Vivekanand College Kolhapur (An Empowered Autonomous *Institute*)

B.Sc. II Semester III

Surprise Test

Paper V: Probability Distributions I

Date: 12/09/2025 **Total Marks: 20M**

Instruction:

Attempt all questions

Q 1 Define Geometric distribution. Hence find its mean and variance. **10 M**

Q 2. State and prove memoryless property of geometric distribution. (5M)

Q 3. Define negative binomial distribution and find its mean. (5M)

20/20

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* Name :- Gautami Namdev Kamble

* Class :- B.Sc - II

* Roll No :- 7755

* Paper Name :- Probability Distributions - I

Q. 1] Define Geometric Distribution, find it's Mean and Variance.

Ans. Geometric Distribution =

A Discrete Random Variable x is said To Follows Geometric Distribution with parameter p. Then it's pmf is Given by,

$$P(X=x) = q^x . p \quad ; \quad x = 0,1,2,\dots ; \quad 0 \le p \le 1 ; \quad p+q = 1$$

$$= 0 \quad ; \quad \text{otherwise.}$$

- Mean of Geometric Distribution =

Let $X \sim G(p)$; Then There Pmf is Given by,

$$P(X=x) = q^x . p \quad ; \quad x = 0, 1, 2, \dots ; \quad 0 \le p \le 1 ; \quad q+p = 1$$

$$= 0 \quad ; \quad \text{Other wise.}$$

By Defination of Mean We Have,

$$\text{Mean} = E(x) = \sum_{x=0}^{\infty} x \cdot P(X=x)$$

$$E(x) = \sum_{x=0}^{\infty} x \cdot q^{x} p$$

$$= p \cdot q \sum_{x=1}^{\infty} x \cdot q^{x-1}$$

$$= pq\,[1 + 2q + 3q^2 + 4q^3 + \cdots]$$

$$= pq\,(1-q)^{-2}$$

$$= \frac{pq}{p^2}$$

$$E(x) = \frac{q}{p}$$

$$\boxed{\text{Mean} = E(x) = \frac{q}{p}}$$

- Variance of Geometric Distribution =

We Know That,

$$E(x) = \frac{q}{p}$$

$$\text{Now, } Var(x) = E(x^2) - [E(x)]^2 \quad \text{——— (I)}$$

consider,

$$E(x^2) = E[x(x-1) + x]$$

$$E(x^2) = E[x(x-1)] + E(x) \quad \text{——— (1)}$$

Now,

$$E[x(x-1)] = \sum_{x=0}^{\infty} x(x-1) \cdot P(X=x)$$

$$= \sum_{x=0}^{\infty} x(x-1) \cdot q^{x} \cdot p$$

$$= pq^2 \sum_{x=2}^{\infty} x(x-1)\, q^{x-2}$$

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$$= pq^2 \, [2 + 6q + 12q^2 + \cdots]$$
$$= 2pq^2 \, [1 + 3q + 6q^2 + \cdots]$$
$$= 2pq^2 \, (1-q)^{-3}$$
$$= 2pq^2 \, (p)^{-3}$$
$$= \frac{2pq^2}{p^3} = \frac{2q^2}{p^2}$$

$\therefore E(x^2) = \frac{2q^2}{p^2}$

$\therefore$ Equation (1) becomes,

$$E(x^2) = E[x(x-1)] + E(x)$$
$$= \frac{2q^2}{p^2} + \frac{q}{p} \quad \text{----} \; (2)$$

Putting The value of Equation (2) in equation (I)

$$\therefore \text{Var}(x) = E(x^2) - [E(x)]^2$$
$$= \frac{2q^2}{p^2} + \frac{q}{p} - \left(\frac{q}{p}\right)^2$$
$$= \frac{2q^2}{p^2} + \frac{q}{p} - \frac{q^2}{p^2}$$
$$= \frac{q^2}{p^2} + \frac{q}{p}$$
$$= \frac{q}{p}\left[\frac{q}{p} + 1\right]$$
$$= \frac{q}{p}\left[\frac{q+p}{p}\right]$$

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$$= \frac{q}{p} \times \frac{1}{p}$$
$$= \frac{p}{q^2}$$

$$\therefore \boxed{\text{Var}(x) = \frac{q}{p^2}}$$

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$\therefore$ Mean of Geometric Distribution is q/p and variance of Geometric Distribution is q/p^2....

