Inferential statistics

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Introduction

- Hypothesis testing is **a form of statistical inference that uses data from a sample to draw conclusions about a population parameter or a population probability distribution**
- the theory, methods, and practice of testing a hypothesis by comparing it with the null hypothesis. The null hypothesis is only rejected if its probability falls below a predetermined significance level, in which case the hypothesis being tested is said to have that level of significance.

Introduction

- Summary of population is called as population parameter
- Summary of sample is called sample statistics
- E.g average result of university is population parameter where as average result of the college is sample statistics.
- Commonly used tests include:
  - Compare sample statistics with the population parameter
  - Compare two data sets

Steps

- State alternative hypothesis
- State null hypothesis
- Select probability of error level or alpha level (α =0.05)
- Calculate the test statistics (t-statistics or z-statistics)
- Critical test statistics
- Interpret the result

Alternative hypothesis

- In a court basic assumption is the person is innocent
  - Null hypothesis: The person is innocent
- The court lawyer is going to prove that person is guilty
  - Alternate hypothesis: The person is guilty
- Statistical terms says
  - Reject the null hypothesis
  - Fail to reject null hypothesis (accept the null hypothesis)

Example 1

- A manufacture company creating a bottle. Each bottle is having volume as 150 cc.
- Every bottle may not be exactly same as 150 cc or there might be some changes in bottle while doing production. This change is mentioned by standard deviation. Here while producing bottles 2cc deviation is accepted.
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- Null hypothesis: the machine is producing bottles of 150 cc
- Alternative hypothesis: machine is not producing bottles of 150 cc.
- Based on the result, judgment can be made as:
  - Machine has changed (stop the machine. Reset the machine)
  - Machine is working as per the settings (no need to change machine settings)

tests

- Lower (One) tail test
  - H0: μ >= 150 cc
  - H1: μ < 150 cc
- Upper (One) tail test
  - H0: μ < = 150 cc
  - H1: μ >150 cc
- Two tail test
  - H0: μ == 150 cc
  - H1: μ != 150 cc

Continue….

- For any test, samples are collected. Based on sample statistics population parameter is calculated.
- Based on samples, prediction about whole population can be calculated.
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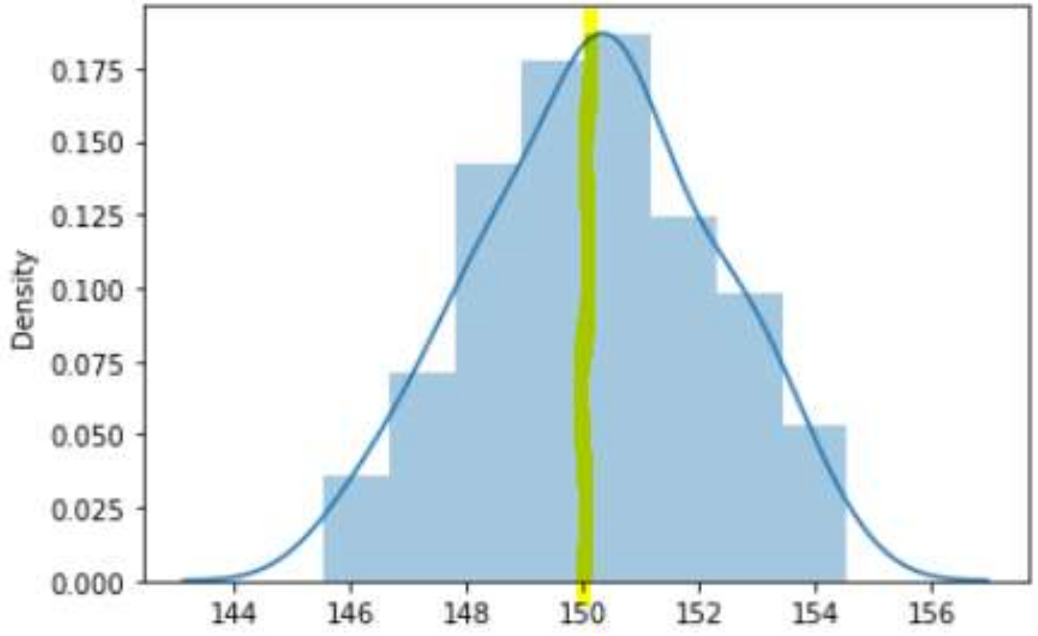

