Vivekanand College Kolhapur (An Empowered Autonomous Institute)

B.Sc. I Semester I

Open Book Test

Paper: Elementary Probability Theory

Date: 13/08/2025 **Total Marks: 10M**

Instructions:

Attempt all questions

Each question carries 2 Marks

Q 1. Define the following terms (4M)

1. Sample space
2. Event
3. Certain event
4. Mutually exclusive event
5. Exhaustive event
6. Compliment of an event
7. Classical definition of Probability
8. Axiomatic definition of Probability

Q2 State & Prove addition theorem for probability of 2 and 3 events. **3M**

Q3. Let A and B be 2 events defined on sample space S then show that

$$P(A' \cap B) = P(B) - P(A \cap B)$$ **3M**

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Q.1) Define with example

i) Sample space
→ The set of all possible distinct outcomes of an experiment is called a sample space.

eg:- i) Tossing of a coin
$S = \{H, T\}$

ii) Blood Group of person
$S = \{A, B, O, AB\}$

ii) Event
Any subject of a sample space is called an event.

eg:- i) The suvan rises in the east and sets in the west.

iii) Certain event
The event which contains all possible outcome of a sample space.

eg:- i) Getting head or tail when we toss a coin $A = \{H, T\}$

ii) Getting a no ≤ 6 if we rolled a die
$S = \{1, 2, 3, 4, 5, 6\}$.

iv) Mutually exclusive events.
Two events A and B are said to be mutually exclusive events if and only if they cannot happen simultaneously.

i.e $A \cap B = \{\} = \phi$

v) Exhaustive events.

Two events A and B are said to be exhaustive if and only if their union is a sample space.

eg:- In tossing of a coin

$S = \{H, T\}$

Event A: getting head

$A = \{H\}$

Event B: getting tail

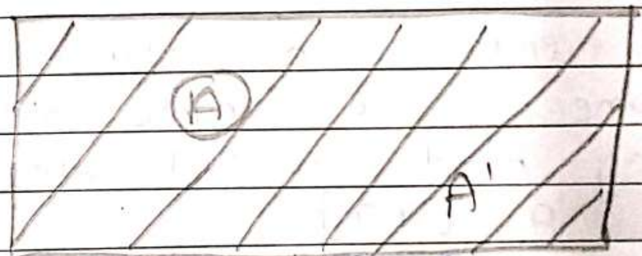
$B = \{T\}$

$\therefore A \cup B = \{H, T\}$

$\therefore A \cup B \cancel{\neq A \cap B} = \Omega$

i.e. A and B are exhaustive events.

vi) Complement of an event



If A is the event of Ω then complement of A is ploted by A' to A'

A' is the event containing all points of sample space Ω which are not in A.

eg. $\Omega = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$

$A' = \{2, 4, 6\}$

viii) Classical Defination of probability

If a random experiment result in 'n' mutually exclusive and equally likely outcomes, out of which 'm' are favourable to the event A, then the probability of occurance of the event is given by,

$$P(A) = \frac{m}{n}$$

$$P(A) = \frac{\text{no. of elements in event } A}{\text{no. of events in sample space}}$$

eg:- i) An integer between 1 and 100 (both inclusive) is selectial of random, find the probability of selecting a perfect square if,

→ $S = \{1, 2, 3, 4 \ldots 100\}$

$n = 10(S) = 100$

Event A: getting perfect square

$A = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

$n(A) = 10$

viii) Axiomatic defination of probability

let Ω be a sample space corresponding to a random experiment let A be any of A i.e P(A) is defined as any ramd. Function of Ω which statisfies the following ~~Actis~~ Axioms.

Axiom 1: $P(A) > 0$

Axiom 2: $P(\Omega) = 1$

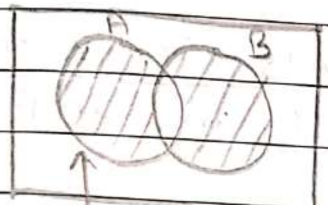
Axiom 3 : let $A_1, A_2, \ldots A_n$ be 'n' mutually exclusive disjoint events on Ω then

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

In particular if A and B are disjoint events define Ω then $P(A \cup B) = P(A) + P(B)$

Q.2] State and prove addition theorem of probability for two events A and B also state the same for three events A, B, C -

→



$(A \cap B)$ $(A' \cap B)$

statement :- let A and B be any two event defined and sample space.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

B is composed of two disjoint events $A \cap B$ and $\cancel{A} \cap B'$

i.e $\cancel{A} = (A \cap B) \cup (A \cap B')$

$P(A) = P(A \cap B) + P(A \cap B')$ — Ax — 3

$\therefore P(A \cap B') = P(A - P(A \cap B)$ — (i)