Q. 2] State And Prove Memoryless property of Geometric Distribution.

Ans. Statement : If $X \sim G(p)$ Then For Any Two Positive integers s and t We Have,

$$P[X \geq s+t \mid X \geq s] = P(X \geq t)$$

Proof : If $X \sim G(p)$ then its pmf is Given by

$$P(X=x) = q^x p \quad ; x = 0, 1, 2, \ldots \quad 0 \leq p \leq 1, \; p+q=1$$
$$= 0 \quad ; \text{otherwise}$$

Now consider,

$$P[X \geq x] = \sum_{X=x}^{\infty} P(X=x)$$

$$= \sum_{X=x}^{\infty} q^x p$$

$$= p[q^x + q^{x+1} + q^{x+2} + \cdots]$$

$$= pq^x[1 + q + q^2 + \cdots]$$

$$= pq^x[1-q]^{-1}$$

$$= pq^x(p)^{-1}$$

$$= \frac{pq^x}{p}$$

$$= q^x$$

$$\therefore P[X \geq x] = q^x$$

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Put $x = s+t$ then
$$P[X \geq s+t] = q^{s+t}$$
Put $x = s$ then,
$$P[X \geq s] = q^{s}$$

Now Consider The LHS,
$$P[X \geq s+t \mid X \geq s]$$

$$= \frac{P[X \geq s+t,\ X \geq s]}{P[X \geq s]}$$

$$= \frac{P[X \geq s+t]}{P[X \geq s]} = \frac{q^{s+t}}{q^{s}} = \boxed{q^t}$$

Which is The same as $P[X \geq t]$

$$\therefore \boxed{P[X \geq s+t \mid X \geq s] = P[X \geq t]}$$

Hence, Here Prove The Memoryless property of Geometric Distribution ....

Q.3] Define Negative Binomial Distribution And find its Mean.

Ans. Negative Binomial Distribution =
Suppose A Trial Result into two out-comes sucess and failure with Constant probability p and q. Resp. Suppose X denotes The no. of Failures before Getting The k^{th}

success. In other Words We Get the k^{th} success at $x+k^{th}$ trial.

A Discrete Random Variable X is said To follows Negative Binomial Distribution with Parameter K. p. Then its Pmf is Given by,

$$P(X=x) = {}^{x+k-1}C_x \cdot p^K \cdot q^K \quad ; \quad x = 0, 1, 2, \cdots$$
$$0 \leq p \leq 1$$
$$q + p = 1$$

$$= 0 \quad ; \quad \text{otherwise}$$

And it can be Denoted by $X \sim NBD(k, p)$

- Mean of Negative Binomial Distribution =

We Have,

$$\sum_{x=0}^{\infty} P(X=x) = 1$$

$$\therefore \sum_{x=0}^{\infty} {}^{x+k-1}C_x \cdot p^x \cdot q^x = 1$$

$$\therefore \sum_{x=0}^{\infty} {}^{x+k-1}C_x \cdot q^x = p^{-K}$$

$$\therefore \sum_{x=0}^{\infty} {}^{x+k-1}C_x \cdot q^x = (1-q)^{-k} \quad \cdots\cdots (1)$$

The Series on L.H.S of equation (1) is Absolutely Conversion hence Term by Term diff. w.r. to q is valid.