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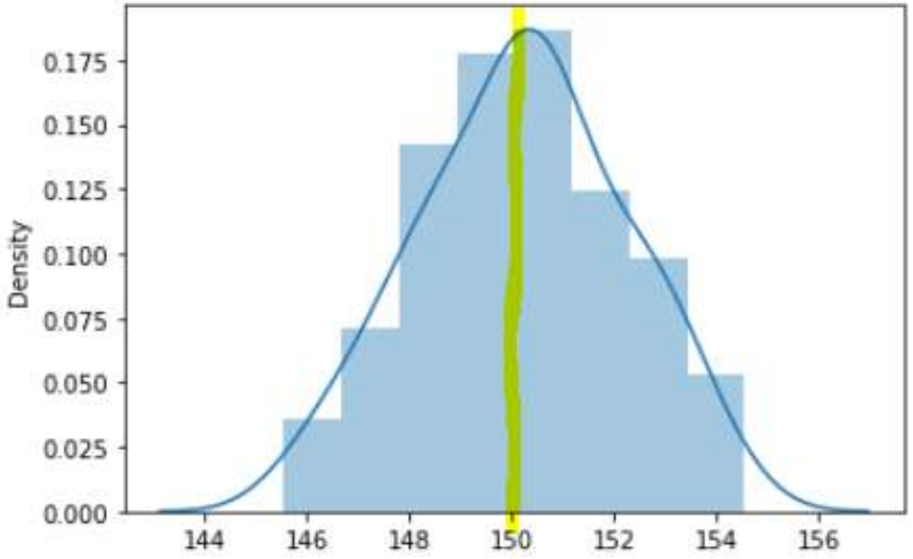
- Normal distribution has observed as per the graph
- Most of the bottles are having volume as 150cc
- Some bottles may have 148cc or some of the bottle may have 152 cc
- Exceptionally volume may recorded as 144cc o 156 cc
- If any exception are recorded then machine setting should not be changed. As it is the part of distribution.



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- Assumption: volume can be recorded between +2σ and -2 σ anything apart from that is rare observation.
- From +2σ and -2 σ 95% of the observation are recorded.
- Anything beyond 2 standard deviation is exception.
- If the volume recorded is more than 2 standard deviation then there is a cause of concern. This leads to take decision to reset the machine.

Type-1 and Type-2 error

- In this example 1 , some sample of bottles are collected.
- Based on sample, null hypothesis and alternate hypothesis is defined.
- There might be a chance of error while working on samples.
- Generally, out of 100 samples 5 error values can be accepted.

		True state of nature	
		H0 is true	Ha is true
Conclusion	Support H0/ reject Ha	Correct conclusion	Type 2 error
	Support Ha/ reject H0	Type 1 error	Correct conclusion

Continue..

H0: $\mu == 150$ cc

H1: $\mu != 150$ cc

- Type 1 error: Bottle are having 150 cc volume is actually true but based on samples we support alternate hypothesis. This scenario leads to resting of the machine and doing changes in the current process. This is a false alarm
- Type 2 error: Alternate hypothesis is true but predicted as null hypothesis.Bottles are not having 150 cc volume. Machine is having some defect. Conclusion based on collected sample, supports Null hypothesis. This leads to huge loss of production.

Difference

	Type 1 error (alpha) α	Type 2 error (beta) β
Name	Producer's risk / significance level	Consumer's risk
1-error	Confidence interval (1-α)	Power of the test (1-β)
Example	False alarm leading to inconvenience	Missed fire leading to disaster
Effects on process	Unnecessary cost increase due to frequent changes	Defects may produced
Control method	Usually fixed at predetermined level 1%, 5% or 10%	Usually controlled to <10% by appropriate sample size.
Simple definition	Innocent declared as guilty	Guilty declared as innocent.

Confidence level

- Confidence level :
- C = 0.90, 0.95, 0.99 which is equivalent to 90%, 95%, 99% respectively.
- Level of significance or type 1 error:
- $\alpha = 1 - C(0.90, 0.95, 0.99)$