Similarly,

B can expressed as

$B = (A \cap B) \cup (A' \cap B)$

$\therefore P(B) = P(A \cap B) + P(A \cap B)$

We can write ~~A∩B~~ $(A \cup B)$ in terms of disjoint events:

$(A \cap B)$, $(A' \cap B)$ $(A \cap B')$

i.e $(A \cup B) = (A \cap B') \cup (A \cap B) \cup (A' \cap B)$

$\therefore P(A \cup B) = P(A \cap B') + P(A \cap B) + P(A' \cap B)$

putting values from eqn (i) and (ii) we get,

$P(A \cup B) = P(A) - P(A \cap B) + P(A \cap B) + P(B) - P(A \cap B)$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

If A, B and C are three events define an sample space Ω the $P(A \cup B \cup C)$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

Proof :- Assume $B \cup C = D$

$\therefore P(A \cup B \cup C) = P(A \cup D)$

$= P(A) + P(D) - P(A \cap D)$ ... condition th^m of probability.

$\therefore P(A) + P(B \cup C) - P(A \cap [B \cup C])$

$= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)]$

$= P(A) + P(B) + P(C) - P(B \cap C) - [P(A \cap B)] + P(A \cap C) - P[(A \cap B) \cap P(A \cap C)]$

$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$

Q.3) let A and B be the events defined on sample space Ω prove that

i) $0 \le P(A \cap B) \le P(A) \le P(A \le P(A) + P(B)$

ii) $P(A' \cap B) = P(B) - P(A \cap B)$

→ i) $(A \cap B) \subset A$

$P(A \cap B) \le P(A)$ — (1)

$A \subset (A \cup B)$

$P(A) \le P(A \cup B)$ — (II)

by addition th^m W.K.T

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) \ge 0$ — Ax - (1)

This implies that,

$P(A \cup B) \leq P(A) + P(B)$

from eq^n 1, 2 and (ii)

$P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$

ii) To prove - $P(A' \cap B) = P(B) - P(A \cap B)$

Proof :- Event B is composed of two disjoint $A \cap B$ and $A' \cap B$

i.e $B = (A \cap B) \cup (A' \cap B)$

$\therefore P(B) = P(A \cap B) + P(A' \cap B)$

$$\boxed{P(A' \cap B) = P(B) - P(A \cap B)}$$

$\frac{10}{10}$ Date 13 / 8 / 23

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Elementary Probability Theory

Q1. Define with example

1] Sample Space
The set of all possible distinct outcomes of an experiment called as sample space
e.g. 1] tossing of the coin
$S = \{H, T\}$

2] Event
The subset of sample space is called as event
e.g. 1] Tossing of the coin.
$S = \{H, T\}$
Event A = getting a head
$A = \{H\}$

3] Certain event
The event which contains all possible outcomes of a sample space is called as Certain event.
e.g. 1] getting head or tail when we toss a coin
$A = \{H, T\}$

4] Mutually exclusive event.
Two events A and B are said to be mutually exclusive event. If and only if they can't happen symaltanously i.e
$A \cap B = \{\} = \emptyset$

eg. 1] In a rolling of a die
$S = \{1, 2, 3, 4, 5, 6\}$

Event A: getting a even no.
$A = \{2, 4, 6\}$

Event B: getting a multiple of 5
$B = \{5\}$

$\therefore A \cap B = \phi$
i.e. A and B are mutually exclusive event.

5] Exhaustive event
Two events A and B are said to be exhaustive event if and only if their union is a sample space

e.g. 1] In rolling of a die
$(\Omega)\ S = \{1, 2, 3, 4, 5, 6\}$

Event A: getting an even numbers.
$A = \{2, 4, 6\}$

Event B: getting an odd numbers.
$B = \{1, 3, 5\}$

$\therefore A \cup B = \{1, 2, 3, 4, 5\}$
$\therefore A \cup B = \Omega$
i.e. A and B are exhaustive events.

6] Complement of an event
The Complement of an event A is the event that A does not occur.
it denoted by A' or A^c
A' is the event containing all points of sample space Ω which are not in A

e.g. $\Omega = \{1,2,3,4,5,6\}$
$A = \{1,3,5\}$
$A^c = \{2,4,6\}$

7] Classical defination of probability
Probability of an event is called as classical defination of probability.
e.g. C : The product of two numbers is not greater than 9

1] Two fair dies are rolled
sample space (Ω) = $\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

Event C : The product of two numbers is not greater than 9.