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$$\sum_{x=0}^{\infty} {}^{x+k-1}C_x \, x \, q^{x-1} = -k(1-q)^{-k-1} \cdot (-1)$$

$$\therefore \sum_{x=0}^{\infty} {}^{x+k-1}C_x \, x \, q^{x-1} = k(1-q)^{-k-1}$$

Multiplying both sides by qp^k We get,

$$\sum_{x=0}^{\infty} {}^{x+k-1}C_x \, x \, q^{x-1} \cdot qp^k = k(1-q)^{-k-1} \cdot qp^k$$

$$\therefore \sum_{x=0}^{\infty} {}^{x+k-1}C_x \, x \cdot q^x \cdot p^k = \frac{kq}{1-q} = \frac{kq}{p}$$

$$\therefore \boxed{\text{Mean} = E(x) = \frac{kq}{p}}$$

$\therefore$ The mean of Negative Binomial Distribution is $\frac{kq}{p}$ ....

Q.4] State And prove The Addetive Property of Negative Binomial Distribution.

Ans. Statement : If $X \sim NBD(k_1, p)$ And $Y \sim NBD(k_2, p)$, X and Y are independent Random variable. Then X+Y has Negative Binomial Distribution with Parameters.

$(k_1 + k_2, p)$

BSc - SY

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Subject :- Probability distribution - I
Roll No. :- 7750.

Q1. Define geometric distribution find it's mean and Variance.

→ A discreate random variable x is said to be follow geometric distribution with parameter 'p' if it's pmf is given by

$$P(X=x) = q^x p \qquad x = 0, 1, 2, 3, \ldots \qquad 0 \le p \le 1$$

$$= 0 \qquad \text{o.w}$$

Mean and variance :-

Let $X \sim G(p)$ the it's pmf is given by

$$P(X=x) = q^x p \qquad x = 0, 1, 2, 3, \ldots \qquad 0 \le p \le 1, \quad q + p = 1$$

by def^n of mean we have

$$\text{mean} = E(x) = \sum x P(x)$$

$$= \sum x q^x \cdot p$$

$$= \sum x \cdot q^{x-1} p$$

$$= p \sum x p^x$$

$$= pq \sum x \, q^{x-1}$$

$$= pq \sum [1 + 2p + 3p^2 + 4p^3 + \ldots +]$$

$$= pq (1-q)^{-2}$$

$$= pq\,p^{-2}$$

$$= \frac{pq}{p^2}$$

mean = $\boxed{E(X) = \frac{q}{p}}$

$Var(X) = E(X)^2 - [E(x)]^2$

Consider,

$$E(x)^2 = E(x(x-1)+x]$$

$$= E[x(x-1)] + E(x) \quad \text{---- (1)}$$

Now consider

$$E[x(x-1)] = \sum x(x-1)\; P(X=x)$$

$$= \sum x(x-1)\, q^x p$$

$$= pq \sum x(x-1)\, q^{x-1} p$$

$$= pq^2 [2 + 6q + 6q^2 + \dots +]$$

$$= 2pq^2 [1 + 3q + 6q^2 + \dots +]$$

$$= 2pq^2 (1-q)^{-3}$$

$$= 2pq^2 (1-q)^{-3}$$

$$= 2pq^2\, q^{-3}$$

$$= \frac{2pq^2}{p^3}$$

$$= \frac{2q^2}{p^2}$$

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$$E(x)^2 = E[x(x-1)] + E(x)$$

$$= \frac{2q^2}{p^2} + \frac{q}{p}$$

putting eq^n ① in eq^n

$$= E(x)^2 - [E(x)]^2$$

$$= 2\frac{q^2}{p^2} + \frac{q}{p} - \left[\frac{q}{p}\right]^2$$

$$= 2\frac{q^2}{p^2} - \frac{q^2}{p^2} + \frac{q}{p}$$

$$= \frac{q^2}{p}[2-1] + \frac{q}{p}$$

$$= \frac{q^2}{p^2} + \frac{q}{p}$$

$$= \frac{p}{q}\left[\frac{q}{p} + 1\right]$$

$$= \frac{q}{p}\left[\frac{q+p}{p}\right]$$

$$= \frac{q}{p} \cdot \frac{1}{p}$$

$$= \frac{q}{p^2}$$

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$$\boxed{Var(x) = \frac{q}{p^2}}$$

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Q2 State and prove memoryless property of geometric distribution.