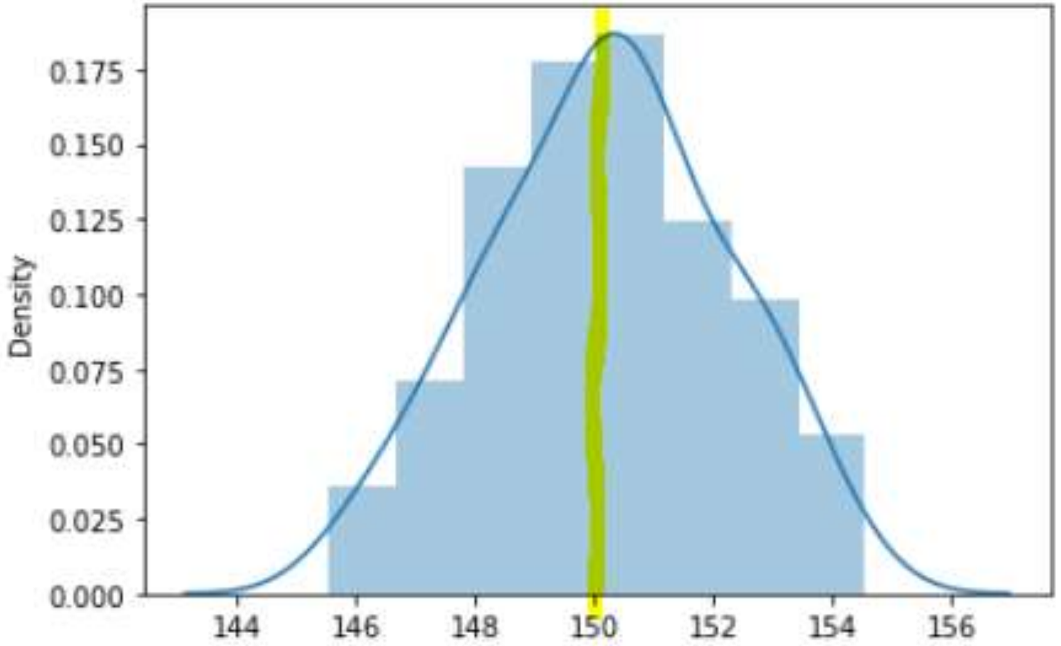
Power of the test

- Power = 1-β (i.e. 1 – type 2 error)
- Type 2 error : failing to reject null hypothesis when null hypothesis is false.
- Power : likelihood of reject null hypothesis when null hypothesis is false.
- Power is the ability of test to correctly reject the null hypothesis.

- Let's pick up 1 bottle from production line and the volume is 153.8cc
- With 95% confidence level we will fail to reject the null hypothesis.

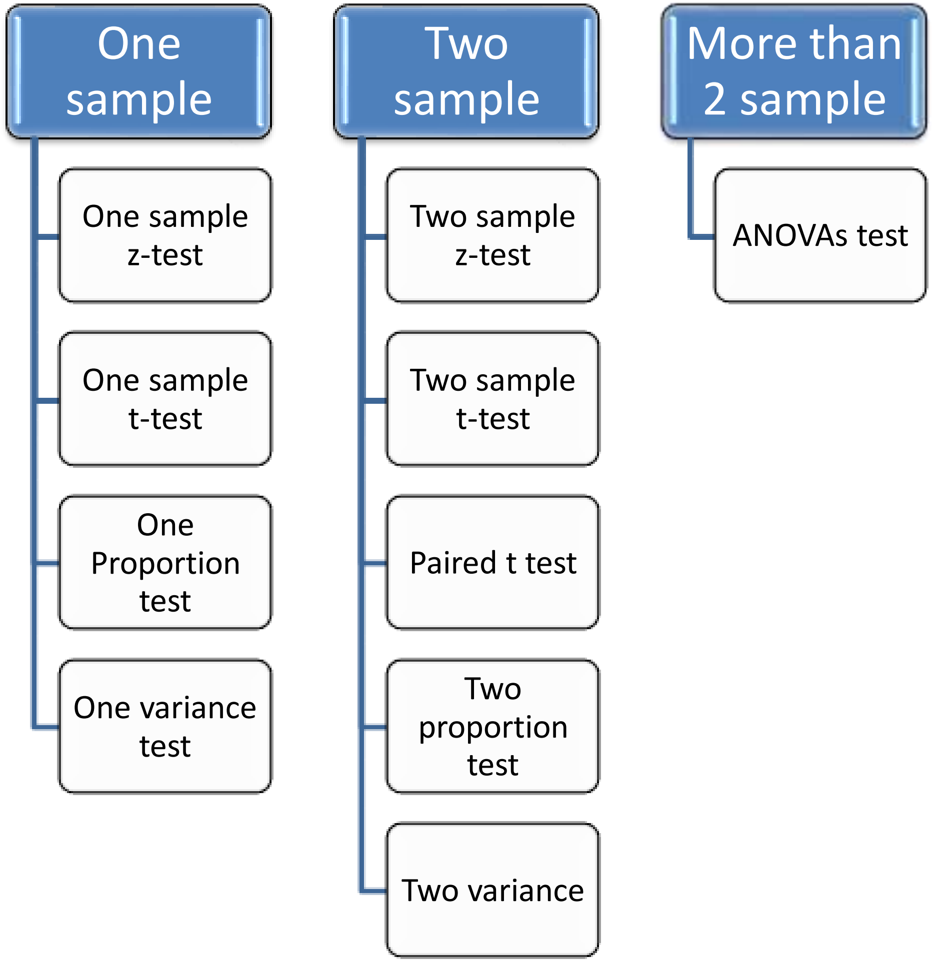
  H0: μ == 150 cc

  H1: μ != 150 cc



- Here sample of 4 bottles are collected. Mean of sample is 153.8
- Zcalc = (X-μ) /σ /sqrt(n)
- Hence Z calc = (153.8-150)/2/sqrt(4) = 3.8
- For α = 0.05 , two tails means 0.025 on both tails Z-critical = 1.96

Test


One sample
One sample z-test
One sample t-test
One Proportion test
One variance test
Two sample
Two sample z-test
Two sample t-test
Paired t test
Two proportion test
Two variance
More than 2 sample
ANOVAs test