$C = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$
$(2,1),(2,2),(2,3),(2,4),\cancel{(2,5)},\cancel{(2,6)}\ (3,1)$
$(3,2),(3,3),(4,1),(4,2),(5,1),(6,1)\}$

$m = n(c) = 17$

$p(c) = \frac{m}{n} = \frac{17}{36}$

8] Axiomatic defination of probability
let Ω be a sample space corresponding to random experiment let A, B any event of Ω probability of 'A' it is denoted by P(A) is defined as any real valued functi on Ω. which satisfies the following axion.

Axiom 1 : $P(A) \geq 0$

Axiom 2 : $P(\Omega) = 1$

Axiom 3 : let $A_1, A_2 \ldots A_n$ be 'n' mutually exclusive. OR

is perticular if A and B are dis joint event It define as the Ω then $P(A) + P(B)$

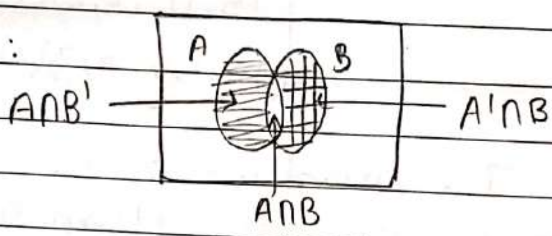
Q2. state and prove

1] addition ~~be~~ theorem of probability for two events. A and B

Statement : let A and B be any two event define an sample space Ω

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Diagram :



Proof : Event A is Composed of two disjoint event $A \cap B$ and $A \cap B'$

i.e $A = (A \cap B) \cup (A \cap B')$

$P(A) = P(A \cap B) + P(A \cap B')$ — Ax 3

$\therefore P(A \cap B') = P(A) - P(A \cap B)$ — I

Similarly B can expressed as

$B = (A \cap B) \cup (A' \cap B)$

$P(B) = P(A \cap B) + P(A' \cap B)$ — Ax 3

$\therefore P(A' \cap B) = P(B) - P(A \cap B)$ — II

We can write $(A \cup B)$ in terms of three disjoint events $(A \cap B), (A' \cap B), (A \cap B')$

i.e $(A \cup B) = (A \cup B') \cup (A \cap B) \cup (A' \cap B)$

$P(A \cup B) = P(A \cap B') + P(A \cap B) + P(A' \cap B)$

Putting values from eq^n I and II

We get. $P(A\cup B) = P(A) - \cancel{P(A\cap B) + P(A\cap B)} + P(B) - P(A\cap B)$

$$= P(A) + P(A\cap B) + P(B)$$

$$\therefore \boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$$

2] Addition theorem of probability for three events A, B, C.

Statement : If A, B, C are defined of sample space of Ω then

$$P(A\cup B\cup B) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$$

Proof : Assume $(B\cup C) = D$

$$\therefore P(A\cup B\cup C) = P(A\cup B)$$

$$= P(A) + P(D) - P(A\cap D)$$

$$= P(A) + P(B\cup C) - P[A\cap(B\cup C)]$$

$$[(A\cap B)\cup(A\cap C)]$$

$$= P(A) + P(B) + P(C) - P(B\cap C) - [P(A\cap$$

$$+ P(A\cap C) - P((A\cap B)\cap(A\cap C))]$$

$$\therefore \boxed{P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(B\cap C) - P(A\cap B) - P(A\cap C) + P(A\cap B\cap C)}$$

mutually exclusive event. If and only if they can't happen symaltanously i.e $A\cap B = \{\} = \emptyset$

Q3. Let A and B be the events defined on Samp
space Ω
Two events A and B be define on Sample space Ω are said to be independant if and only if $P(A\cap B) = P(A).P(B)$

2) Prove that

1) $0 \leq P(A\cap B) \leq P(A) \leq P(A\cup B) \leq P(A) + P(B)$

proof : $(A\cap B) \subset A$

$$P(A\cap B) \leq P(A) \quad \text{--- I}$$

$$A \subset (A\cup B)$$

$$\therefore P(A) \leq P(A\cup B) \quad \text{--- II}$$

we know that, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$$P(A\cap B) \geq 0 \quad \text{--- Ax-I}$$

$$P(A\cup B) \leq P(A) + P(B) \quad \text{--- III}$$

From eq^n I, II, III

$$\boxed{0 \leq P(A\cap B) \leq P(A) \leq P(A\cup B) \leq P(A) + P(B)}$$

2) $P(A'\cap B) = P(B) - P(A\cap B)$

proof : Event B is Composed of two disjoin events $(A\cap B) \cup (A'\cap B)$

i.e $B = (A\cap B) \cup (A'\cap B)$

$$\therefore P(B) = P(A\cap B) + P(A'\cap B) \quad \text{--} \quad Ax.3$$

$$\therefore \boxed{P(A'\cap B) = P(B) - P(A\cap B)}$$