→ If $X \sim G(p)$ there for any two positive integer s & t we have

$$P[X \geq s+t / X \geq s] = P[X \geq t]$$

Proof :-

if $X \sim G(p)$ then it pmf is given by

$$P(X=x) = q^x p \qquad x = 0, 1, 2, 3, \dots \quad 0 \leq p \leq 1$$

Consider

$$P[X \geq x] = \sum P \; P(X=x)$$

$$= \sum_{x=x}^{\infty} q^x p$$

$$= p[q^x + q^{x+1} + q^{x+2} + \dots +]$$

$$= pq^x [1 + q + q^2 + \dots +]$$

$$= pq^x [1-q]^{-1}$$

$$= \frac{pq^x}{p} = q^x$$

$$= P[X \geq x] = q^x \quad \text{---- (1)}$$

put $x = s+t$

$$P[X \geq s+t] = q^{s+t}$$

$$P[X \geq s] = q^s$$

Now consider,

$$= \frac{P[X \geq s+t / X \geq s]}{P[X \geq s)}$$

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$$= \frac{P[X \geq s+t]}{P[X \geq s]}$$

$$= \frac{q^{s+t}}{q^{s}}$$

$$= q^{t}$$

which is same as $P[X \geq t]$

$$\therefore P[X \geq s+t / X \geq s] = P[X \geq t]$$

Q3 Define negative binomial distribution and it's mean.

A discreate random variable x is said to Follow geometric distribution with parameter ('k and p' if it's pmf is given by

$$P(X=x) = {}^{x+k-1}C_{x} \; p^{k} \; q^{x} \qquad x = 0,1,2,3,\ldots$$

$$0 \leq p \leq 1$$

$$p+q=1$$

$$= 0$$

Mean of Negative binomial distribution.

We have

$$= \sum_{x=0}^{\infty} P(X=x) = 1$$

$$= \sum {}^{x+k-1}C_{x} \; p^{k} \; q^{x} = 1$$

$$= \sum {}^{x+k-1}C_{x} \; p^{k} = (1-p)^{-x}$$

$$= \sum {}^{x+k-1}C_{x} \; q^{x} = p^{-k}$$

$$\sum {}^{x+k-1}C_x \, q^x = (1-q)^{-k} \quad \text{----} \quad (1)$$

The series of L.H.S on eqⁿ (1) is absolutly Conversion hence term by term differentiation w.r.t 'q' is Valid.

$$\sum {}^{x+k-1}C_x \, x \, q^{x-1} = -k(1-q)^{-k-1}(-1)$$

$$\sum {}^{x+k-1}C_x \, x \, q^{x-1} = k(1-q)^{-k-1}$$

Multiply both side by $q \cdot p^k$ we get

$$\sum {}^{x+k-1}C_x \, x \cdot q^{x-1} \, q p^k = k(1-q)^{-k-1} \cdot q p^k$$

$$\sum {}^{x+k-1}C_x \, q^x \, p^k = \frac{kq}{1-q}$$

$$\sum x \cdot P(X=x) = \frac{kq}{p}$$

$$\text{mean} = E(X) = \frac{kq}{p}$$

Vivekanand College Kolhapur (An Empowered Autonomous Institute)

B.Sc. II Semester IV

Surprise Test

Paper VII: Testing of Hypothesis

Date: 05/01/2026 **Total Marks: 20M**

Instructions: Attempt all questions

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1 Define the terms (5 M)

a) Simple hypothesis

b) Null hypothesis

2. Define P value and power of test. Explain general steps in procedure of testing of hypothesis. (5 M)

3. Define the terms: (5M)

a) Critical region

b) Central limit theorem

4 Define the terms: (5M)

a) Standard error

b) Alternative hypothesis and its types

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Godane Poonam Sachin

|| ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ||

– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

15/20

कोल्हापूर १९६४

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :	
Seat No./ Roll No. :	7739
Seat No./ Roll No. In words	
	01958

Unit Test - I

प्र. क्र. Q. No.

