11)$$

where, $\xi = z/R_d$ known as dimensionless propagation distance and $R_d = k\,r_0^2$ is known as Rayleigh diffraction length. Equation (11) solved numerically using proper boundary condition f = 1 and $\partial f/\partial z = 0$.

3. Result, Discussion, and Conclusions:

The nonlinear second order, differential equation (11) governs propagation dynamics of q-Gaussian laser beam. Equation (11) is solved numerically using the parameters: ω_p= 1.7760×10^{15} rad/s, r_0 = 20×10^{-4} cm, c = 3×10^{10} cm/s, n_0 = 10^{18} cm^{-3}.
It is crucial to mention here that as $q \to \infty$ the equations (11) becomes

$$\frac{d^2 f}{d\xi^2} = \left(\frac{1}{f^3}\right) - \left(\frac{2\,p\,f\,\rho^2}{(p + 2\,f^2)^2}\right) \qquad (12)$$

where, $p = \frac{\alpha E_0^2}{f}$ and $\rho = \frac{\omega_p\, r_0}{\mathrm{c}}$. The equation (12) is equivalent to the previous equation obtained by Sodha *et al.* [9] as well as Valkunde *et al.* [24] for the Gaussian laser beam propagating in the collisional plasma. Under critical conditions f = 1, $d^2 f/d\xi^2 = 0$. R.H.S. of equation (11) reduces to

$$F(p_0) = \left(\frac{4+q}{q}\right) - \left(\frac{280.3712\;p_0}{(p_0 + 2)^2}\right) \qquad (13)$$

where, p_0 and ρ_0 are critical beam power and critical beam radius, respectively. Equation (13) shows dependence of $F(p_0)$ on p_0. Figure 1 shows 3 distinct regions of propagation dynamics as mentioned in table I. From Figure 1, it has been seen that the interval of p_0 for self focusing and defocusing increases with increase in the values of q.

For $q = \infty$,

I) Self-focusing region: $F(p_0) < 0$ for $0.014474 < p_0 < 276.357$
II) Defocusing region: $F(p_0) > 0$ for $276.357 < p_0 < 0.014474$
III) Self-trapping points: $F(p_0) = 0$ for $p_0 = 0.014474$ and $p_0 = 276.357$

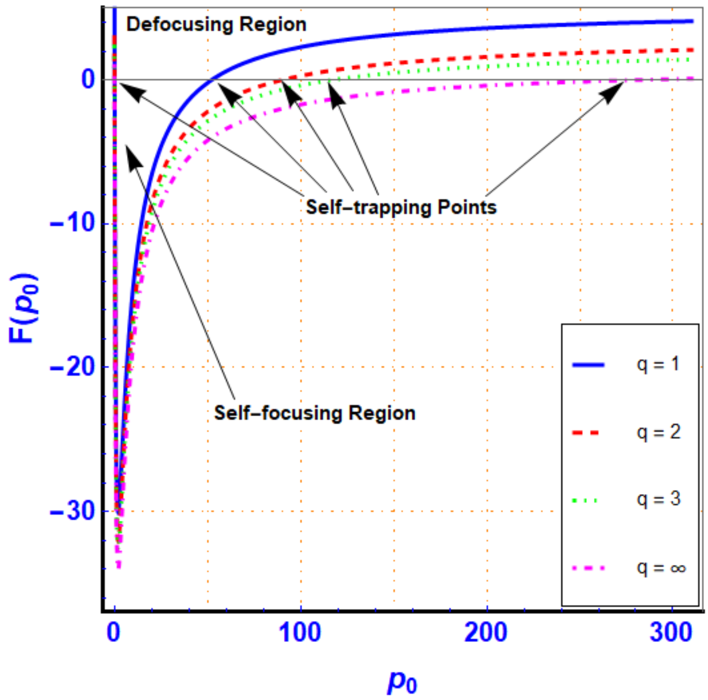

Figure 1: Variation of $F(p_0)$ with respect to critical beam power p_0 for various values for q (q = 1, 2, 3 and ∞)

Figure 1 depicts the limits of the critical beam power for three different regions by analytical investigation, as mentioned in table I.

Table I

q	Self-focusing region $F(p_0) < 0$	Defocusing region $F(p_0) > 0$	Self-trapping points $F(p_0) = 0$
1.	$0.0769271 < p_0 < 51.9973$	$51.9973 < p_0 < 0.0769271$	$p_0 = 0.0769271$ and $p_0 = 51.9973$
2.	$0.0447366 < p_0 < 89.4123$	$89.4123 < p_0 < 0.0447366$	$p_0 = 0.0447366$ and $p_0 = 89.4123$
3.	$0.0344457 < p_0 < 116.125$	$116.125 < p_0 < 0.0344457$	$p_0 = 0.0344457$ and $p_0 = 116.125$
∞	$0.014474 < p_0 < 276.357$	$276.357 < p_0 < 0.014474$	$p_0 = 0.014474$ and $p_0 = 276.357$

Analytical investigation of Self-focusing, Defocusing, Self-trapping for various values for q (q = 1, 2, 3 and ∞)