Unit : Testing of Hypothesis - I

Date: 24/12/2025

Time: 30 minutes

Marks: 20

Q.1]

1) Simple Hypothesis :-

A statistical Hypothesis which can be completely specifies the probability Distribution or Parameter of The probability Distribution, are called as simple Hypothesis

ex.., i) Let $x_1, x_2, \ldots, x_n$ be the Random sampling of sample size n from $N(\mu, \sigma^2)$ then the simple Hypothesis is,

$H_0: \mu = \mu_0 \;, \sigma^2 = \sigma_0^2$

2) Null Hypothesis :-

A Hypothesis with no difference is

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is called as Null Hypothesis.

- Which is Denoted by Ho.
- ex., i) let $H_0: P_1 = P_2$ i.e there is no significance Difference between Two Population Praportion.

Q.2] 1) P-value : ~~Maximu~~ minimum Prob. of rejecting null Hypothesis Ho when null Hypothesis Ho is True.

2) Power of Test :-

The Probability of Rejecting Null Hypothesis Ho When We Accept the Alternative Hypothesis H1, is called as Power of Test.

i.e.

$$\text{Power of Test} = P[\text{reject } H_0 \mid \text{Accept } H_1]$$
$$= 1 - P[\text{Accept } H_0 \mid \text{Accept } H_1]$$
$$= 1 - P[\text{Type II Error}]$$
$$= 1 - \beta$$

$\therefore \boxed{\text{Power of Test} = 1 - \beta}$

3) General steps in Testing of Hypothesis

Ans :

Step 1 : To set up The Null Hypothesis Ho

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and null Hypothesis and Alternative Hypothesis.

Step 2: To choose The correct Appropriate level of significance α.

Step 3: To Compute the value of Test statistics Z under the null Hypothesis Ho.

Step 4: To Compute the critical value of $Z\alpha/2$ for Two sided Alternative Hypothesis and $Z\alpha$ for one sided Alternative Hypothesis, from the level of significance α.

Step 5: Now, To compare The Z with $Z\alpha/2$ or $Z\alpha$ and Take Decision, eighter we reject Ho or Accept Ho.

Step 6: state conclusion About the claim from the Problem.

i] If $|Z| \geq Z\frac{\alpha}{2}$ then we reject null Hypothesis Ho, at α % level of significance. From the Alternative Hypothesis.

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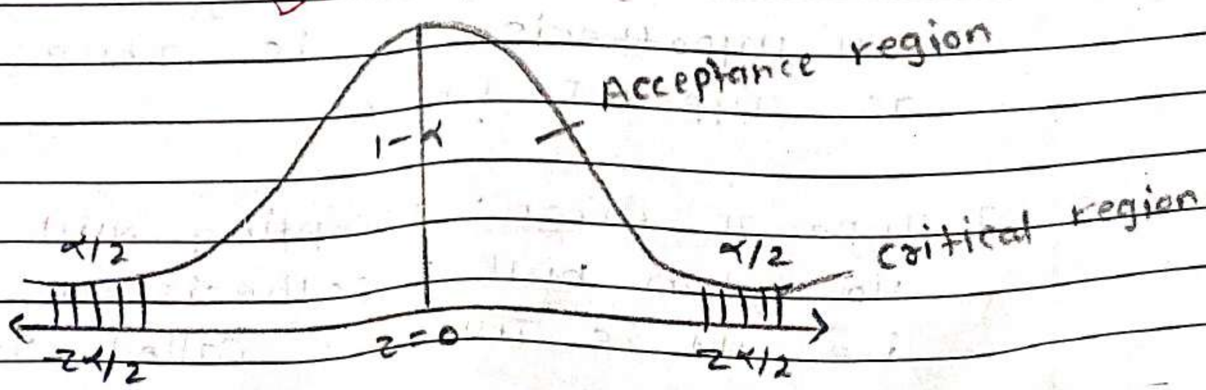