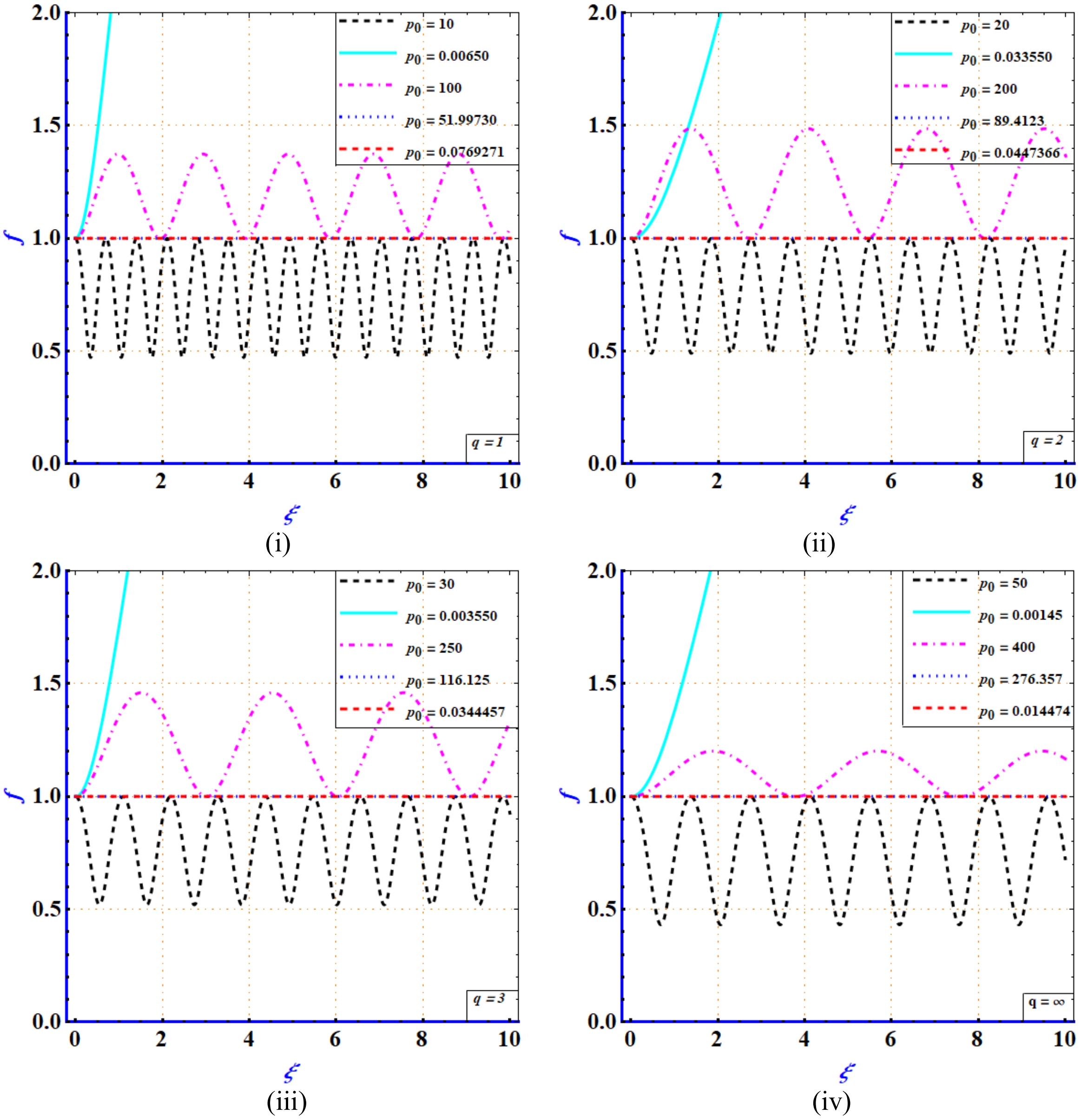

Figure 2: BWP f versus propagation distance ξ in isotropic homogeneous collisional plasma for various values for q [(i) $q = 1$, (ii) $q = 2$, (iii) $q = 3$ and (iv) $q = \infty$]

From Figure 2, oscillatory self-focusing is noticed within the self-focusing interval of $\boldsymbol{p_0}$ as mentioned in table I. In Figure 2, within the defocusing interval of $\boldsymbol{p_0}$, for smaller values of $\boldsymbol{p_0}$ ($\boldsymbol{p_0} < 0.0769271$, $\boldsymbol{p_0} < 0.0447366$, $\boldsymbol{p_0} < 0.0344457$, $\boldsymbol{p_0} < 0.014474$ respectively) steady defocusing is observed while for large values of $\boldsymbol{p_0}$ ($\boldsymbol{p_0} > 51.9973$, $\boldsymbol{p_0} > 89.4123$, $\boldsymbol{p_0} > 116.125$, $\boldsymbol{p_0} > 276.357$ respectively) oscillatory defocusing is observed. Figure 2, represents propagation dynamics of q-Gaussian laser beams at distinct intervals of critical beam power mentioned in table I. Finally, in conclusion the present analysis represents that propagation dynamics of a q-Gaussian laser beam in isotropic, homogeneous and collisional plasma can be explored very effectively, under the pre-conditioning of the critical beam power $\boldsymbol{p_0}$.

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