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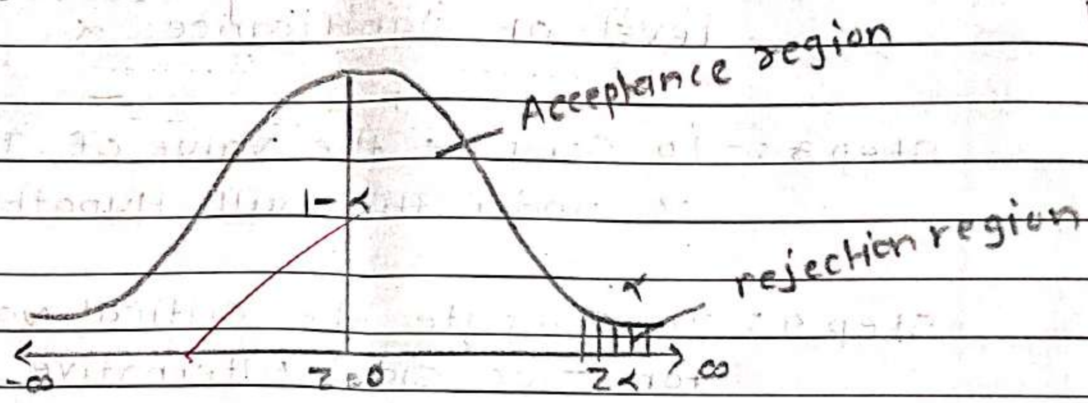
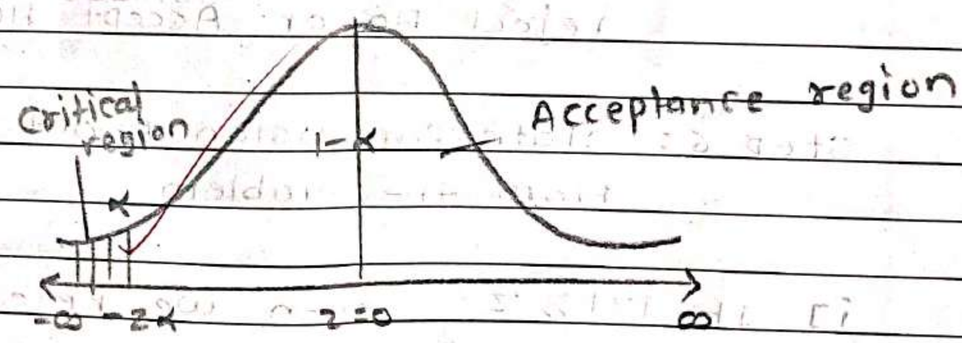
Q. No.

ii) If $Z > Z_\alpha$ then we reject Ho at α % level of significance from Alternative Hypothesis.



iii) If $Z < -Z_\alpha$ the we reject Ho at α %. Level of significance from Alternative Hypothesis.



4] Type I Error:
Rejecting Null Hypothesis Ho when Null Hypothesis Ho is True, is called as Type I Error.

5] Type II Error: Accepting Null Hypothesis Ho when null Hypothesis Ho is false i.e H1 is True is called Type II Error.

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॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
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Q. No.

Q.3) 1) Critical region :-

A region in which Rejected Null Hypothesis Ho, is called as critical region.

- critical region can be denoted by W or C.

2) Central limit Theorem :-

Let $x_1, x_2, \ldots, x_n$ be the random sampling of size n drawn from the Probability Distribution from $N(\mu, \sigma^2)$ i.e. follows Normal Dist^n with mean μ and finite variance σ^2 then.

$$Z = \frac{\bar{x} - E(\bar{x})}{S.E(\bar{x})} \sim N(0,1) \text{ , for large } n$$

$$\text{i.e} \quad Z = \frac{\bar{x} - E(\bar{x})}{\sqrt{var(\bar{x})}} \quad \text{where } \bar{x} = \frac{\Sigma x_i}{n}$$

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Q. No.

In other words,

$$\boxed{z = \frac{T - E(T)}{\sqrt{Var(T)}}}$$

Where, $T = T(x_1, x_2, \ldots, x_n)$

Hence, central limit Theorem is proved.

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Assignment

13/20 Ashna

Unit :- Testing of Hypothesis I

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BSc. II	01960

प्र. क्र. Q. No.

Q.1. Define the following terms
1] Simple Hypothesis
2] Null Hypothesis

Ans: 1] Simple Hypothesis :-
The hypothesis which completely specifies the probability distribution or parameter of probability distribution is called as simple Hypothesis.
If $x_1, x_2, \ldots, x_n$ are the sample of n size taken from $N(\mu, \sigma^2)$ then some simple hypothesis are given as
$H_0 : \mu = \mu_0, \ \sigma^2 = \sigma_0^2$
~~$H_0 : P_1 =$~~

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2] Null Hypothesis :-
The hypothesis with no difference is called as Null Hypothesis
or
The hypothesis which is tested for possible rejection under the assumptions that it is true is called Null Hypothesis.

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Denoted by Ho

Q.4 Define the following terms
a] Standard error
b] Alternative hypothesis, also define types of alternative hypothesis.

Ans :-

a] Standard Error :-
Standard deviation of statistics is called standard error
It is abbrivatted as, S.E.
If $T = T(x_1, x_2, x_3, \ldots, x_n)$ is statistics then S.E is given as,
$S.E. = \sqrt{V(T)} = \sqrt{E[T - E(T)]^2}$

b] Alternative Hypothesis :-
The hypothesis which is complementary to Null hypothesis is called Alternative Hypothesis.
or
The hypothesis which is accepted in the case of rejection of Null Hypothesis is called alternative hypothesis.
Denoted by H_1.

Types of Alternative Hypothesis :-
1] Two Sided (Tailed) Hypothesis :-
2] One Sided (Tailed) Hypothesis :-

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Two Sided Hypothesis :-
The hypothesis of type $H_1 : \mu \neq \mu_0$, $H_1 : P_1 \neq P_2$, $H_1 : \delta \neq \delta_0$ are called two sided hypothesis.

One Sided Hypothesis :-
The hypothesis of type $H_1 : \mu > \mu_0$, $H_1 : \mu < \mu_0$, $H_1 : P_1 > P_2$, $H_1 : P_1 < P_2$ are called one sided hypothesis.

Q.2 Define Type Ist Error & Type IInd Error & power of the test & Explain general steps in testing of hypothesis.

Ans :- **Type Ist Error :- [α]**
Rejecting Null Hypothesis Ho, when Ho is true is called as Type Ist Error.

Type IInd Error [β]
Accepting Null Hypothesis Ho, when Ho is false is called Type IInd Error.

Power of the Test :-
Probability of rejecting null hypothesis Ho, when alternative Hypothesis H_1 is true is called power of the Test.

Power of the Test = P [Reject Ho | Ho ~~is false~~]

= 1 − P [Accept Ho | Accept H_1]

= 1 − P [Type IInd Error]

= 1 − β

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Here are the steps of testing the Hypothesis :-

Step I :- To set up Null Hypothesis H_0 & Alternative Hypothesis H_1.

Step II :- To choose appropriate level of significance [α]